U1D10_T_1.6 Analyse Optimum Volume and Surface Area

Tuesday, February 13, 2018 7:31 PM

U1D9_T_1.
6 Analyse ...

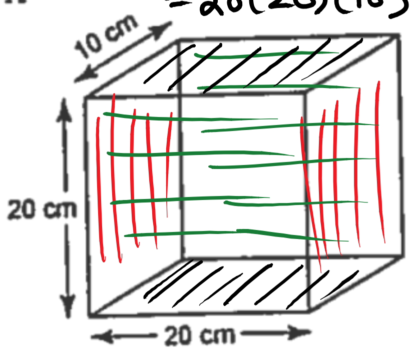
MAP 4CI

1.6 Analyse Optimum Volume and Surface Area

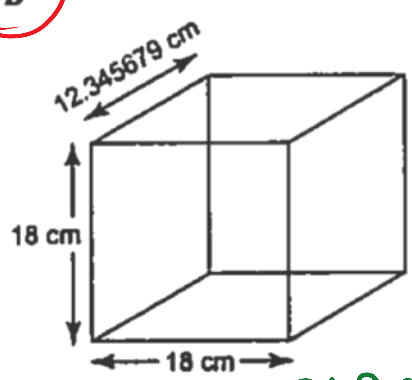
1. Each box/rectangular prism has the same volume $\underline{4000\text{ cm}^3}$.
Calculate the surface area of each box. Include the bottom of each box. List them in order from greatest to least surface area. D C A B
Which is the optimum shape (minimum surface area for the given volume)? B is closest to a cube
OPTIMAL is 1536.90cm^2

$V = lwh$
$= 20(20)(10)$

A



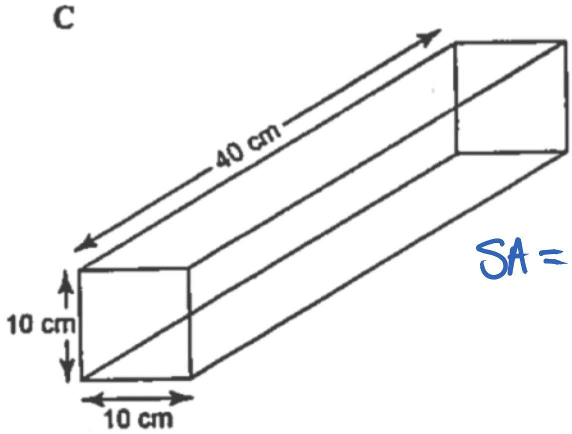
B



$SA = 1536.9\text{ cm}^2$

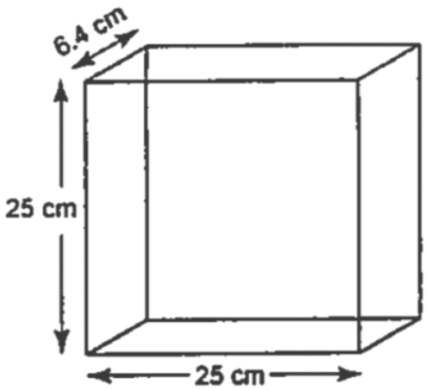
$$SA = 2lw + 2lh + 2wh$$
$$= 2(20)(10) + 2(20)(20) + 2(10)(20)$$
$$= 1600\text{ cm}^2$$

C



SA = 1800 cm^2

D



SA = 1890 cm^2

Summary: Optimizing the dimensions of a rectangular prism.

Among all rectangular prisms with a given volume, a **cube** has the minimum surface area.

Among all rectangular prisms with a given surface area, a **cube** has the maximum volume

∴ The minimum surface area of a square-based prism occurs when the **height** is equal to the side length of the base.

Special Case: When the bottom is not included, the minimum surface area for a given volume occurs when the **side length** of the base is equal to **twice the height of the square-based prism.**

Optimizing with constraints

There may be constraints on the prism you are optimizing:

- The dimensions may have to be whole numbers, or
- The dimensions may have to be multiples of a given number.

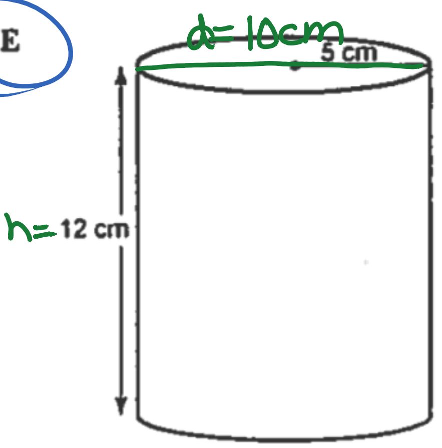
In these cases it may not be possible to form a cube. The maximum volume or minimum surface area occurs when **the dimensions are closest in value**.

2. Each cylindrical container has the same surface area. Calculate the volume of each. List them in order from minimum to maximum volume. Which cylinder is the optimum shape (maximum volume for a given surface area)?

GHFE

E is OPTIMAL

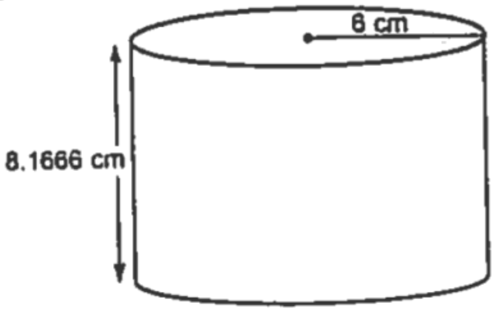
E



$$SA = 2\pi r^2 + 2\pi r h$$
$$= 2\pi(5)^2 + 2\pi(5)(12)$$
$$\doteq 534 \text{ cm}^2$$

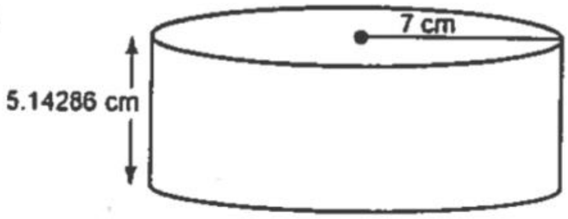
$$V = \pi r^2 h$$
$$= \pi(5)^2(12)$$
$$\doteq 942.5 \text{ cm}^3$$

F



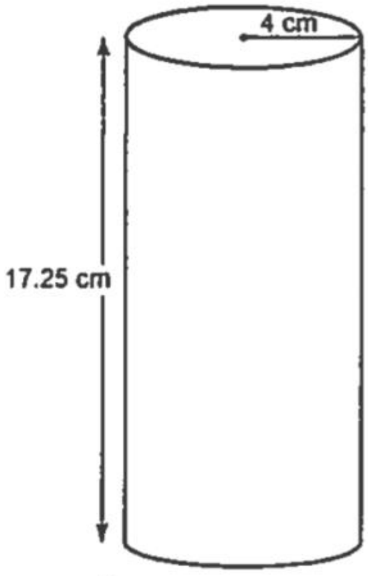
$$V = 923.6 \text{ cm}^3$$

G



$V \doteq 791.7\,cm^3$

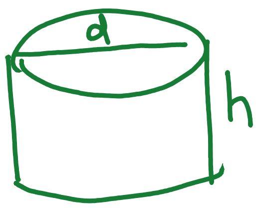
H



$V \doteq 867.1\,cm^3$

Summary: For a cylinder, the maximum volume for a given surface area occurs when the **height is equal to the diameter**.

For a cylinder, the minimum surface area for a given volume occurs when the **height is equal to the diameter.**



$d = h$

Ex. 1 A rectangular prism has a maximum volume and uses 384 m^2 of cardboard. What are the dimensions and volume?

* CUBE

$SA = 384$

So, $6x^2 = 384$

$$\frac{6x^2}{6} = \frac{384}{6}$$

$$x^2 = 64$$

$$x = \sqrt{64}$$

$$x = 8$$

$$V = x^3$$

$$V = 8^3$$

$$V = 512$$

∴ a cube 8m × 8m × 8m will have the maximum volume of 512 m^3.

GIVEN SA, MAXIMIZE V PRISM

Ex. 2 A rectangular prism has a volume of 1331 m^3.
What dimensions give a minimum surface area?
Calculate the surface area.

$V = 1331$

So, $x^3 = 1331$

$x = \sqrt[3]{1331}$

$x = 11$

$SA = 6x^2$

$SA = 6(11)^2$

$SA = 726$

$\therefore$ the minimum surface area of $726\,m^2$ occurs with a cube $11\,m \times 11\,m \times 11\,m$.

Given Volume, Minimize SA.
PRISM

Ex. 3 What are the optimum dimensions (i.e., minimum surface area) for a cylindrical can that can hold 750 mL of juice? $(1\,mL = 1\,cm^3)$

$V = 750\,cm^3$

So, $\frac{2\pi r^3}{2\pi} = \frac{750}{2\pi}$

$r^3 = \frac{750}{2\pi}$

$r = \sqrt[3]{\frac{750}{(2\pi)}}$

$r = \sqrt[3]{119.3662\ldots}$

$r = 4.923725\ldots$

$r \doteq 4.9\,cm$

$h = 2r$

So, $h = 9.8\,cm$

OPTIMIZING CYLINDER

$\therefore$ radius 4.9 cm, height 9.8 cm results in the optimal cylinder.

Maximizing Volume & Minimizing Surface Area Summary
All of these – provided all sides are enclosed

	Given Surface Area, Maximizing Volume	Given Volume, Minimizing Surface Area
Square Based Prism Enclosing all sides. (optimal is a cube with side length x)	Surface Area = $6x^2$ *Solve for x.* Volume = x^3	Volume = x^3 *Solve for x.* Surface Area = $6x^2$
Cylinder Enclosing all sides. (optimal has height = diameter or h = 2r)	Surface Area = $6\pi r^2$ *Solve for r.* Volume = $2\pi r^3$	Volume = $2\pi r^3$ *Solve for r.* Surface Area = $6\pi r^2$

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