U9D1- MCR Exam Review Solutions

U9D1-T
MCR Exa...

Exam Review Day 1
Work through these questions at
whiteboards

1. **Simplify Completely. Be sure to state restrictions on the variable.**

$$\frac{x^2 - 9}{4x + 12} + \frac{8x^2 + 16x}{x^2 - x - 6} \div \frac{2x}{x - 3}$$

$$= \frac{(x-3)(x+3)}{4(x+3)} + \frac{8x(x+2)}{(x-3)(x+2)} \times \frac{(x-3)}{2x}$$

$$= \frac{x-3}{4} + \frac{8x}{x-3} \times \frac{x-3}{2x}$$

*can only cross reduce across a multiplication sign.

$$= \frac{x-3}{4} + \frac{4}{1} \frac{\times 4}{\times 4}$$

$$= \frac{x-3+16}{4}$$

$$= \frac{x+13}{4} \qquad x \neq \pm 3, -2, 0$$

means number of items produced ... *x*-value of vertex

2. **Determine the production level required for maximum profit. What is the maximum profit?** ← *y*-value of vertex

partially factored

Given: $P(x) = -3x(x-50)+1000$, **where x is the number of items produced and P(x) is the profit in dollars.**

Graph goes through (0,1000) and (50,1000)

So axis of symmetry is $x = 25$

$P(25) = -3(25)(25-50)+1000$

$= -3(25)(-25)+1000$

$= 3(625)+1000$

$= 1875+1000$

$= 2875$ ← when 25 items are produced

∴ a profit of $2875 occurs

3. Given the parent function, $f(x) = \frac{1}{x}$ and the transformation described as $y = -2f(x + 4) - 5$
Write the image equation and state the domain and range of both the parent function and the image equation.

$$y = -2\left(\frac{1}{x+4}\right) - 5$$

$$y = \frac{-2}{x+4} - 5$$

D: $\{x \in \mathbb{R}, x \neq -4\}$

R: $\{y \in \mathbb{R}, y \neq -5\}$

for $y = \frac{1}{x}$, D: $\{x \in \mathbb{R}, x \neq 0\}$

R: $\{y \in \mathbb{R}, y \neq 0\}$

4. Simplify, leave no negative exponents.

$$\left(\sqrt[5]{\dfrac{-32x^{-4}}{x^{21}}} \right)^3$$

$$= \left[\left(-32x^{-4-21} \right)^{\frac{1}{5}} \right]^3$$

$$= \left[\left(-32x^{-25} \right)^{\frac{1}{5}} \right]^3$$

$$= \left[\sqrt[5]{-32} \; x^{\frac{-25}{5}} \right]^3$$

$$= \left(-2x^{-5} \right)^3$$

$$= (-2)^3 \left(x^{-5} \right)^3 \qquad = \dfrac{-8}{x^{15}}$$

$$= -8x^{-15}$$

5. Find all values of A, given

$$-180° \leq A \leq 720°$$

take note!

$$cscA = -\sqrt{2}$$

note:
$$\frac{1}{CSCA} = SinA$$

$$SinA = \frac{-1}{\sqrt{2}}$$

related acute angle is 45°

S   A

(T)
$$A = 180° + 45°$$

(C)
$$A = 360° - 45°$$
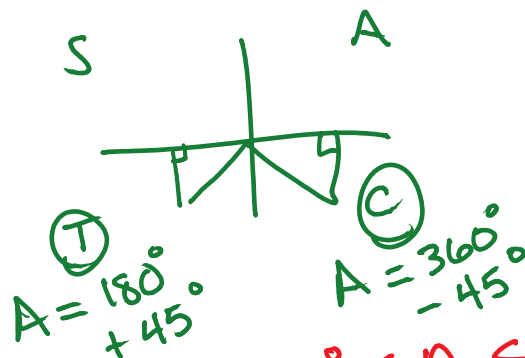
from $0° \leq A \leq 360°$

$$A = 225° \text{ OR } 315°$$

from $-180° \leq A \leq 720°$,

$225° - 360°, 225°, 225° + 360°,$
$315° - 360°, 315°, 315° + 360°$

∴ $A = -135°, -45°, 225°, 315°, 585°$ OR $675°$

6. **High tide is at 4 a.m. when the water is 6 m deep. Low tide is at 8 a.m. when the water is 1 m deep.**

   a) **Determine the following:**

   Maximum: 6   **Minimum:** 1
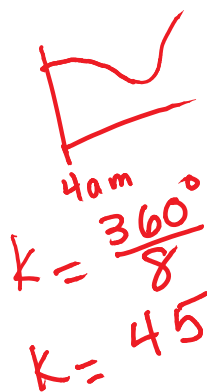
   $\frac{6-1}{2}$ ← **Amplitude:** 2.5   **Period:** 4 hrs × 2 = 8 hours

   Equation of Sinusoidal Axis: $y = 1 + 2.5$

   Phase Shift: b) no phase shift $y = 3.5$
   c) see below

   b) ~~Construct~~ **a model for the height of the function over time using a cosine function, where _t_ is the time in hours since 4 a.m. and _H(t)_ is the height of the water, in metres.**

   4 am

   $k = \frac{360°}{8}$

   $k = 45$

   $$H(t) = 2.5\cos 45t + 3.5$$

   c) **Construct a model for the height of the function over time using a sine function, where _t_ is the time in hours since 12 a.m. and _H(t)_ is the height of the water, in metres.**

   Phase shift right 2
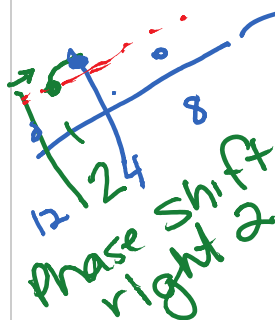
   $$H(t) = 2.5\sin 45(t-2) + 3.5$$

7. In a geometric sequence, $t_5 = 162$ and $t_{10} = 39366$. Find an expression for the nth term. $\quad t_n = ar^{n-1}$

$$\frac{ar^9}{ar^4} = \frac{39366}{162}$$

$$r^5 = 243$$

$$r = \sqrt[5]{243}$$
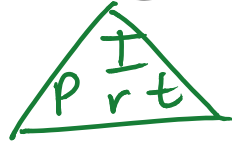
$$r = 3$$

$ar^4 = 162,$

$r = 3$

So,

$a(3)^4 = 162$

$a = \dfrac{162}{81}$

$a = 2$

$$\therefore t_n = 2(3)^{n-1}$$

8. $440 grew to $505.45 at 3.5%/a simple interest. For how long was the money invested?

$$I = Prt$$

Triangle: $\dfrac{I}{P \quad r \quad t}$

$$t = \dfrac{I}{Pr}$$

$$I = 65.45 \qquad P = 440 \qquad r = 0.035$$

$$t = \dfrac{65.45}{440(0.035)}$$

$$t = 4.25$$

note:
$\frac{1}{4}$ year is 3 months

∴ the money was invested for 4 years, 3 months.

$$A = P(1+i)^n \qquad A = P + I$$

9. $5000 is invested at 4.3%/a compounded weekly for 18 months. How much interest is earned on the investment?

$$A = ? \qquad P = 5000 \qquad i = \frac{0.043}{52}$$

$$n = \frac{18}{12} \times 52$$

$$n = 52 + 26$$

$$n = 78$$

$$I = ?$$

$\hookrightarrow$ for compound interest, must find A first.

$$A = 5000(1 + 0.043 \div 52)^{78}$$

$$A \doteq 5332.99$$

$$I = 5332.99 - 5000$$

$$I = 332.99$$

$\therefore$ $332.99 interest is earned.