

# U8D4\_T Perimeter and Area Relationships of Rectangl

Friday, May 11, 2018 5:34 PM



U8D4\_T  
Perimeter...

MPM 1D1 U8D4

## Warm up:

- a) What is the ideal shape for maximizing the area of a rectangle when given a fixed perimeter?

A square is the optimal shape when fencing four sides.

- b) How can you predict the dimensions of a rectangle with a maximum area if you know the perimeter?



The width of the square is the Perimeter divided by four.

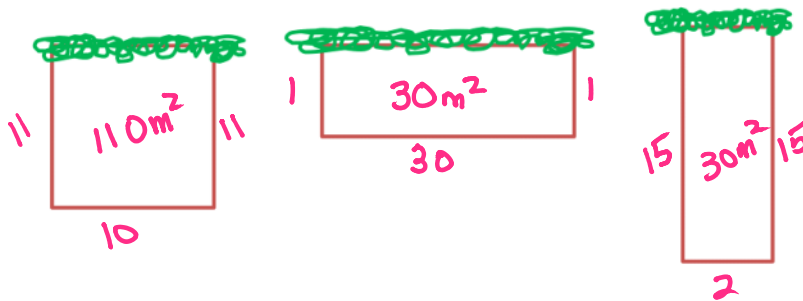
$$w = \frac{P}{4}, \quad l = w$$

## Day 4: Perimeter & Area Relationships of Rectangles(3-sided)

Investigation A: **How can you model the maximum area of a rectangle with a fixed sum of the lengths of only three sides?**

**EX.** Ian's customer decides to use an existing hedge as one of the boundaries for the pigpen enclosure. This means that he will only use the prefabricated fencing on three sides of the rectangular pen. The client still wants the pen to have the greatest area possible.

1. Ian has 32 m of prefabricated fencing.
  - a) Sketch rectangles to determine the dimensions of the rectangle that has the maximum area.



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b) Record your results in the table below.

Width (m)	2w	Length (m)	Sum of the lengths of three sides (m)	Area (m <sup>2</sup> ) $A=lw$
6	12	20	32	$6(20) = 120$
7	14	18	32	$7(18) = 126$
8	16	16	32	$8(16) = 128$
9	18	14	32	$9(14) = 126$
10	20	12	32	$10(12) = 120$

## 2. REFLECT:

a) What are the dimensions of the rectangular pen with maximum area?

$$16\text{m} \times 8\text{m}$$

b) Examine the length and width of the enclosure with maximum area. Do any relationships exist between these dimensions and the dimensions found for a four-sided enclosure with same perimeter?

$$W = \frac{P}{4} \quad \leftarrow \text{width is the same as a four-sided enclosure.}$$

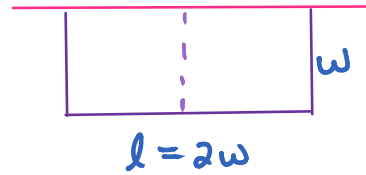
$$W = \frac{32}{4}$$

$$W = 8$$

c) Will the hedge allow Ian to enclose more, less or the same amount of area as before?  
*more - this is double the area.*

d) How can you predict the dimensions of a 3-sided rectangle with a maximum area given perimeter?


$$w = \frac{P}{4}, \quad l = 2w$$



## Investigation B: How can you model the minimum perimeter of a 3-sided rectangle given a fixed area?

Ian's customer decides to use the side of the barn as one of the boundaries for the chicken enclosure. He still needs  $36 \text{ ft}^2$  to comply with regulations for his free-range chickens, and still wants to keep his cost for fencing to a minimum.

- Complete the table below, testing different possible dimensions that comply with the given criteria.

Width (m)	Length (m)	Area ( $\text{m}^2$ )	Perimeter (m) 
1	36	36	$2+36 = 38$
2	18	36	$4+18 = 22$
3	12	36	$6+12 = 18$
4	9	36	$8+9 = 17$ ⊗
6	6	36	$12+6 = 18$

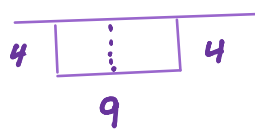
- REFLECT:** What did you find?
  - What are the dimensions of the rectangle with the minimum perimeter?  $9\text{m} \times 4\text{m}$

b) Examine the length and width of the enclosure with minimum perimeter. Do any relationships exist between these dimensions and the dimensions found for a four sided enclosure with same area?

4-sided  $w = \sqrt{A}$   
 $w = 6, l = 6$

$P = 24m$

3-sided

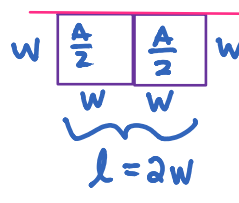


c) Will the hedge allow Ian to use more, less or the same amount of fencing as before?

*less*

d) How can you predict the dimensions of a 3-sided rectangle with a minimum perimeter given area?

\* Divide the area into two squares.



$w = \sqrt{\frac{A}{2}}$ ,  $l = 2w$

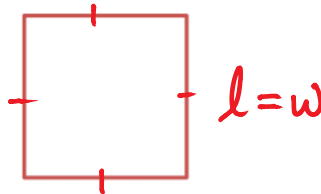
Summary:

When optimizing the area and perimeter of a rectangle, there are two possibilities:

1. **Four-sided shape:** Make the figure in the shape of a square, with all sides equal length.

$$P = 4w \text{ and } A = w^2$$

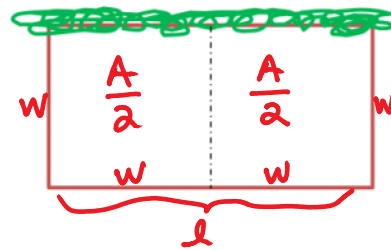
$$w = \frac{P}{4} \quad w = \sqrt{A}$$



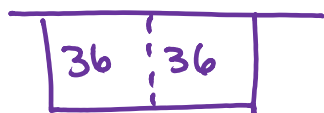
2. **Three-sided shape:** Use an existing “wall” to create one side of the rectangle. For the remaining three sides, the length is twice as long as the widths.

$$P = 4w \quad l = 2w \text{ and } A = 2w^2$$

$$w = \frac{P}{4} \quad w = \sqrt{\frac{A}{2}}$$



**EX. 1.** Sir Adam Beck PS is adding a rectangular kindergarten playground to the side of the school. The school will form one side of the rectangle. The area of the playground is to be  $72 \text{ m}^2$ . Minimizing the perimeter will minimize the cost of the fence. What dimensions use the minimum length of fence?



When fencing 3 sides,  
make 2 squares —  
each has an area of  
 $36 \text{ m}^2$ .

$$\begin{aligned} W &= \sqrt{36} & l &= 2w \\ &= 6 & &= 12 \end{aligned}$$

$\therefore$  the playground should be  
 $12 \text{ m} \times 6 \text{ m}$ .



**EX. 2.** Dave has prepared a proposal for a client.

In the proposal, he reports how 80 m of fencing can be used to fence an enclosure on

- Four sides
- Three sides, using a wall at the back of the property as the fourth side
- Two sides, using the wall at the back and an existing hedge on an adjacent side.

Draw diagrams for each scenario and calculate the maximum area that can be enclosed in each case. Which proposal should the client choose?

Explain.

4-sided



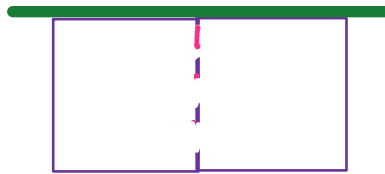
$$W = \frac{80}{4}$$

$$W = 20\text{m}$$

$$l = 20\text{m}$$

$$A = 400\text{m}^2$$

3-sided

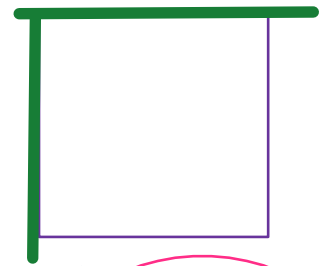


$$W = \frac{80}{4}$$

$$W = 20\text{m} \quad l = 40\text{m}$$

$$A = 800\text{m}^2$$

2-sided



try  $w=40,$   
 $l=40$   
 $A=1600\text{m}^2$

try  $w=20,$   
 $l=60$   
 $A=1200\text{m}^2$

Fencing two sides with length and width both 40m will give the greatest area.