

Perimeter & Area Relationships of Rectangles (4-sided)**KEY TERM:**

Optimization:

**EX.** Ian has a summer job at a fencing company. A customer has purchased 32 sections of prefabricated fencing, each 1 m in length, and wants Ian to create a rectangular pigpen with the largest area possible.

Investigation A: **How can you model the maximum area of a rectangle with a fixed perimeter?**

1. Complete the table below, testing different possible dimensions. To complete the table.
  - a) Determine the dimensions of 4 different rectangles that Ian could use for this fence.  
Recall:  $Perimeter = 2(l + w)$
  - b) Calculate the area of each rectangle. Recall:  $A = lw$

| Width (m) | Length (m) | Perimeter (m) | Area (m <sup>2</sup> ) |
|-----------|------------|---------------|------------------------|
| 2         |            | 32            |                        |
| 4         |            | 32            |                        |
| 6         |            | 32            |                        |
| 8         |            | 32            |                        |

2. **REFLECT:** What did you find?
  - a) What are the dimensions of the rectangle with the maximum, or optimal value?
  - b) What is the maximum area?
  - c) What happened to the area as the length and width became closer in value?
  - d) Describe the shape of the rectangle with maximum area.
  - e) How can you predict the dimensions of a rectangle with a maximum area if you know the perimeter?
3. Suppose the customer decides to use 40 m of fencing instead of 32 m.
  - a) Predict the dimensions of the rectangular pen with the maximum area.
  - b) Draw rectangles and find their areas to test your hypothesis.

Investigation B: **How can you model the minimum perimeter of a rectangle with a fixed Area?**

Ian has another customer who needs  $36 \text{ ft}^2$  to comply with regulations for his free range chickens, but wants to keep his cost for fencing to a minimum.

1. Complete the table below, testing different possible dimensions that comply with the given criteria.
  - a. Determine the dimensions of 5 different rectangles that Ian could use for this fence.
  - b. Calculate the perimeter of each rectangle.

| Width (m) | Length (m) | Area (m <sup>2</sup> ) | Perimeter (m) |
|-----------|------------|------------------------|---------------|
| 1         |            | 36                     |               |
| 2         |            | 36                     |               |
| 3         |            | 36                     |               |
| 4         |            | 36                     |               |
| 6         |            | 36                     |               |

**2. REFLECT:** What did you find?

- a. What are the dimensions of the rectangle with the minimum, or optimal value?
- b. What is the minimum perimeter?
- c. What happened to the perimeter as the length and width became closer in value?
- d. What is the ideal shape for minimizing the perimeter of a rectangle when given a fixed area?
- e. How can you predict the dimensions of a rectangle with a minimum perimeter if you know the area?

**EX. 1.** a) Determine the dimensions of a rectangle with maximum area that has a perimeter of 60 m.

a) Determine the minimum perimeter of a rectangle that has an area of  $49 \text{ cm}^2$ .

**EX. 2.** Sir Adam Beck PS is adding a rectangular kindergarten playground to the yard. The area of the playground is to be  $72 \text{ m}^2$ . Minimizing the perimeter will minimize the cost of the fence. What whole number dimensions use the minimum length of fence?