U7D4 Present Value Annuities

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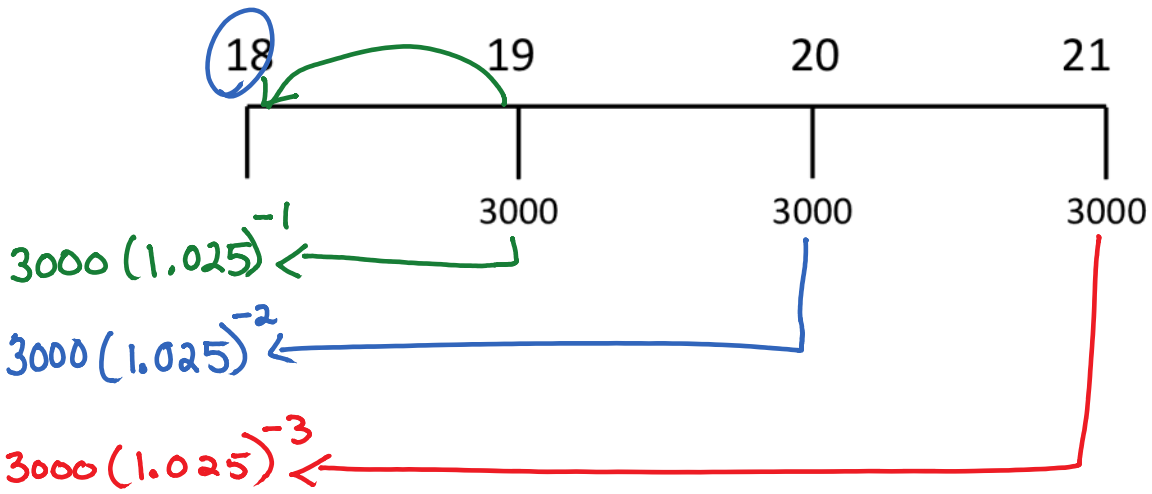
U7D4_T-Present V...

U7D4 **Present Value Annuities**

Example 1:

How much money do you need to have saved at age 18 if you want to be able to withdraw $3000 every year from age 19 to age 21, inclusive? Assume an interest rate of 2.5%/a compounded annually. Use a timeline. $i = 0.025$

NOTE:



8568.07

∴ he must invest $8568.07

Example 2: (Same as Ex. 1 with formula)

How much money do you need to have saved at age 18 if you want to be able to withdraw $3000 every year from age 19 to age 21, inclusive? Assume an interest rate of 2.5%/a compounded annually.

Use the formula: $PV = \frac{R[1-(1+i)^{-n}]}{i}$

PV is the Present Value of the annuity (the amount of money needed at age 18 to make the withdrawals in the future)

R = 3000 i = 0.025 n = 3 PV = ?

$$PV = \frac{3000\left[1-(1+0.025)^{-3}\right]}{0.025}$$

$$PV \doteq 8568.07$$

∴ you would need $8568.07 at age 18.

Example 3. Calculate your car payments if you have borrowed $10,000 for 4 years, with an interest rate of 4.8%/a compounded monthly and your payments are monthly at the end of the month.

Use the formula: $PV = \frac{R[1-(1+i)^{-n}]}{i}$ where you are solving for R, PV is the Present Value of the annuity (the amount of money borrowed in this case). Check with technology. (google fncalculator)

R = ? i = $\frac{0.048}{12}$ n = 4x12 = 48 PV = 10 000

$$R = \frac{PV(i)}{[1-(1+i)^{-n}]}$$

$$= \frac{10000 \times 0.048 \div 12}{[1-(1+0.048 \div 12)^{-48}]}$$

$$= 229.388$$

$$\doteq 229.39$$

∴ the payment should be $ 229.39

Example 4. You want to retire with $650000. Find the amount you must deposit monthly for 40 years if your retirement investment fund (RIF) earns 6.4%/a compounded monthly. Assume you are depositing your money at the end of the month. Use the accumulated value annuity formula and solve for R. Check with technology. (google fncalculator)

Recall: $A = \frac{R[(1+i)^n - 1]}{i}$ $\quad R = \frac{Ai}{[(1+i)^n - 1]}$

$A = 650\,000 \quad i = \frac{0.064}{12} \quad n = 40 \times 12$

$n = 480$

$R = ?$

$$R = \frac{650000 \times 0.064 \div 12}{[(1 + 0.064 \div 12)^{480} - 1]}$$

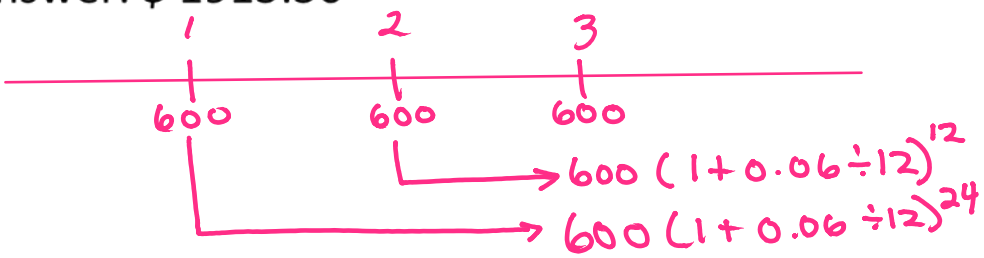
$R \doteq 292.59$

$\therefore$ you would need to make monthly deposits of $292.59.

Solve numbers 1 and 2 with a timeline and 3 and 4 with the annuity formulas.

1. Find the future value of $600 deposited at the beginning of each year for 3 years at an annual interest rate of 6% compounded monthly.

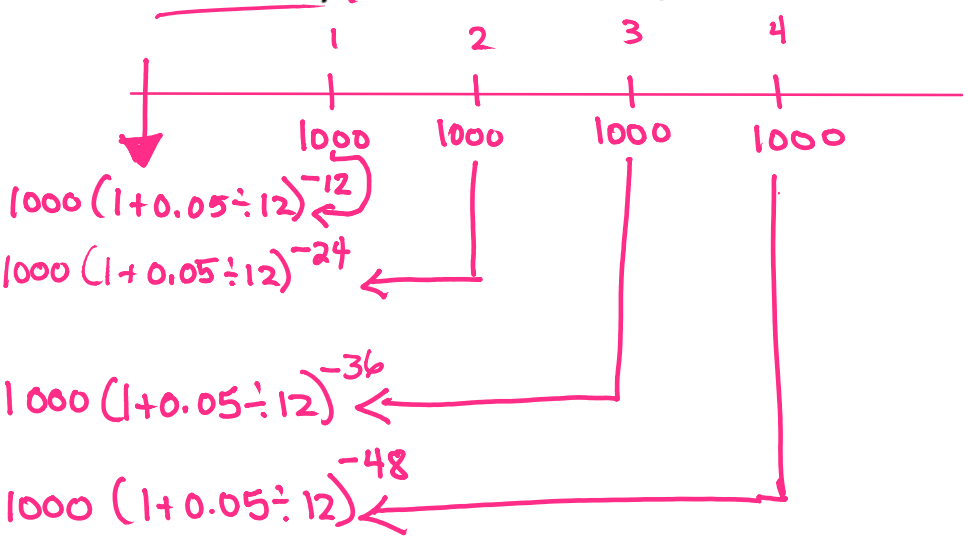
Answer: $ 1913.30



1913.30

∴ the future value is $1913.30.

2. Find the present value of a series of regular $1000 withdrawals from an investment account earning 5%/a compounded monthly. Assume you are withdrawing $1000 per year for 4 years at the end of the year. Answer: $3536.40



$3536.40

∴ the present value is $3536.40.

3. Find the amount borrowed (present value) if you are making car loan payments of $350/month for 5 years. The interest rate is 1%/a compounded monthly.

Answer: $20 475.32

$P = ?$ $R = 350$ $i = \frac{0.01}{12}$ $n = 5 \times 12 = 60$

$$P = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{350\left[1-(1+0.01\div 12)^{-60}\right]}{(0.01\div 12)}$$

$$= \$20\,475.32$$

∴ $20475.32 was borrowed to buy the car.

4. If your parents deposit $50 every month into an education fund, for 15 years, how much will be in the account at the time of the last deposit. Interest rate is 6%/a compounded monthly. → A = ?

Answer: $14 540.94

$R = \$50$ $\quad i = 0.06 \div 12$ $\quad n = 15 \times 12$

$i = 0.005$ $\quad n = 180$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{50\left[(1.005)^{180} - 1\right]}{0.005}$$

$$= \$14\,540.94$$

∴ the education fund will be worth $14540.94