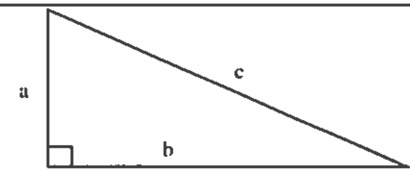
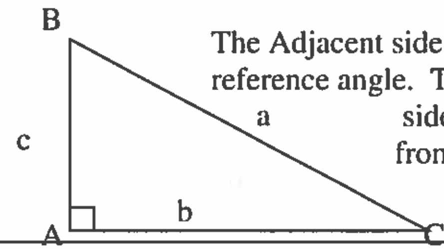
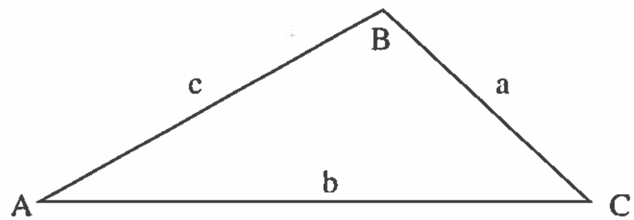
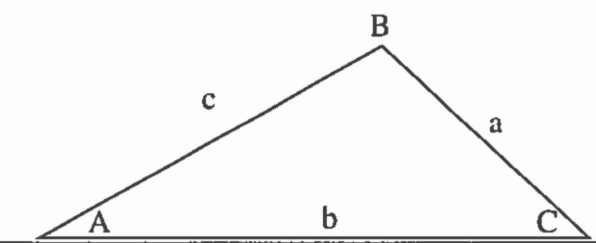
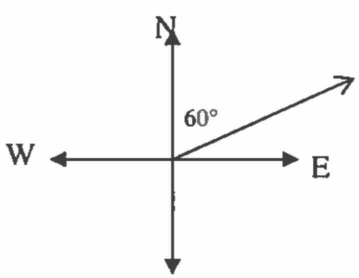
MAP 4CI Trigonometry Reference Sheet

Formula	Picture	When to use	
Pythagorean $a^2 + b^2 = c^2$		Right angle triangle - given 2 sides	- asked to find third side
Trig Ratios SOHCAHTOA $sin\theta = \frac{O}{H}, cos\theta = \frac{A}{H}, tan\theta = \frac{O}{A}$ In standard position, $r = \sqrt{x^2 + y^2}$ $sin\theta = \frac{y}{r}, cos\theta = \frac{x}{r}, tan\theta = \frac{y}{x}$	 The Adjacent side is beside the reference angle. The Opposite side is across the from the reference angle.	Right angle triangle - given two sides - given one side and an angle	- asked to find angle - asked to find side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$		No right angle - given two angles and one opposite side - given two sides and one opposite angle	- asked to find other opposite side - asked to find other opposite angle
Cosine Law $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$		No right angle - given two sides & a contained angle - given three sides	- calculate the third side - can calculate angle

Angle of elevation is always measured UP from the HORIZONTAL. Angle of depression always measured DOWN from the HORIZONTAL.



Bearing 060° is the same as N60°E
Bearing is measured clockwise from North.
So a bearing of 200° is the same as S20°W.

You will be given a copy of this reference sheet for your quiz and your test.

QUIZ DATE: FRI. MAR. 2

TEST DATE: WED. MAR. 21

UNIT 2 QUIZ DATE:_______________ UNIT 2 TEST DATE:____________________

NOTE: Notes will NOT be allowed for quiz/test. You will have a copy of the reference sheet attached

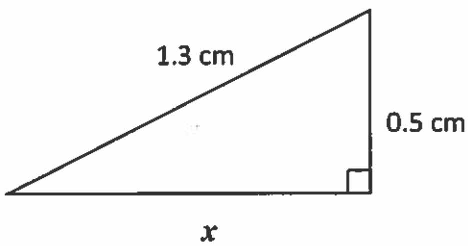
U2D1 MAP 4CI **Trigonometry Intro** Date:_________

Set your calculator to **DEGREE mode

1. **Pythagorean Theorem.** Draw a right triangle. Label the sides a, b and c (c must be the longest side). Side c is called the ____________________.

Now draw a square on each side of the triangle. State the relationship between the squares on the sides of the right triangle. __________________________

Ex. 1 Determine the length of the indicated side.



Ex. 2 Brad walks 1.7 km North and then 1.5 km East along the sides of a park. Dan starts at the same point and takes a shortcut along the diagonal. How much shorter is Dan's walk?

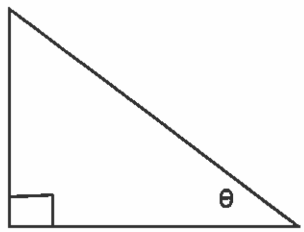
2. **Solving Equations.**

Ex. 1 Solve for x to the nearest tenth.

a) $\frac{12}{x} = \frac{20}{3}$

b) $\frac{6.7}{2.8} = \frac{x}{4.2}$

3. **Primary Trig Ratios.** Given a right triangle with angle θ (theta), label the sides "hypotenuse", side "opposite" to angle θ, and side "adjacent" to angle θ.



To remember the 3 primary trig. ratios of the sides of a right triangle relative to angle θ use ______________________________

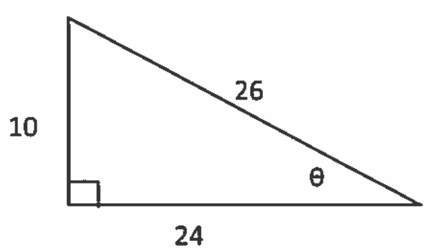
The 3 primary trig ratios are:

sine θ =

cosine θ =

tangent θ =

Ex. 1 Write the 3 primary trig ratios relative to θ.



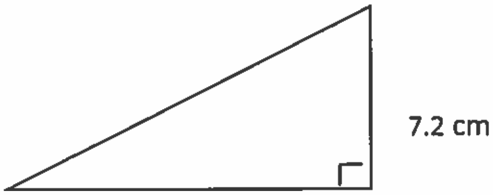
Ex. 2 Evaluate to four decimal places.

a) $\sin 54 =$ b) $\cos 14 =$ c) $\tan 61 =$

U2D1 Practice: Pg 72 # 4 - 7, 9, 10 ab Check Answers: Pg. 540

U2D2 MAP 4CI **Determining Lengths of Sides in Right Triangles**

Example 1: Determine the length of side x, to the nearest tenth.
Given $\Delta XYZ,\ z = 7.2\ cm,\ Z = 35°, Y = 90°$



Recall:
Angle of elevation/inclination is always measured UP from the HORIZONTAL.

Angle of depression always measured DOWN from the HORIZONTAL.

Example 2: Tanya is standing 7.92 m from the flagpole. She is holding a clinometer at eye level 1.6 m above the ground. How tall is the flagpole if she measures a 50° angle of elevation?

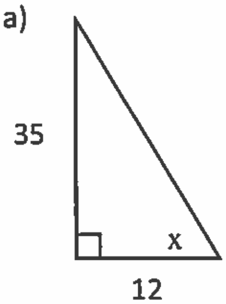
U2D2 Practice: Page 80 #1a, 2a, 3, 4, 9, 10, 12

U2D3 **Determining Measures of Angles in Right Triangles**

Trig ratios can also be used to find the measures of angles of a right triangle that are not known.
Examples: For the following triangles, identify the trig ratio to use, write the equation and solve it to one decimal place using the INVERSE TRIG buttons on your calculator.

$\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$

a)

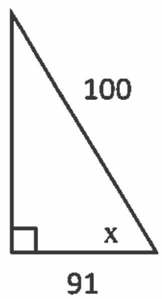


Have:

Need:

Use:

b)

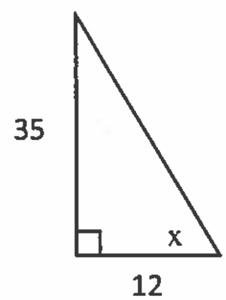


Have:

Need:

Use:

c)



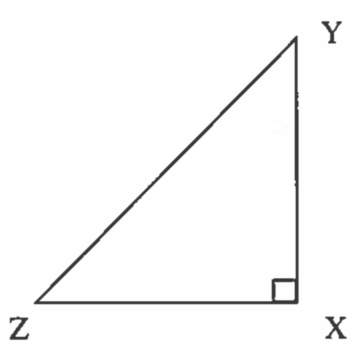
Have:

Need:

Use:

Ex. 2 Solve Δ XYZ given that $\angle X = 90°$, $x = 8.2\,cm$, $z = 6.0\,cm$.

To solve means ____



U2D3 Practice: Pg. 80 # 5, 6, 7, 11, 13 ✓ Answers Pg. 540 2.1

U2D4a: Investigating Obtuse Angles

Introduction to the Activity:

In this activity, you will use your calculator and the following chart to investigate the trigonometric ratios of obtuse angles. Then, you will analyze the results to determine any patterns.

Performing the Activity

1) Refer to the chart that follows. For each of the listed angles, use your calculator to determine the value of each primary trigonometric ratio in the chart.
2) After you have completed the chart, answer the questions that follow.

Round values to 3 decimal places. There will be some rounding error.

Primary Angle, *B*	***sin B***	***cos B***	***tan B***
5°	$\frac{opp}{hyp} \approx 0.087$	$\frac{adj}{hyp} \approx 0.996$	$\frac{opp}{adj} \approx 0.087$
10°			
25°			
30°			
89°			
91°			
150°			
155°			
170°			
175°			

Investigating Obtuse Angles (Continued)

After you have completed the chart, answer the following questions.

1) What do you notice about the signs (positive? negative?) of the values of *sin B*? Be as specific as possible. Why does this happen?

2) What do you notice about the signs (positive? negative?) of the values of *cos B*? Be as specific as possible. Why does this happen?

3) What do you notice about the signs (positive? negative?) of the values of *tan B*? Be as specific as possible. Why does this happen?

4) Write down pairs of $\angle B$ that have approximately the same value for *sin B*. Verify that the values are actually the same using your calculator. For example, check that $\sin 5^\circ$ and $\sin 175^\circ$ give the same value. How are the angles related to each other?

Using the same pairs of angles, what do you notice about the values of *cos B*? (Verify on your calculator if needed.)

Using the same pairs of angles, what do you notice about the values of *tan B*? (Verify on your calculator if needed.)

5) Use $\sin^{-1}$ on your calculator to solve for angle B in $\sin B = 0.5$. What value does your calculator give?

What other value for B is possible?

How can you quickly determine the value of the second angle?

Complete the following using a calculator and what you have learned:

$\sin B \approx 0.7660$ B ≈ ____________ or B ≈ ____________

$\sin B \approx 0.9205$ B ≈ ____________ or B ≈ ____________

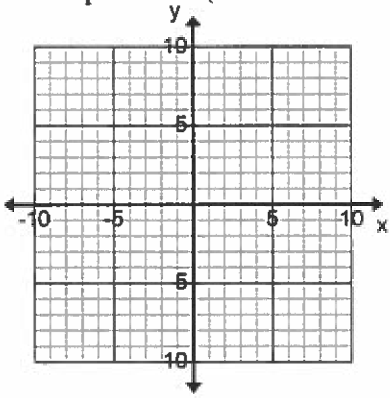
MAP 4CI Trigonometric Ratios with Obtuse Angles in Standard Position Date:__________

U2D4b

1. The terminal arm of an angle, θ, in standard position passes through A(2, 4).

a) Sketch a diagram for this angle in standard position. (see instructions below)

b) Determine the length of OA

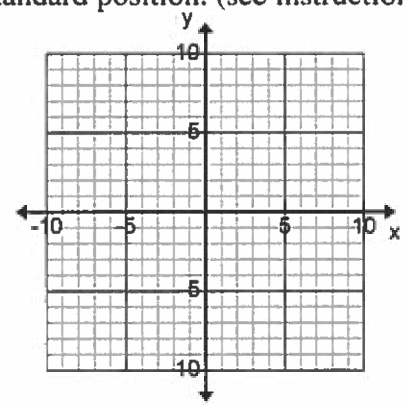


c) Determine the primary trigonometric ratios to three decimal places.

2. The terminal arm of an angle, θ, in standard position passes through B(-5, 6).

a) Sketch a diagram for this angle in standard position. (see instructions below)

b) Determine the length of OB



c) Determine the primary trigonometric ratios to three decimal places.

OBTUSE ANGLES IN STANDARD POSITION

Angles in standard position:

- You will be given an ordered pair.
- Plot that point on the Cartesian Plane
- Join that point to the origin (this line segment is called the "terminal arm")
- Draw the "initial arm" on the positive x-axis beginning at the origin.
- θ is measured from the initial arm, counter-clockwise to the terminal arm.

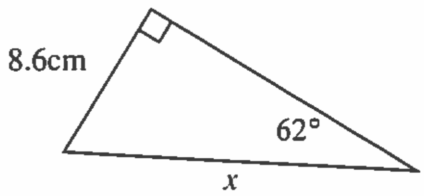
To find the primary trig ratios, drop a vertical line segment from the plotted point to the x-axis. This will form a right triangle.

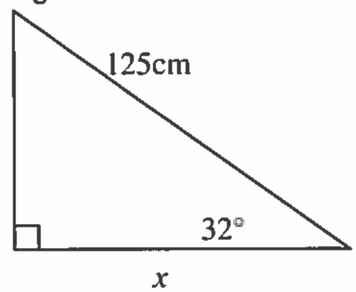
U2D4 Practice: Page 93 #3, 6

U2D5 (NEXT CLASS): QUIZ

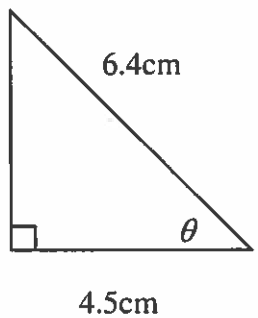
MAP 4CI **Trig Mid Unit Review** Name:________________________

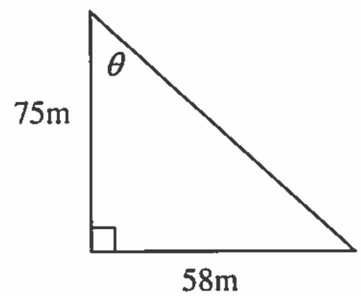
1. Find the side "x" to the nearest tenth in each of the following triangles.





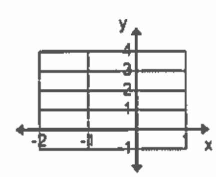
2. Find the angle θ to the nearest degree for each of the following triangles.





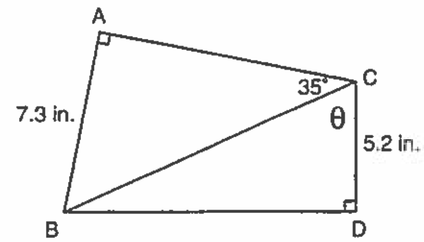
3. Solve ΔABC, a=5.0cm, b=12.0cm, angle $C = 90°$, Include a labeled diagram with your answer. (Round angles to the nearest degree and sides to nearest tenth).

4. The terminal arm of an angle, θ, in standard position passes through A(-1, 3).
   a) Determine the length of OA.



   b) Determine the 3 primary trigonometric ratios to three decimal places.

5. Determine the measure of ∠BCD.



6. For each trig. ratio below, determine whether the angle is obtuse, acute or could be either.
   a) tanA = -1.6 b) cosB = 0.9945 c) sinC = 0.35 d) cosD = - 0.7

7. Determine all possible values for angle Z (Z is between 0 and 180°).
   a) cosZ = - 0.93 b) sinZ = 0.73

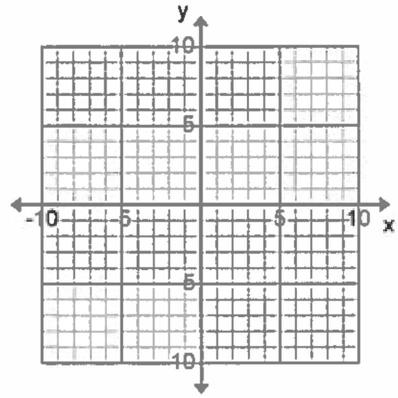
Answers: 1.a) 9.7 cm b) 106.0 cm 2. a) $\theta = 45°$ b) 38° 3. a) c = 13, A = 23°, B = 67°
4. a) $\sqrt{10}$ b) $\sin\theta = 0.949$, $\cos\theta = -0.316$, $\tan\theta = -3$ 5. h = 12.7 in., $\theta = 66°$
6. a) obtuse b) acute c) could be either d) obtuse 7. a) 158° b) 47° or 133°

MAP 4CI Trigonometric Ratios for Obtuse Angles in Standard Position Date:________
Unit 2 Day 6

1. The sine of an obtuse angle, θ, in standard position is $\frac{3}{5}$.

a) Identify the coordinates of a point that lies on the terminal arm of $\angle\vartheta$.

b) Sketch a diagram of $\angle\vartheta$.



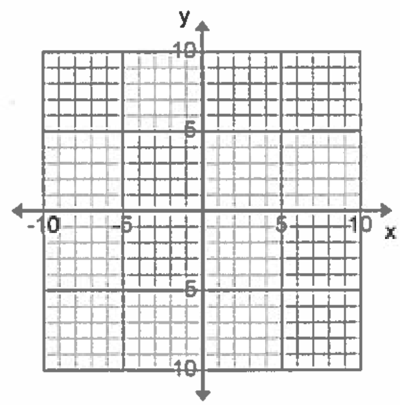
c) Determine cos θ and tan θ.

d) Determine the measure of $\angle\vartheta$, using a calculator.

2. The tangent of an obtuse angle, θ, in standard position is -1.

a) Identify the coordinates of a point that lies on the terminal arm of $\angle\vartheta$.

b) Sketch a diagram of $\angle\vartheta$.



c) Determine sin θ and cos θ. Round your answers to three decimal places.

d) Determine the measure of $\angle\vartheta$, using a calculator.

U2D6 Practice: page 93 #10, 11, 12

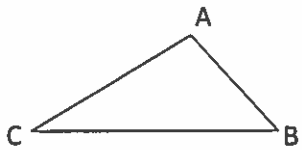
U2D7 SINE LAW

SINE LAW: If looking for an angle: $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

To use the sine law you need one complete side-angle pair.

If looking for a side: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

Example 1: Calculate the value of angle A and angle B. Round to one decimal place.



a = 14, b = 13, c = 15, ∡C = 67.4°

Example 2: In $\Delta DEF, E = 108°, F = 32°, e = 7.5\ cm.$ Determine the length of 'd' to one decimal place.

U2D7 Practice: page 101 #1a,2,4a,6,7a,8

U2D8

APPLICATIONS OF SINE LAW

1. Two people stand approximately 50 m apart on level ground. One person measures the angle of elevation of a hot air balloon to be 58º. The other person measures the angle of elevation to be 41°. How far is each person from the hot air balloon?

U2D8 Practice: Page 102 #9, 10, 11, 14 (Sine Law Applications)

Cosine Law:

The Cosine Law can be used to solve for an unknown side, if you are given two sides and a contained angle: $a^2 = b^2 + c^2 - 2\,bc\,\text{Cos}\,A$

It can also be re-arranged to solve for an unknown angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Example 1: Determine the length of side 'c' to the nearest tenth.
Given $\Delta ABC, C = 110°, b = 15\ mm, a = 8\ mm$

Example 2: Determine the value of angle D to the nearest degree.
Given $\Delta DEF, d = 10\ cm, e = 15\ cm, f = 17\ cm$

U2D9 Practice: page 110 # 1a, 3, 4a, 5, 6, 7a

Unit 2 Day 10: Cosine Law Applications

Review Cosine Law:

The Cosine Law can be used to solve for an unknown side, if you are given two sides and a contained angle:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

It can also be re-arranged to solve for an unknown angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Bearings: Direction can be written in several ways

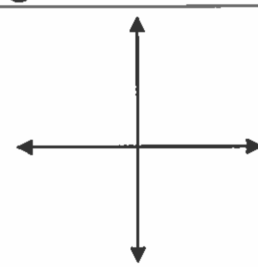
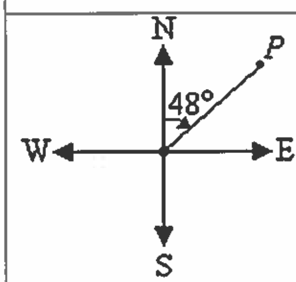
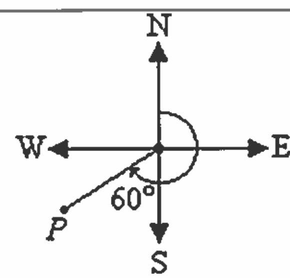
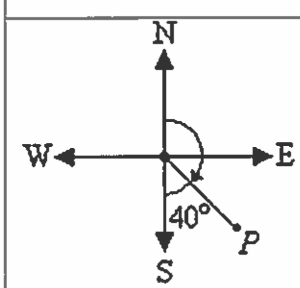
Direction	bearing	Diagram
N60°E	060°	

Diagram	Bearing	Direction	Diagram	Bearing	Direction
					
			Provide a sketch here.	235°	

1. A harbour master uses radar to monitor two ships. B and C, as they approach the harbour, H. One ship is 5.3 miles from the harbour on a bearing of 032°. The other ship is 7.4 miles away from the harbour on a bearing of 295°. How far apart are the two ships?

2. An aircraft navigator knows that town A is 71 km due north of the airport, town B is 201 km from the airport, and towns A and B are 241 km apart. On what bearing should she plan the course from the airport to town B?

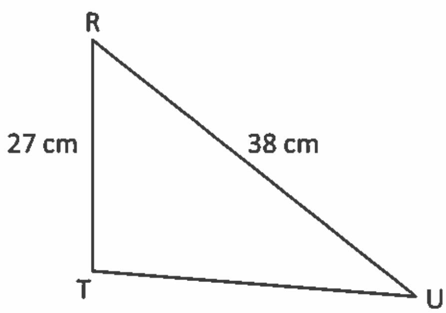
U2D10 Practice: page 110 # 7b, 9,10,11,12

Applications of Trigonometry

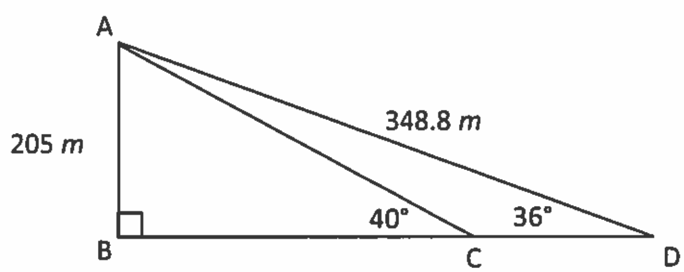
Date:________________

Ex. 1 From the top of a vertical cliff a person measures the angle of depression of a boat as 9°. The height of the cliff is 142 m. How far is the boat from the base of the cliff? Round your answer to the nearest m.

Ex. 2 Find the length of TU to the nearest tenth.



Ex. 3 A smokestack, **AB**, is 205m high. From two points C and D on the **same side** of the smokestack's base B, the angles of elevation to the top of the smokestack are 40° and 36° respectively. The distance from the top of the smokestack to point D is 348.8 m. Find the distance between C and D to the nearest metre.



Ex. 4 Two guy-wires are anchored at the same point. The first guy-wire is 12 m in length and is attached to the top of a tower. The second guy-wire is 9 m in length and is attached to a point 5 m below the top of the tower. How far are the wires anchored from the base of the tower? Round your answer to the nearest tenth of a metre.

U2D11 Practice: Pg 126 # 1, 2, 3, 5, 7, 9, 11 ✓ Answers Pg 542 2.5

Unit 2 Day 12: Review

Obtuse Angles

Obtuse angle - $90° \le \theta \le 180°$

Supplementary Angles A+B=180°

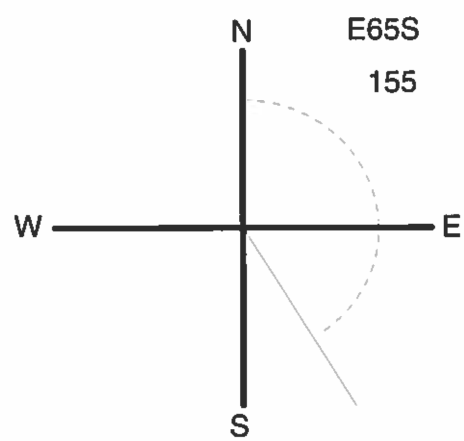
$$\sin A = \sin B, \qquad \cos A = -\cos B, \qquad \tan A = -\tan B$$

The primary trigonometric ratios of an angle, θ, in standard position are defined in terms of the coordinates of a point, (x, y), on the terminal arm, as follows:

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x} \quad \text{where} \quad r = \sqrt{x^2 + y^2}$$

Bearing and Directions

Bearings - 050°, Directions – N50°E



Types of Problems

Directions,

Solve a Triangle

Area

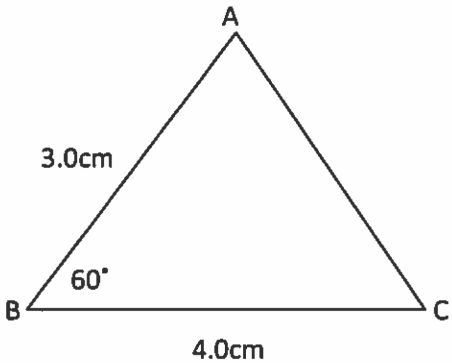
Practice Drawing Triangles.

Draw the following triangles, state unknowns and approach to solving (You do not need to solve):

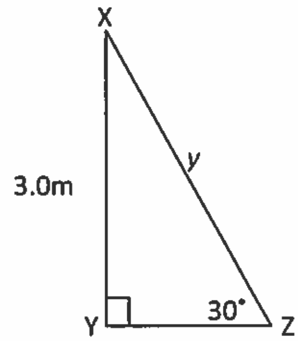
1. Triangle ABC, where a=3m, b=4m, A=90°
2. Triangle XYZ, where X=108°, z=27mm, y=12mm.
3. Triangle PQR, where P=43°, R=118°, q=50m.

Example #1 : Calculate the length of the unknown side in each triangle.

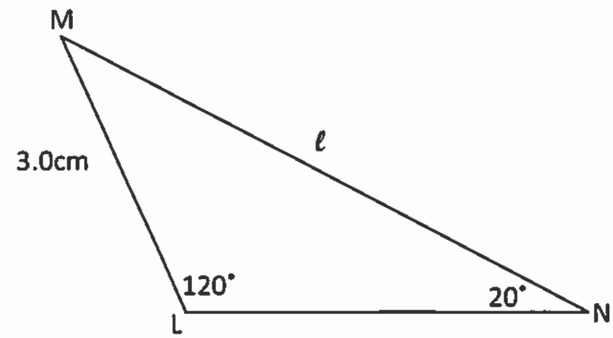
a.



b.

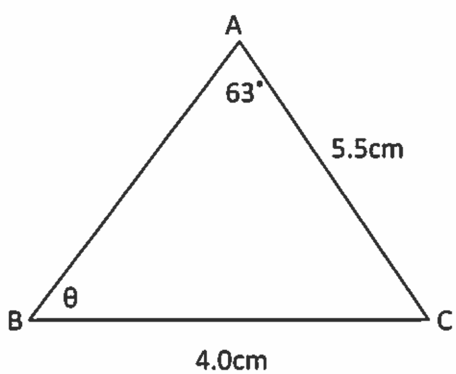


c.

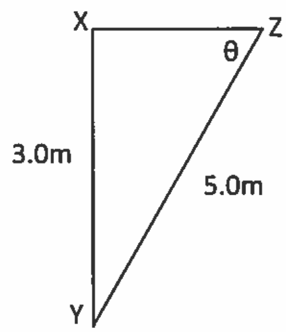


Example #2 : Calculate the indicated angle in each triangle.

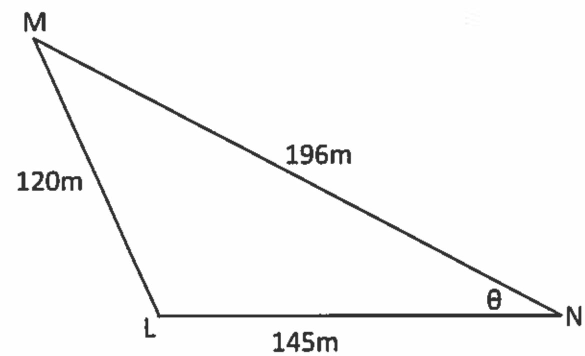
a.



b.



c.



Example #3 **CHALLENGE QUESTION**

A boat is proceeding on a bearing of 045° at 12 km/hr. At 3:00PM the captain sees a navigation buoy at 020°. He sights the same buoy at 230° at 4:15. How many km's is the boat from the buoy at 4:15PM?

a. Draw the figure
b. Determine what Trig Rules to use
c. Solve for unknown.

U2D12 Review: page 132 #1-12 and attached "practice test questions"