MCR 3UI - EXAM REVIEW: FULL SOLUTIONS

Unit 1

1. (a) $(4x^2-7x-7)-(8x^2-5x-9)$
$= 4x^2-7x-7-8x^2+5x+9$
$= -4x^2-2x+2$
\# No restrictions

(b) $2(x-3)^2-(2x+1)(3x+2)$
$= 2(x-3)(x-3)-(6x^2+4x+3x+2)$
$= 2(x^2-3x-3x+9)-6x^2-7x-2$
$= 2x^2-12x+18-6x^2-7x-2$
$= -4x^2-19x+16$
\# no restrictions

(c) $\frac{3x-3y}{5x-5y}$
$= \frac{3\cancel{(x-y)}}{5\cancel{(x-y)}}$
$= \frac{3}{5}$
R: $x-y\neq 0$
$\therefore x\neq y$

(d) $\frac{x^2-16}{x^2-x-12}$
$= \frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)}$
$= \frac{x+4}{x+3}$
R: $x-4\neq 0$
$x\neq 4$
$x+3\neq 0$
$x\neq -3$
$\therefore x\neq -3, 4$

(e) $\frac{x^2+2x-3}{x^2+6x+8}\times\frac{x^2+2x-8}{x^2+x-6}$
$= \frac{\cancel{(x+3)}(x-1)}{\cancel{(x+4)}(x+2)}\times\frac{\cancel{(x+4)}\cancel{(x-2)}}{\cancel{(x+3)}\cancel{(x-2)}}$
$= \frac{x-1}{x+2}$
R: $x+4\neq 0$ $\quad x+2\neq 0$
$x\neq -4$ $\quad x\neq -2$
$x+3\neq 0$ $\quad x-2\neq 0$
$x\neq -3$ $\quad x\neq 2$
$\therefore x\neq -4, -3, -2, 2$

(f) $\frac{2x^2-x-1}{3x^2+x-2}\div\frac{2x^2-3x-2}{3x^2-11x+6}$
$= \frac{(2x+1)(x-1)}{(3x-2)(x+1)}\div\frac{(2x+1)(x-2)}{(3x-2)(x-3)}$
$= \frac{\cancel{(2x+1)}(x-1)}{\cancel{(3x-2)}(x+1)}\times\frac{\cancel{(3x-2)}(x-3)}{\cancel{(2x+1)}(x-2)}$
$= \frac{(x-1)(x-3)}{(x+1)(x-2)}$

R: $3x-2\neq 0$ $\quad x+1\neq 0$
$x\neq 2/3$ $\quad x\neq -1$
$2x+1\neq 0$ $\quad x-2\neq 0$
$x\neq -1/2$ $\quad x\neq 2$
$3x-2\neq 0$ $\quad x-3\neq 0$
$x\neq 2/3$ $\quad x\neq 3$
$\therefore x\neq -1, -1/2, 2/3, 2, 3$

(g) $\frac{x+2}{3}+\frac{2x-1}{4}-\frac{3x+1}{2}$

$=\frac{4(x+2)}{12}+\frac{3(2x-1)}{12}-\frac{6(3x+1)}{12}$

$=\frac{4(x+2)+3(2x-1)-6(3x+1)}{12}$

$=\frac{4x+8+6x-3-18x-6}{12}$

$=\frac{-8x-1}{12}$

$=\frac{-(8x+1)}{12}$

$=-\frac{8x+1}{12}$

(h) $\frac{4}{2x-3}-\frac{1}{3-2x}$

$=\frac{4}{2x-3}-\frac{1}{-(2x-3)}$

$=\frac{4}{2x-3}+\frac{1}{2x-3}$

$=\frac{5}{2x-3}$

R: $2x-3\neq 0$

$x\neq 3/2$

(i) $\frac{2}{x^2+5x+4}-\frac{3}{x^2-3x-4}$

$=\frac{2}{(x+4)(x+1)}-\frac{3}{(x-4)(x+1)}$

$=\frac{2(x-4)}{(x+4)(x+1)(x-4)}-\frac{3(x+4)}{(x+4)(x-4)(x+1)}$

$=\frac{2(x-4)-3(x+4)}{(x+4)(x+1)(x-4)}$

$=\frac{2x-8-3x-12}{(x+4)(x+1)(x-4)}$

$=\frac{-x-20}{(x+4)(x+1)(x-4)}$

R: $x\neq -4,-1,4$

(j) $\frac{x+1}{3x^2+4x+1}+\frac{2x-1}{3x^2-5x-2}$

$=\frac{x+1}{(3x+1)(x+1)}+\frac{2x-1}{(3x+1)(x-2)}$

$=\frac{(x+1)(x-2)+(2x-1)(x+1)}{(3x+1)(x+1)(x-2)}$

$=\frac{x^2-2x+x-2+2x^2+2x-x-1}{(3x+1)(x+1)(x-2)}$

$=\frac{3x^2-3}{(3x+1)(x+1)(x-2)}$

$=\frac{3(x^2-1)}{(3x+1)(x+1)(x-2)}$

$=\frac{3\cancel{(x+1)}(x-1)}{(3x+1)\cancel{(x+1)}(x-2)}$

$=\frac{3(x-1)}{(3x+1)(x-2)}$

R: $3x+1\neq 0$ $\quad x+1\neq 0$

$x\neq -1/3$ $\quad x\neq -1$

$x-2\neq 0$

$x\neq 2$

$\therefore x\neq -1,-1/3,2$

1k) $\dfrac{2x+2}{x^2-1} + \dfrac{x^2-1}{2x^2-x-1} \div \dfrac{9x+6}{12x+6}$

$= \dfrac{2(x+1)}{(x+1)(x-1)} + \dfrac{(x+1)(x-1)}{(2x+1)(x-1)} \times \dfrac{6(2x+1)}{3(3x+2)}$

$= \dfrac{2}{x-1} + \dfrac{x+1}{2x+1} \times \dfrac{2(2x+1)}{3x+2}$

$= \dfrac{2}{x-1} + \dfrac{x+1}{1} \times \dfrac{2}{3x+2}$

$= \dfrac{2}{x-1} + \dfrac{2(x+1)}{3x+2}$

$= \dfrac{2(3x+2) + 2(x+1)(x-1)}{(x-1)(3x+2)}$

$= \dfrac{6x+4+2x^2-2}{(x-1)(3x+2)}$

$= \dfrac{2x^2+6x+2}{(x-1)(3x+2)} \qquad x \neq \pm 1, -\frac{1}{2}, -\frac{2}{3}$

1l) $\dfrac{2x+4}{4x} - \dfrac{7x+7}{3x} \times \dfrac{5x^2}{14x^2+14x}$

$= \dfrac{2(x+2)}{2(2x)} - \dfrac{7(x+1)}{3x} \times \dfrac{5x^2}{14x(x+1)}$

$= \dfrac{x+2}{2x} - \dfrac{\cancel{x+1}}{3} \times \dfrac{5}{2(\cancel{x+1})}$

$= \dfrac{x+2}{2x} - \dfrac{5}{6}$

$= \dfrac{3(x+2) - 5(x)}{6x}$

$= \dfrac{3x+6-5x}{6x}$

$= \dfrac{-2x+6}{6x}$

$= \dfrac{2(-x+3)}{6x}$

$= \dfrac{-x+3}{3x}, \quad x \neq -1, 0$

2 (a) $\sqrt{50}$
$= \sqrt{25 \times 2}$
$= \sqrt{25} \times \sqrt{2}$
$= 5\sqrt{2}$

(b) $\sqrt{44}$
$= \sqrt{4 \times 11}$
$= 2\sqrt{11}$

(c) $2\sqrt{3} \times \sqrt{6}$
$= 2\sqrt{18}$
$= 2\sqrt{9 \times 2}$
$= 2(3)\sqrt{2}$
$= 6\sqrt{2}$

(d) $\frac{\sqrt{72}}{\sqrt{6}}$
$= \sqrt{12}$
$= \sqrt{4 \times 3}$
$= 2\sqrt{3}$

(e) $5\sqrt{10} \times 3\sqrt{2}$
$= 15\sqrt{20}$
$= 15\sqrt{4 \times 5}$
$= 15(2)\sqrt{5}$
$= 30\sqrt{5}$

(f) $(2\sqrt{5})^2$
$= 2^2(\sqrt{5})^2$
$= 4(5)$
$= 20$

(g) $\frac{8-\sqrt{40}}{2}$
$= \frac{8-\sqrt{4 \times 10}}{2}$
$= \frac{8-2\sqrt{10}}{2}$
$= \frac{\cancel{2}(4-\sqrt{10})}{\cancel{2}}$
$= 4-\sqrt{10}$

(h) $\frac{15\sqrt{48}}{5\sqrt{3}}$
$= \frac{15}{5}\sqrt{\frac{48}{3}}$
$= 3\sqrt{16}$
$= 3(4)$
$= 12$

(i) $\sqrt{48} - \sqrt{27} + \sqrt{12}$
$\sqrt{16 \times 3} - \sqrt{9 \times 3} + \sqrt{4 \times 3}$
$= 4\sqrt{3} - 3\sqrt{3} + 2\sqrt{3}$
$= 3\sqrt{3}$

(j) $\sqrt{6}(3\sqrt{2} + 2\sqrt{8})$
$= 3\sqrt{12} + 2\sqrt{48}$
$= 3\sqrt{4 \times 3} + 2\sqrt{16 \times 3}$
$= 3(2)\sqrt{3} + 2(4)\sqrt{3}$
$= 6\sqrt{3} + 8\sqrt{3}$
$= 14\sqrt{3}$

(k) $(2-\sqrt{3})(1+3\sqrt{3})$
$= 2 + 6\sqrt{3} - \sqrt{3} - 3(3)$
$= 2 + 5\sqrt{3} - 9$
$= 5\sqrt{3} - 7$

(l) $\frac{2}{\sqrt{7}}$
$= \frac{2}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)$
$= \frac{2\sqrt{7}}{7}$

(m) $\frac{3}{\sqrt{3}-4}$
$= \frac{3}{\sqrt{3}-4}\left(\frac{\sqrt{3}+4}{\sqrt{3}+4}\right)$
$= \frac{3(\sqrt{3}+4)}{3-16}$
$= \frac{3(\sqrt{3}+4)}{-13}$

(n) $\frac{5}{2\sqrt{6}+\sqrt{3}}$
$= \frac{5}{2\sqrt{6}+\sqrt{3}}\left(\frac{2\sqrt{6}-\sqrt{3}}{2\sqrt{6}-\sqrt{3}}\right)$
$= \frac{5(2\sqrt{6}-\sqrt{3})}{4(6)-3}$
$= \frac{5(2\sqrt{6}-\sqrt{3})}{24-3}$
$= \frac{5(2\sqrt{6}-\sqrt{3})}{21}$

3. (a) $2x^2 - 7x = 4$

$2x^2 - 7x - 4 = 0$

$(2 \quad +1)\ 2$

$(1 \quad -4)\ 2$

$(2x+1)(x-4) = 0$

$2x+1 = 0$ or $x - 4 = 0$

$x = -\tfrac{1}{2}$ $\qquad x = 4$

$\therefore x = -\tfrac{1}{2}, 4$

(b) $3x^2 = 6 - 7x$

$3x^2 + 7x - 6 = 0$

$(3 \quad -2)$

$(1 \quad 3)$

$(3x-2)(x+3) = 0$

$\therefore x = \tfrac{2}{3}, -3$

4. (a) $x^2 - 5x = 13$

$x^2 - 5x - 13 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-13)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 + 52}}{2}$$

$$= \frac{5 \pm \sqrt{77}}{2}$$

(b) $3x^2 = -3x + 7$

$3x^2 + 3x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(-3)^2 - 4(3)(-7)}}{2(3)}$$

$$= \frac{-3 \pm \sqrt{9 + 84}}{6}$$

$$= \frac{-3 \pm \sqrt{93}}{6}$$

5. (a) $y = (x^2 - 7x) + 2$

$= (x^2 - 7x + \frac{49}{4} - \frac{49}{4}) + \frac{2}{1}$

$= (x - \frac{7}{2})^2 - \frac{49}{4} + \frac{8}{4}$

$= (x - \frac{7}{2})^2 - \frac{41}{4}$

$\therefore$ minimum of $-\frac{41}{4}$ when $x = \frac{7}{2}$

(b) $y = (-4x^2 - 8x) + 5$

$= -4(x^2 + 2x) + 5$

$= -4(x^2 + 2x + 1 - 1) + 5$

$= -4(x^2 + 2x + 1) + 4 + 5$

$= -4(x+1)^2 + 9$

$\rightarrow$ maximum of 9 when $x = -1$

(c) $y = (-2x^2 + 5x) + 5$

$= -2(x^2 - \frac{5}{2}x) + 5$

$= -2(x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16}) + \frac{5}{1}$

$= -2(x - \frac{5}{4})^2 + \frac{25}{8} + \frac{40}{8}$

$= -2(x - \frac{5}{4})^2 + \frac{65}{8}$

$\therefore$ maximum of $\frac{65}{8}$ when $x = \frac{5}{4}$

(d) $y = (\tfrac{1}{2}x^2 - 4x) + 6$

$= \tfrac{1}{2}(x^2 - 8x) + 6$

$= \tfrac{1}{2}(x^2 - 8x + 16 - 16) + 6$

$= \tfrac{1}{2}(x^2 - 8x + 16) - 8 + 6$

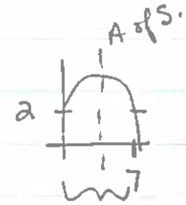
$= \tfrac{1}{2}(x - 4)^2 - 2$

$\rightarrow$ minimum of -2 when $x = 4$

by Partial Factoring ← this page is all

5. a) $y = x^2 - 7x + 2$

$y = x(x-7) + 2$



A of S: $x = \frac{7}{2}$

$y = \frac{7}{2}\left(\frac{7}{2} - 7\right) + 2$

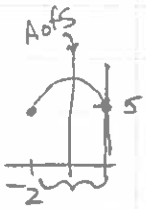
$y = \frac{7}{2}\left(\frac{-7}{2}\right) + 2$

$y = \frac{-49}{4} + \frac{8}{4}$

$y = -\frac{41}{4}$ $\therefore$ minimum of $-\frac{41}{4}$ occurs at $x = \frac{7}{2}$.

5 b) $y = -4x^2 - 8x + 5$

$y = -4x(x+2) + 5$



A of S: $x = \frac{-2}{2}$

$x = -1$

$y = -4(-1)(-1+2) + 5$

$y = -4(-1)(1) + 5$

$y = 9$ $\therefore$ max of 9 occurs at $x = -1$.

5c) $y = -2x^2 + 5x + 5$

$y = -x(2x-5) + 5$

OR $y = -2x\left(x - \frac{5}{2}\right) + 5$

A of S: $x = \frac{5}{4}$ $\frac{1}{2}$ of $\frac{5}{2}$

$y = -2\left(\frac{5}{4}\right)\left(\frac{-5}{4}\right) + 5$

$y = \frac{25}{8} + \frac{40}{8}$

$y = \frac{65}{8}$

$\therefore$ max of $\frac{65}{8}$ at $x = \frac{5}{4}$.

5d) $y = \frac{1}{2}x^2 - 4x + 6$

$y = \frac{1}{2}x(x-8) + 6$

A of S: $x = 4$ $\frac{1}{2}$ of 8

$y = \frac{1}{2}(4)(-4) + 6$

$y = -8 + 6$

$y = -2$

$\therefore$ min of -2 occurs at $x = 4$.

6 A. a) set $h(t) = 0$

$$\div(2) \quad -4t^2 + 20t + 2 = 0$$
$$2t^2 - 10t - 1 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

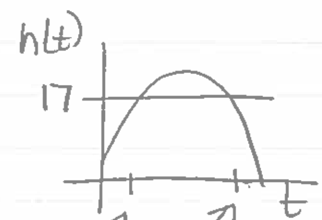
$$t = \frac{10 \pm \sqrt{100 - 4(2)(-1)}}{4}$$

$$t = \frac{10 \pm \sqrt{108}}{4}$$

$$t = {}^{-}0.098\ldots \text{ OR } t = 5.098\ldots$$
$$\doteq 5.1 \text{ seconds}$$

$\therefore$ the ball was in the air about 5.1 seconds

6A b) set $h(t) = 17$



$$-4t^2 + 20t + 2 = 17$$
$$-4t^2 + 20t - 15 = 0$$
$$4t^2 - 20t + 15 = 0$$

$$t = \frac{20 \pm \sqrt{400 - 4(4)(15)}}{8}$$

$$t = \frac{20 \pm \sqrt{160}}{8}$$

$$t = 0.91886\ldots \text{ OR } t = 4.0811\ldots$$

$$4.0811\ldots - 0.91886\ldots$$
$$\doteq 3.2 \text{ seconds}$$

$\therefore$ the ball was in the air for about 3.2 seconds.

$$h(t) = -4t^2 + 20t + 2$$

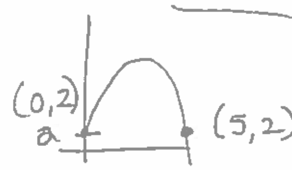
6A c) Max height – complete square or Partial factor

$$h(t) = -4(t^2 - 5t \qquad) + 2$$
$$h(t) = -4\left(t^2 - 5t + \tfrac{25}{4} - \tfrac{25}{4}\right) + 2$$
$$h(t) = -4\left(t - \tfrac{5}{2}\right)^2 + 25 + 2$$
$$\boxed{h(t) = -4\left(t - \tfrac{5}{2}\right)^2 + 27}$$

completing the square.

Partial Factor

$$h(t) = -4t(t-5) + 2$$



when $t=0$, $t=5$ the height is 2 m

Axis of Symmetry at $x = \frac{0+5}{2}$

$$x = \frac{5}{2}$$

$$h\left(\frac{5}{2}\right) = -4\left(\frac{5}{2}\right)\left(\frac{5}{2} - 5\right) + 2$$
$$= -4\left(\frac{5}{2}\right)\left(-\frac{5}{2}\right) + 2$$
$$= 25 + 2$$
$$= 27$$

By Partial Factoring.

$\therefore$ the maximum height of 27 m is reached after 2.5 seconds.

6. B set $h(t) = 10$

$15t - 4t^2 = 10$

$4t^2 - 15t + 10 = 0$

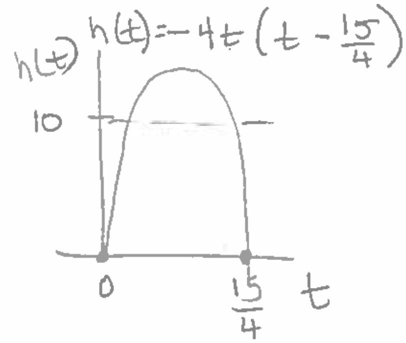
$D = b^2 - 4ac$

$= 15^2 - 4(4)(10)$

$= 225 - 160$

> 0

There are two real solutions for $h(t) = 10$. since the discriminant > 0.



C. Break even is when $P(x) = 0$

$-3x^2 + 7x + 9 = 0$

$3x^2 - 7x - 9 = 0$

$x = \frac{7 \pm \sqrt{49 - 4(3)(-9)}}{6}$

$x = \frac{7 \pm \sqrt{157}}{6}$

$x = \frac{(7 + \sqrt{157})}{6}$ OR $x = \frac{7 - \sqrt{157}}{6}$

$\doteq 3.254994$ < 0 so inadmissible.

in thousands

$\therefore$ a production level of 3255 items would result in a break-even.

7. $kx^2 - 3x + k = 0$

$D = b^2 - 4ac$

$D = 9 - 4(k)(k)$

$D = 9 - 4k^2$

a) $D = 0$

$9 - 4k^2 = 0$

$4k^2 = 9$

$k^2 = \frac{9}{4}$

$k = \frac{\pm 3}{2}$

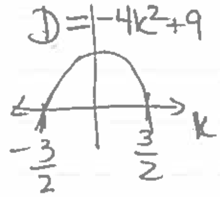
$\therefore$ there is one root when $k = \frac{3}{2}$ or when $k = \frac{-3}{2}$

7b) 2 roots

$D > 0$

$9 - 4k^2 > 0$

$-4k^2 + 9 > 0$



$\left\{ -\frac{3}{2} < k < \frac{3}{2} \right\}$

8. $g(x) = x + 2 \qquad f(x) = 2x^2 - 2x - 3$

$f(x) = g(x)$

$2x^2 - 2x - 3 - x - 2 = 0$

$2x^2 - 3x - 5 = 0$

$D = 9 - 4(2)(-5)$

$= 9 + 40$

$= 49$

$D > 0$ so two points of intersection.

$x = \frac{3 \pm \sqrt{49}}{4}$

$x = \frac{3 \pm 7}{4}$

$x = \frac{10}{4}$ OR $x = \frac{-4}{4}$

$x = \frac{5}{2} \qquad x = -1$

$g\left(\frac{5}{2}\right) = \frac{5}{2} + \frac{4}{2}$

$= \frac{9}{2}$

$g(-1) = -1 + 2$

$= 1$

$\therefore$ Points of intersection are $(-1, 1), \left(\frac{5}{2}, \frac{9}{2}\right)$

9. $y = a(x^2 - Sx + P)$

$S = (3 + \sqrt{5}) + (3 - \sqrt{5})$

$S = 6$

$P = (3 + \sqrt{5})(3 - \sqrt{5})$

$P = 9 - 5$

$P = 4$

family of quadratics with roots $3 \pm \sqrt{5}$ → $y = a(x^2 - 6x + 4)$

substitute in $(3, 10)$ & solve for 'a' for specific quadratic.

$10 = a(3^2 - 6(3) + 4)$

$a = \frac{10}{9 - 18 + 4}$

$a = -2$

$\therefore y = -2x^2 + 12x - 8$

10. (a) Domain: $\{2, 3, 7\}$ Not a function
Range: $\{4, 5, 9, -5, -7\}$ (one x maps to 2-y's)

(b) Domain: $\{-1, 0, 1, 2\}$ A Function
Range: $\{6, -6\}$

(c) Domain: $-3 \le x \le 2, x \in R$ Not a function
Range: $-4 \le y \le 5, y \in R$ (fails vertical line test)

(d) Domain: $0 \le x \le 4, x \in R$ Not a function
Range: $-2 \le y \le 2, y \in R$ (fails vertical line test)

11. $f(x) = 3 - 2x^2$

(a) $f(5) = 3 - 2(5)^2$
$= 3 - 2(25)$
$= 3 - 50$
$= -47$

(b) $f(-\frac{1}{2}) = 3 - 2(\frac{1}{2})^2$
$= 3 - 2(\frac{1}{4})$
$= 3 - \frac{1}{2}$
$= 2\frac{1}{2} = \frac{5}{2} = 2.5$

12. (a) $y = f(x-2) - 3$
→ shift right 2 units
→ shift down 3 units

(b) $y = -f(x+5) - 1$
→ reflect over x-axis
→ shift left 5 units
→ shift down 1 unit

(c) $y = \frac{1}{3}f(-3x) + 5$
→ vertically compress by a factor of $\frac{1}{3}$
→ horizontally compress by a factor of $\frac{1}{3}$
→ reflect over y-axis
→ shift up 5 units

(d) $y = -2f(2(x+3)) + 6$
→ reflect over x-axis
→ vertically stretch by a factor of 2
→ horizontally compress by a factor $\frac{1}{2}$
→ shift left 3 units
→ shift up 6 units

13. → reflect over x-axis
→ vertically compress by a factor of $\frac{1}{4}$
→ horizontally stretch by a factor of 2
→ translate right 8 units

14. (a) $y=3x-5$

For inverse,

$x=3y-5$

$x+5=3y$

$y=\frac{x+5}{3}$

$y=\frac{1}{3}x+5/3$

Yes, inverse is a function (a line)

(b) $y=x^2-7$

For inverse

$x=y^2-7$

$x+7=y^2$

$y=\pm\sqrt{x+7}$

Not a function
→ two values of y for many x-values.
(A full parabola on its side)

(c) $y=(x+2)^2$

For inverse

$x=(y+2)^2$

$\pm\sqrt{x}=y+2$

$-2\pm\sqrt{x}=y$

Not a function
→ two values of y for many x-values.
(a full parabola on its side)

(d) $y=\sqrt{x-3}$

For inverse

$x=\sqrt{y-3}$

$x^2=y-3$

$x^2+3=y$

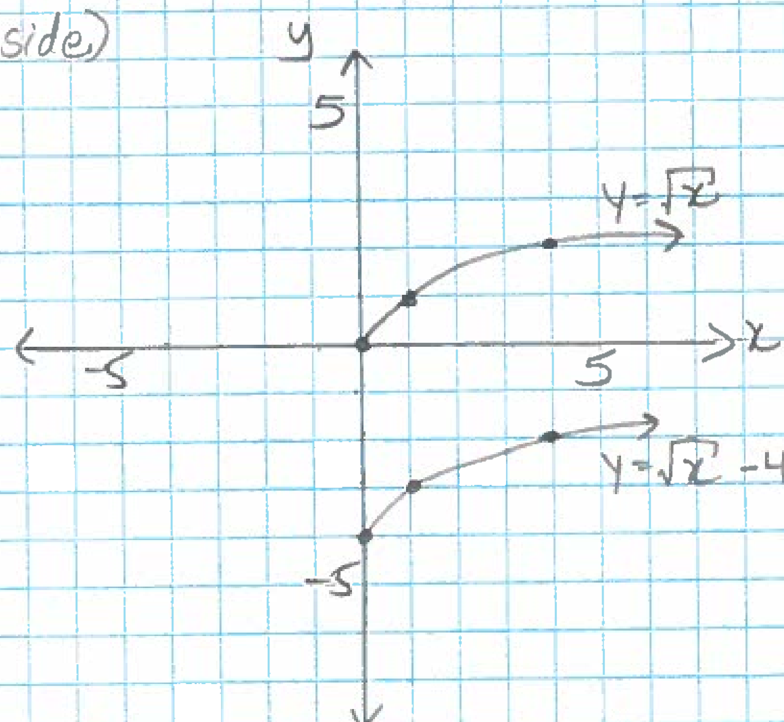
Yes a function
→ a parabola opening up.

15. (a) $y=\sqrt{x}$

$y=\sqrt{x}-4$
→ shift down 4 units

Domain: $\{x\in\mathbb{R} \mid x\geq 0\}$

Range: $\{y\in\mathbb{R} \mid y\geq -4\}$



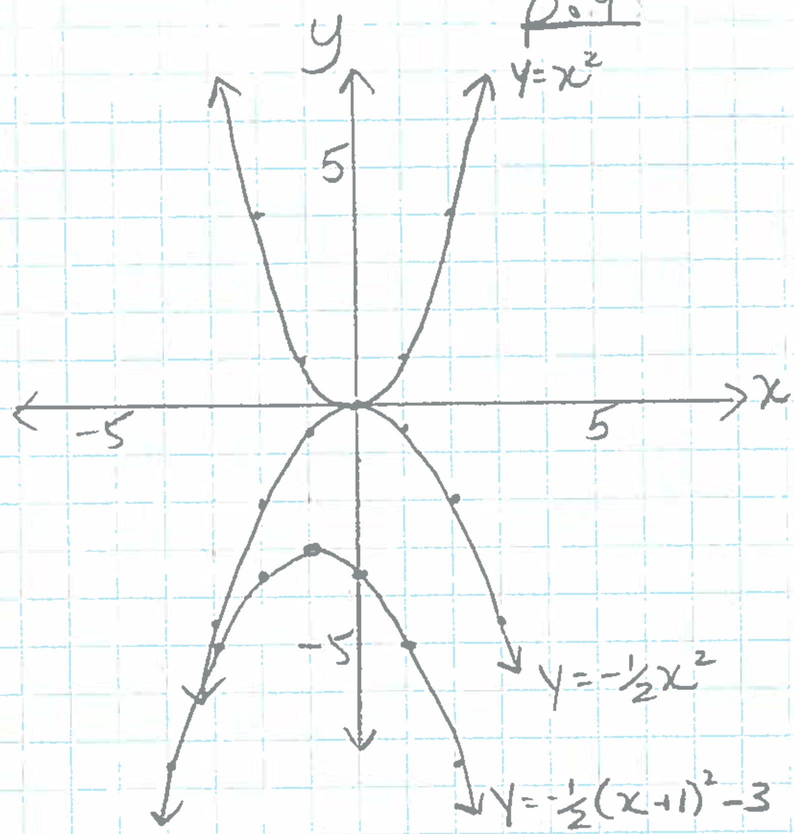
P.9

(b) $y=x^2$
$y=-\frac{1}{2}(x+1)^2-3$

→ reflect over x-axis
→ vertically compress by a factor of ½
→ shift left 1 unit
→ shift down 3 units

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \le -3\}$

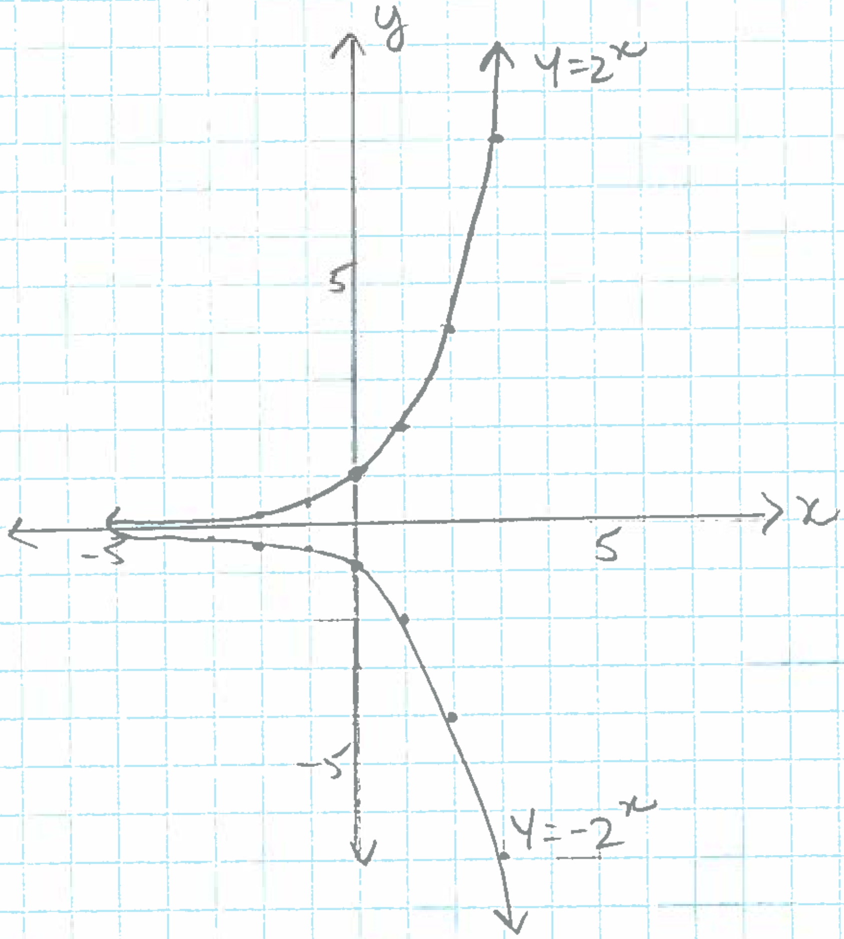


(c) $y=2^x$
$y=-2^x$

→ reflection over x-axis

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y \le 0\}$



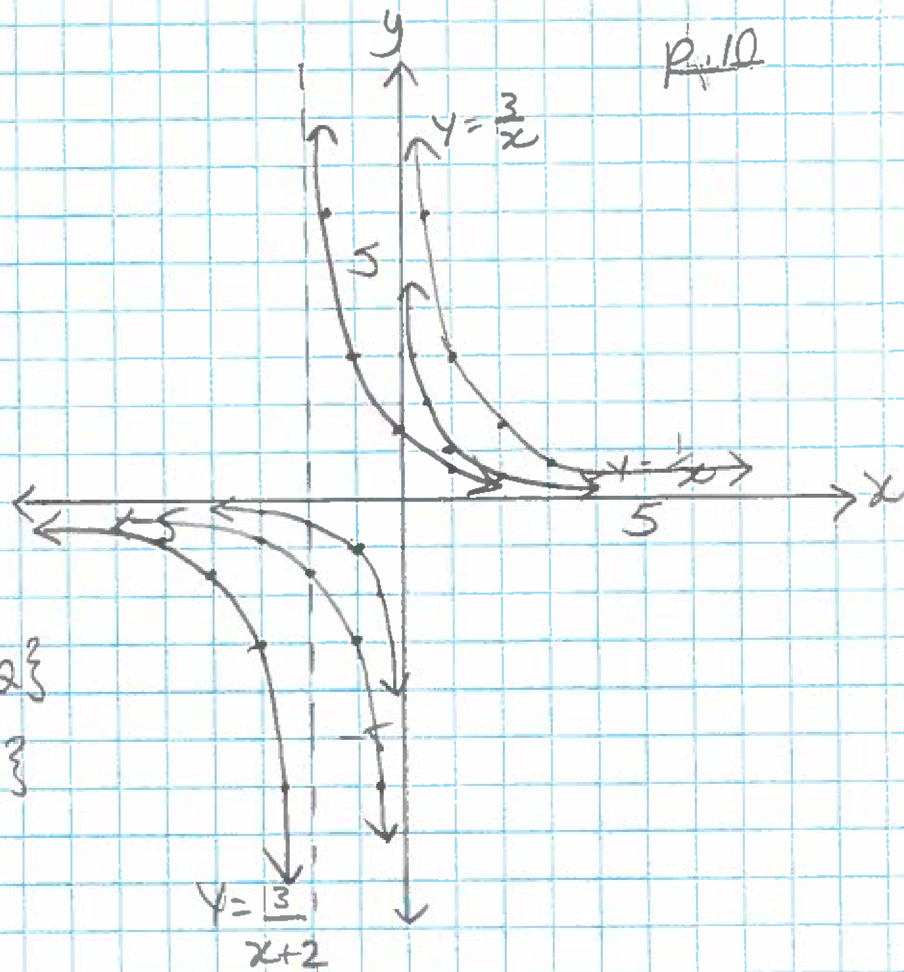
Pg. 10

(d) $y = \frac{1}{x}$

$y = \frac{3}{x+2}$

→ Vertically stretch by a factor of 3

→ shift left 2 units



Domain: $\{x \in R \mid x \neq -2\}$

Range: $\{y \in R \mid y \neq 0\}$

16. $y = x^2 \rightarrow y = 2x^2 \rightarrow y = 2(x+3)^2 \rightarrow y = 2(x+3)^2 + 4$

- $y = 2(x+3)^2 + 4$ is the transformed function

- Domain: $x \in R$

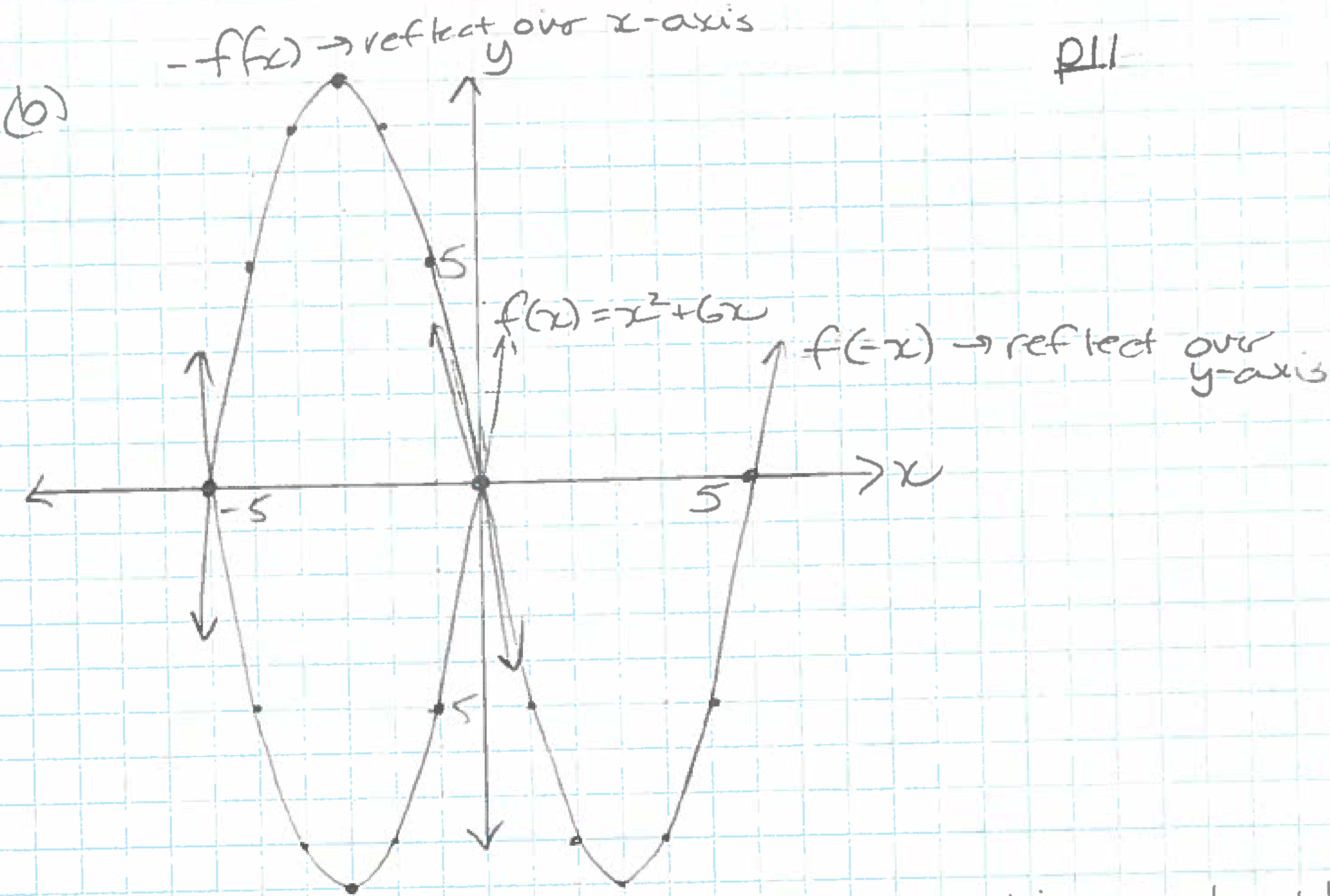
Range: $y \geq 4,\ y \in R$

17. (a) $f(x) = x^2 + 6x$

$-f(x) = -x^2 - 6x$

$f(-x) = (-x)^2 + 6(-x)$
$= x^2 - 6x$

(b)



(c) $f(x) \rightarrow -f(x)$ (reflected in x-axis so x-intercepts are invariant)

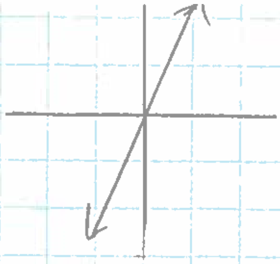
Invariant pts → $(-6,0)$, $(0,0)$

$f(x) \rightarrow f(-x)$ (reflection in y-axis so y-intercepts are invariant).

Invariant pts → $(0,0)$

* invariant points are on the axis of symmetry

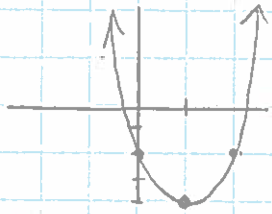
18. a) $y = 3x$



D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R}\}$

yes!

b) $y = 2(x-1)^2 - 4$



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq -4\}$

Yes!

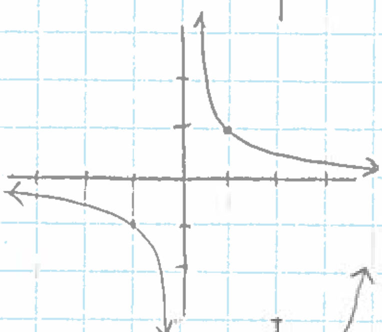
c) $y = -\sqrt{x} + 2$

D: $\{x \in \mathbb{R} \mid x \geq 0\}$

R: $\{y \in \mathbb{R} \mid y \leq 2\}$

yes!

d) $y = \frac{1}{x}$

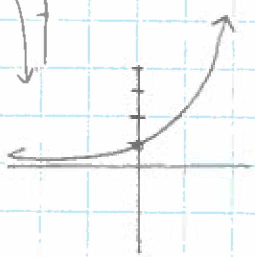


D: $\{x \in \mathbb{R} \mid x \neq 0\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

Yes!

e) $y = 3^x$

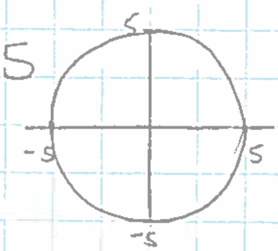


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y > 0\}$

YES!

f) $x^2 + y^2 = 25$



D: $\{x \in \mathbb{R} \mid -5 \leq x \leq 5\}$

R: $\{y \in \mathbb{R} \mid -5 \leq y \leq 5\}$

No! (fails VLT or has more than one y value for an x-value)

Unit 4

19. (a) $x^{-1} \cdot x^{-3} \cdot x^{2}$
$= x^{-1+(-3)+2}$
$= x^{-2}$
$= \frac{1}{x^2}$

(b) $(x^{-1}y^{2})^{-2}$
$= x^{2}y^{-4}$
$= \frac{x^2}{y^4}$

(c) $5x^4 \cdot 3x^2$
$= 15x^6$

(d) $(6x^{-1}y^{2})(-x^{-3}y^{-4})$
$= -6x^{-4}y^{-2}$
$= \frac{-6}{x^4y^2}$

(e) $\frac{3xy^3 \times 10x^4y^2}{15x^2y^6}$
$= \frac{30x^5y^5}{15x^2y^6}$
$= \frac{2x^3}{y}$

(f) $\left(\frac{4x^{-3}y^{4}}{8x^{2}y^{-2}}\right)^{-2}$
$= \left(\frac{8x^{2}y^{-2}}{4x^{-3}y^{4}}\right)^{2}$
$= \left(\frac{2x^5}{y^6}\right)^{2}$
$= \frac{4x^{10}}{y^{12}}$

20. (a) 5^{-2}
$= \frac{1}{5^2}$
$= \frac{1}{25}$

(b) 6^{0}
$= 1$

(c) $(-3)^{-4}$
$= \frac{1}{(-3)^4}$
$= \frac{1}{81}$

(d) $\frac{x^0 + 3^2}{2^4 - y^0}$
$= \frac{1+9}{16-1}$
$= \frac{10}{15}$
$= \frac{2}{3}$

(e) $25^{1/2}$
$= \sqrt{25}$
$= 5$

(f) $\left(\frac{1}{27}\right)^{1/3}$
$= \sqrt[3]{\frac{1}{27}}$
$= \frac{1}{3}$

(g) $(-32)^{4/5}$
$= \sqrt[5]{(-32)^4}$
$= \left(\sqrt[5]{-32}\right)^4$
$= (-2)^4$
$= 16$

(h) $\left(\frac{81}{16}\right)^{5/4}$
$= \left(\sqrt[4]{\frac{81}{16}}\right)^5$
$= \left(\frac{3}{2}\right)^5$
$= \frac{243}{32}$

(i) $\left(\frac{27}{125}\right)^{-2/3}$
$= \left(\frac{125}{27}\right)^{2/3}$
$= \left(\sqrt[3]{\frac{125}{27}}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

21. (a) $\sqrt[3]{-x}$
$= (-x)^{\frac{1}{3}}$

(b) $\sqrt[3]{\sqrt{x^2}}$
$= ((x^2)^{\frac{1}{2}})^{\frac{1}{3}}$
$= (x)^{\frac{2}{6}}$
$= (x)^{\frac{1}{3}}$

(c) $(\sqrt{x^3})(\sqrt{x})$
$= (x^3)^{\frac{1}{2}}(x)^{\frac{1}{2}}$
$= (x^{\frac{3}{2}})(x^{\frac{1}{2}})$
$= (x)^{\frac{4}{2}}$
$= x^2$

22. (a) C

(b) $p(n) = 50 \times 3^n$ ← number of days

initial population

"multiplying factor"
(this is a rate of increase of 200%)
$3 = (1+2)$
$= (100\% + \underline{\underline{200\%}})$

23. (a)

$V(t) = 20\,000\,(0.7)^t$ ← number of years

initial value of car

percent of value carried to the next year

(b) (i) When $t=1$

$V(t) = 20000(0.7)^1$
$= 14\,000$

$\therefore$ The car is worth \$14000 after one year.

(ii) When $t=2$

$V(t) = 20000(0.7)^2$
$= 9800$

$\therefore$ After 2 yrs the car is worth \$9800

(c) 10% of 20000 $\rightarrow 0.10 \times 20000 = 2000$

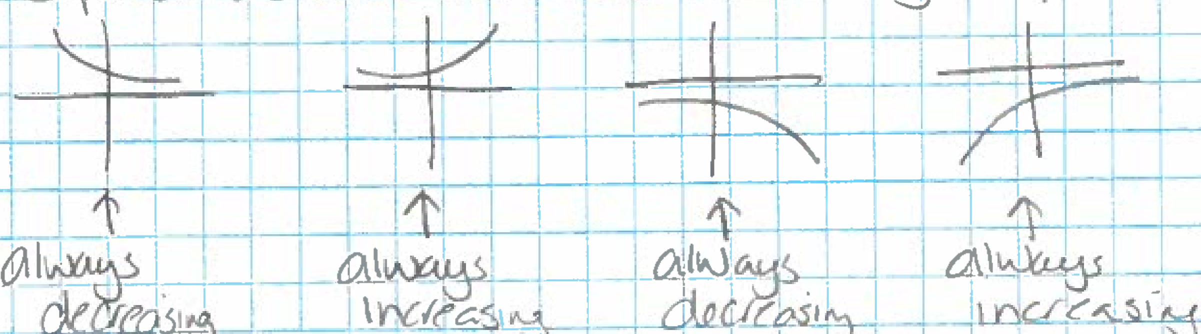
Find t when $V(t) = 2000$

$2000 = 20000(0.7)^t$
$0.10 = 0.7^t$

Trial & Error $\rightarrow$ $t=3 \rightarrow 0.7^3 = 0.343$
$t=5 \rightarrow 0.7^5 = 0.168$
$t=7 \rightarrow 0.7^7 = 0.082$
$t=6 \rightarrow 0.7^6 = 0.1176$
$t=6.5 \rightarrow 0.7^{6.5} = 0.984$

$\therefore$ It will take about 6.5 yrs.

24. (a) Exponential Functions have the following shapes:



$\therefore$ Yes, they are always either increasing or decreasing

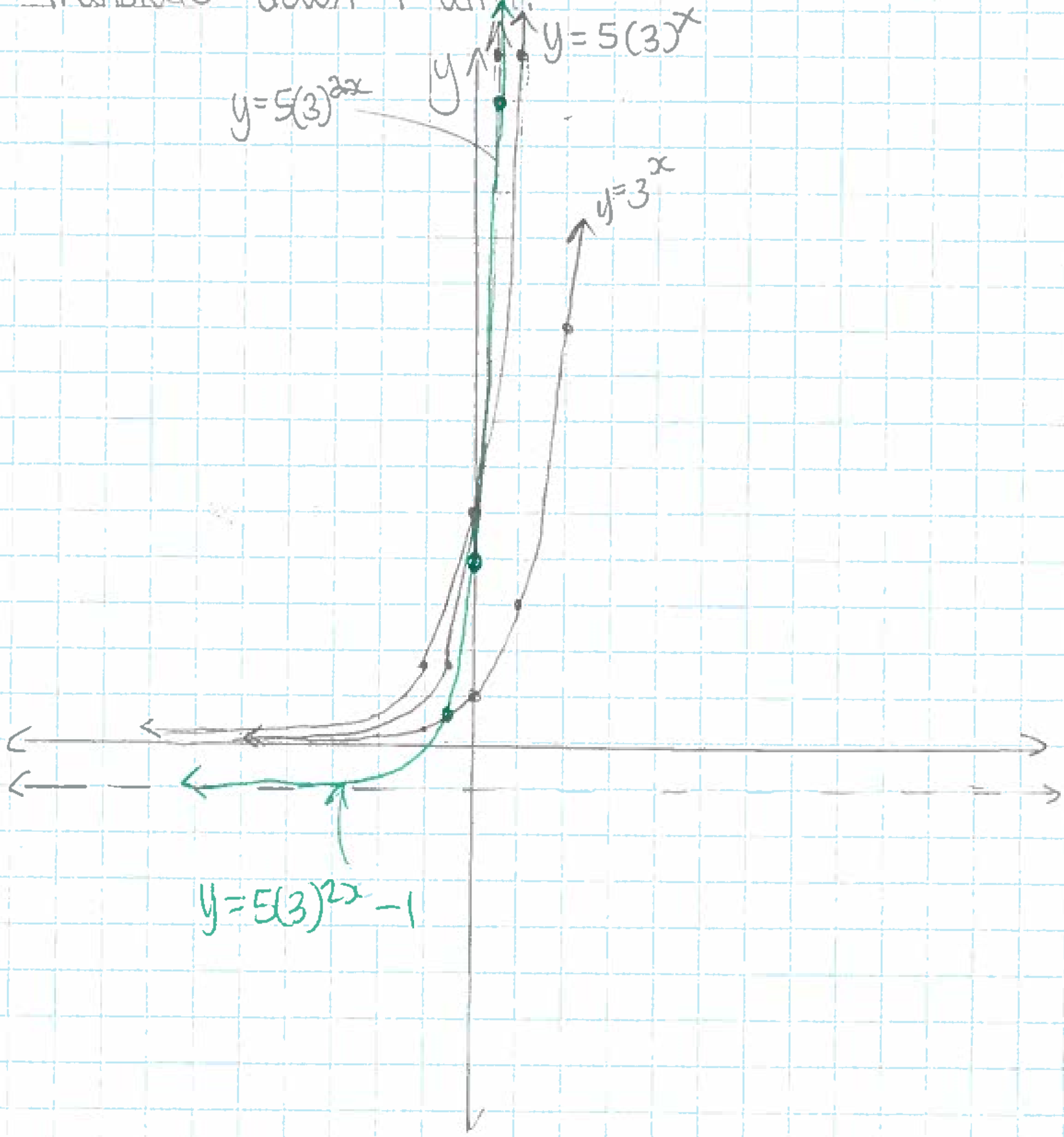
(b) An exponential of the form $y=ab^x$ will have one of the above shapes. All of these have the x-axis as a horizontal asymptote and therefore never cross the x-axis.

25. (a) $y = 10^{\frac{1}{3}x}$ → not showing
(b) A $y = 10^x + 3$
(c) B $y = 10^{x+3}$
(d) G $y = (\frac{1}{3})10^x$
(e) $y = 3(10^x)$ → not showing
(f) $y = 10^{x-3}$ → not showing
(g) C $y = -10^x$

26. (a)(i) $y = 5(3)^{2x} - 1$

→ Vertical stretch by a factor of 5
→ horizontal compression by a factor of $\frac{1}{2}$
→ translate down 1 unit.

(b)



(c) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y > -1\}$

Asymptote: $y = -1$

x-intercept: $x \doteq 0.73$ (by looking at graph & trial & error)

2(b)(ii)(a) $3^{x} = \left(\frac{1}{3}\right)^{-x}$ So; $y = -\left(\frac{1}{3}\right)^{12-3x} + 2$

$\because \left(\frac{1}{3}\right)^{-1} = 3,$

$3^{-1} = \frac{1}{3}$

can be written as

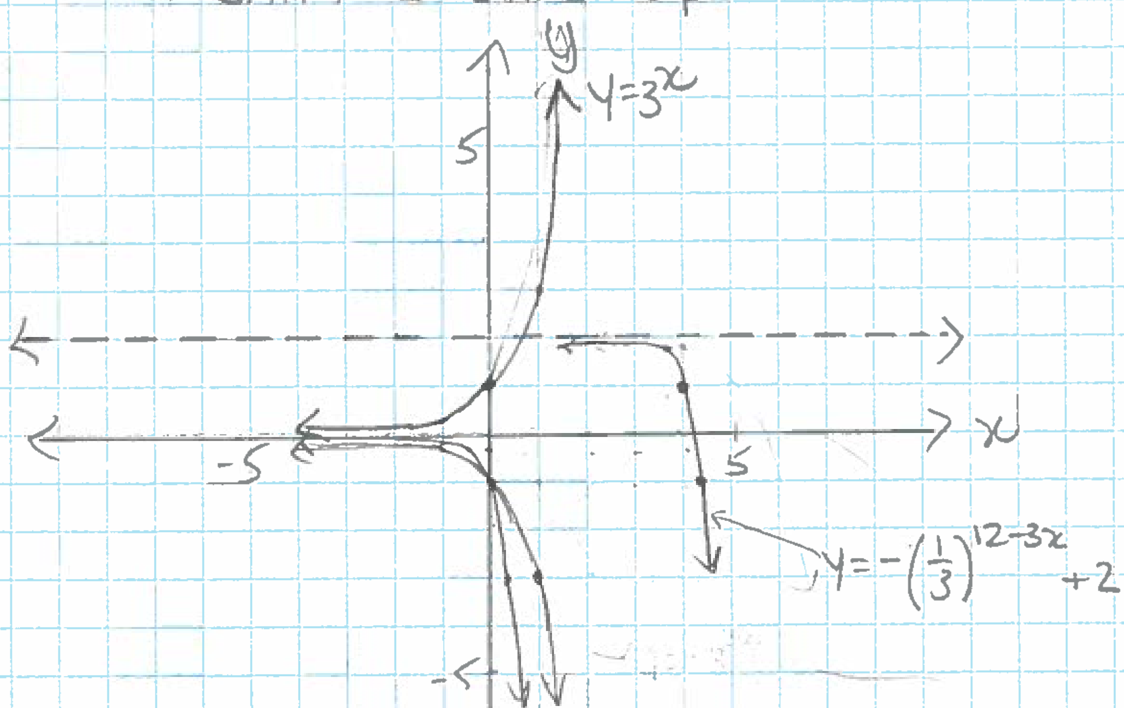
$y = -(3)^{-(12-3x)} + 2$

$= -3^{3x-12} + 2$

$= -3^{3(x-4)} + 2$

→ reflection over the x-axis
→ horizontal compression of factor ⅓
→ shift 4 units right
→ shift 2 units up.

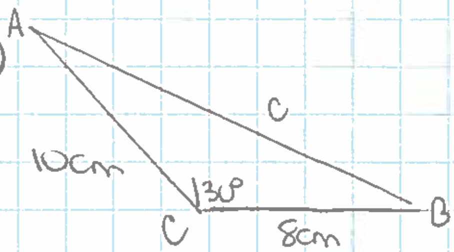
(b)



(c) (i) Domain: $\{x \in \mathbb{R}\}$
(ii) Range: $\{y \in \mathbb{R} \mid y < 2\}$
(iii) Asymptote: $y = 2$
(iv) x-intercept: $x \doteq 4.21$ (by looking at graph and/or trial & error)

UNIT 5

27 (a)



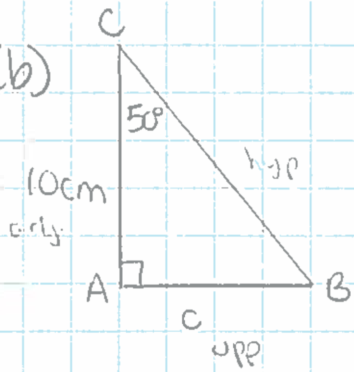
$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 8^2 + 10^2 - 2(8)(10)\cos 130$

$c = \sqrt{(64 + 100 - 160\cos 130)}$

$c \doteq 16.3$ cm

(b)



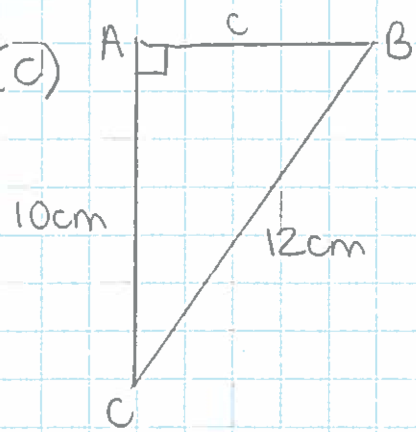
$\tan A = \frac{opp}{adj}$

$\tan 50 = \frac{c}{10}$

$c = 10\tan 50$

$c \doteq 11.9$ cm

(c)



$a^2 = b^2 + c^2$

$12^2 = 10^2 + c^2$

$144 = 100 + c^2$

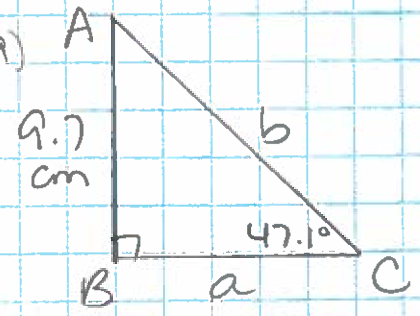
$c^2 = 144 - 100$

$c^2 = 44$

$c = \sqrt{44}$

$c \doteq 6.6$ cm

28 (a)



$\sin 47.1 = \frac{9.7}{b}$

$b = \frac{9.7}{\sin 47.1}$

$\doteq 13.2$

$\tan 47.1 = \frac{9.7}{a}$

$a = \frac{9.7}{\tan 47.1}$

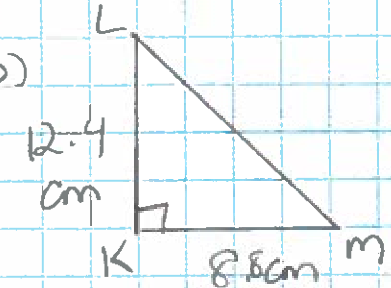
$a \doteq 9.01$

$\angle A \doteq 180 - 90 - 47.1$

$\angle A \doteq 42.9°$

$\therefore \angle A \doteq 42.9°,\ a \doteq 9.01\text{cm},\ b \doteq 13.2\text{cm}$

(b)



$k^2 = 12.4^2 + 8.8^2$

$= 153.76 + 77.44$

$= 231.2$

$k \doteq 15.2$

$\tan M = \frac{12.4}{8.8}$

$\tan M \doteq 1.4091$

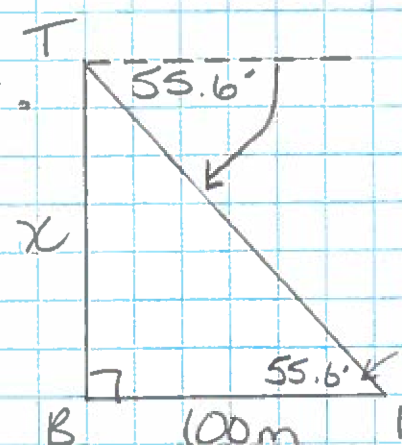
$M \doteq 54.6°$

$\angle L \doteq 180 - 90 - 54.6$

$= 35.4°$

$\therefore \angle M = 54.6°,\ \angle L \doteq 35.4°,\ k \doteq 15.2\text{cm}$

29.



By z rule (Alternate angles are equal when a transversal cuts through parallel lines)

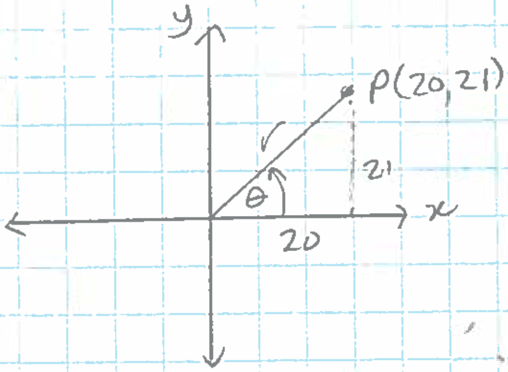
$\tan 55.6 = \frac{x}{100}$

$x = 100 \tan 55.6$

$\doteq 146.0$

$\therefore$ The building is 146m high

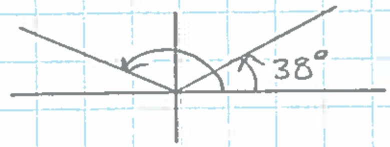
30.



$$r^2 = 20^2 + 21^2$$
$$= 400 + 441$$
$$= 841$$
$$r = 29$$

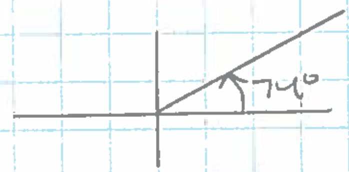
$\therefore \sin\theta = \boxed{21}/29 \qquad \cos\theta = \frac{20}{29}$

31. (a) $\sin A = 0.6157$
$A \doteq 38°$
or $A \doteq 180 - 38 = 142°$

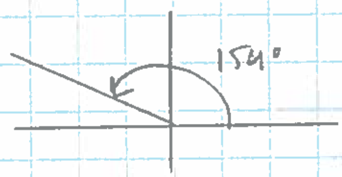


(b) $\cos A = 0.2756$
$A \doteq 74°$

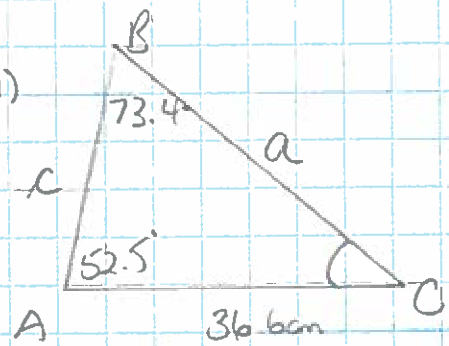
$0° \leq \theta \leq 180°$
(only one arm!)



(c) $\cos A = -0.8988$
$A \doteq 154°$



32. a)



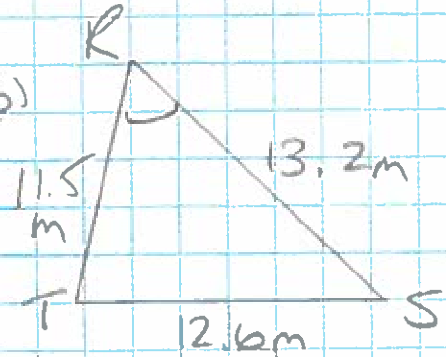
$\angle C = 180 - 73.4 - 52.5$
$= 54.1$

$$\frac{a}{\sin 52.5} = \frac{36.6}{\sin 73.4}$$
$$a = \frac{36.6 \sin 52.5}{\sin 73.4}$$
$$\doteq 30.3$$

$$\frac{c}{\sin 54.1} = \frac{36.6}{\sin 73.4}$$
$$c = \frac{36.6 \sin 54.1}{\sin 73.4}$$
$$\doteq 30.9$$

$\therefore \angle C = 54.1°$, $a \doteq 30.3$cm, $c \doteq 30.9$cm

32 (b)



$12.6^2 = 13.2^2 + 11.5^2 - 2(13.2)(11.5)\cos R$

$158.76 = 174.24 + 132.25 - 303.6\cos R$

$-147.73 = -303.6 \cos R$

$\cos R = \dfrac{-147.73}{-303.6}$

$\cos R \doteq 0.48659$

$R \doteq 60.9°$

If using Sine Law,
Solve for S first so there is no need to consider ambiguous case ∴ largest side is t so largest angle is T.

$\dfrac{\sin S}{11.5} = \dfrac{\sin 60.9}{12.6}$

$\sin S = \dfrac{11.5 \sin 60.9}{12.6}$

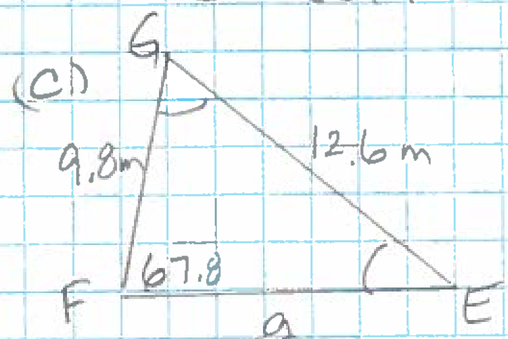
$\sin S = 0.7975$

$S \doteq 52.9$

$\angle T = 180 - 60.9 - 52.9$
$= 66.2$

$\therefore \angle R \doteq 60.9°, \angle S \doteq 52.9°, \angle T \doteq 66.2°$

(c)



Cannot be ambiguous when solving for E since e < f.

$\dfrac{\sin E}{9.8} = \dfrac{\sin 67.8}{12.6}$

$\sin E = \dfrac{9.8 \sin 67.8}{12.6}$

$= 0.7201$

$E = 46.1°$

$\angle G = 180 - 67.8 - 46.1$
$= 66.1°$

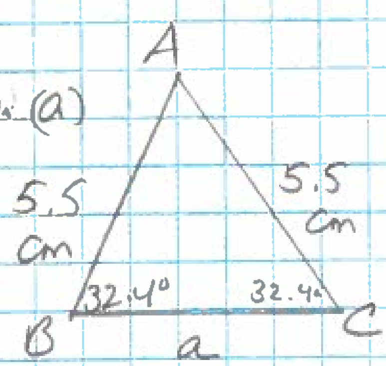
$\dfrac{g}{\sin 66.1} = \dfrac{12.6}{\sin 67.8}$

$g = \dfrac{12.6 \sin 66.1}{\sin 67.8}$

$= 12.4$

$\therefore \angle G = 66.1°, g = 12.4 m, \angle E = 46.1°$

33. (a)



$\angle A = 180 - 32.4 - 32.4$
$= 115.2$

$\dfrac{a}{\sin 115.2} = \dfrac{5.5}{\sin 32.4}$

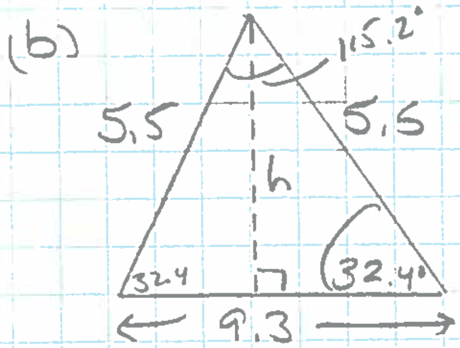
$a = \dfrac{5.5 \sin 115.2}{\sin 32.4}$

$= 9.3$

$P \doteq 5.5 + 5.5 + 9.3$
$= 20.3$

∴ The perimeter is 20.3 cm

(b)



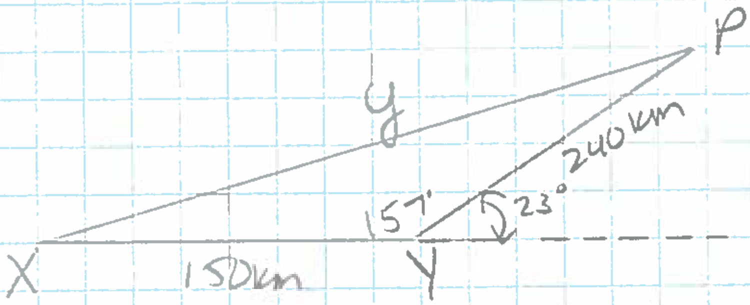
$\sin 32.4 = \dfrac{h}{5.5}$

$\therefore h = 5.5 \sin 32.4$
$= 2.9$

$A = \frac{1}{2}bh$
$= \frac{1}{2}(9.3)(2.9)$
$= 13.5$

$\therefore$ The area of the triangle is $13.5\ cm^2$

34.

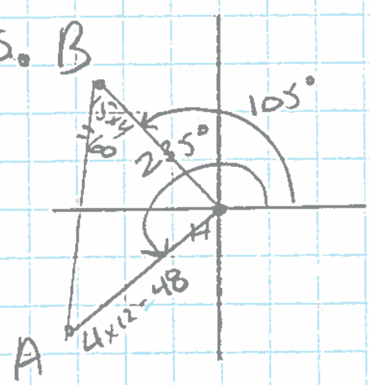


$\angle Y = 180 - 23$
$= 157°$

$y^2 = 150^2 + 240^2 - 2(150)(240)\cos 157°$
$= 22500 + 57600 - 72000(-0.9205)$
$= 80100 + 66276$
$= 146376$
$y = \sqrt{146376}$
$= 382.6$

$\therefore$ The airplane is 383 km away from airport X.

35.



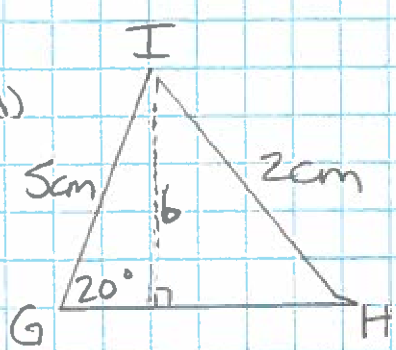
$\angle H = 235 - 105 = 130°$

$a = 60\ km$
$b = 48\ km$

$h^2 = 60^2 + 48^2 - 2(60)(48)\cos 130°$
$= 3600 + 2304 - 5760(-0.6428)$
$= 5904 + 3702.528$
$= 9606.528$
$h = 98.01$

$\therefore$ The ships are 98 km apart.

36.(a)

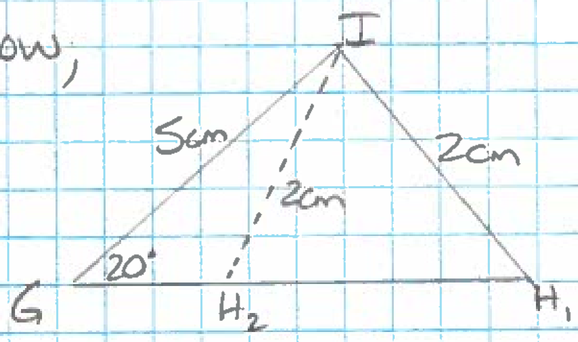


$\sin 20 = \frac{b}{5}$

$b = 5\sin 20$
$= 1.71$
$1.71 < 2$

$\therefore$ There are 2 triangles

Now,



$\frac{\sin H_1}{5} = \frac{\sin 20}{2}$

$\sin H_1 = \frac{5\sin 20}{2}$

$\sin H_1 = 0.8551$

$H_1 = 58.8°$

$\angle I_1 = 180° - 20 - 58.8°$
$= 101.2°$

$\frac{i_1}{\sin 101.2} = \frac{2}{\sin 20°}$

$i_1 = \frac{2\sin 101.2}{\sin 20°}$

$= 5.7$

or $H_2 = 180 - 58.8$
$= 121.2°$

$I_2 = 180 - 121.2 - 20$
$= 38.8°$

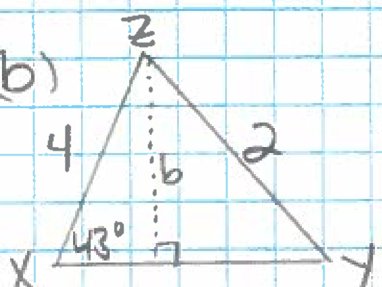
$\frac{i_2}{\sin 38.8} = \frac{2}{\sin 20}$

$i_2 = \frac{2\sin 38.8}{\sin 20}$

$= 3.7$

$\therefore$ ① $\angle I = 101.2$, $i = 5.7$, $\angle H = 58.8°$
② $\angle I = 38.8°$, $i = 3.7$, $\angle H = 121.2°$

(b)

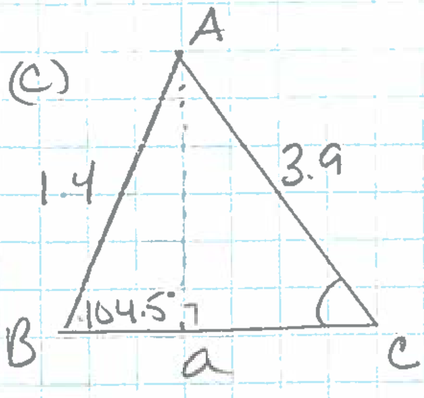


$b = 4\sin 43$
$= 2.7$

$b > 2$

$\therefore$ no triangle is possible.

36. (c)



$b > c$

$\therefore$ Only one triangle is possible

$$\frac{\sin C}{1.4} = \frac{\sin 104.5}{3.9}$$

$$\sin C = \frac{1.4 \sin 104.5}{3.9}$$

$$= 0.3475$$

$$C = 20.3$$

$$\angle A = 180 - 104.5 - 20.3$$
$$= 55.2$$

$$\frac{a}{\sin 55.2} = \frac{3.9}{\sin 104.5}$$

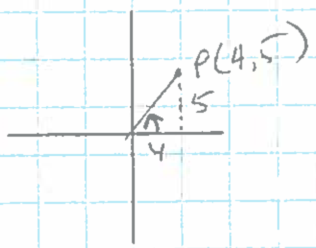
$$a = \frac{3.9 \sin 55.2}{\sin 104.5}$$

$$= 3.3$$

$\therefore \angle A = 55.2°, \angle C = 20.3°,$
$a = 3.3\text{m}$

Unit 6

37. (a)



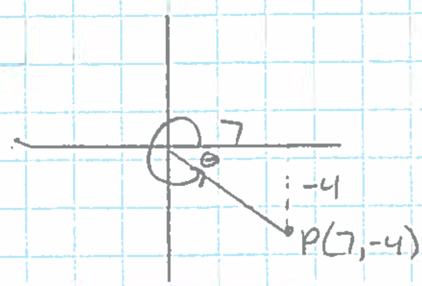
$$r^2 = 5^2 + 4^2$$
$$= 25 + 16$$
$$= 41$$
$$r = \sqrt{41}$$

$$\sin\theta = \frac{5}{\sqrt{41}}$$

$$\cos\theta = \frac{4}{\sqrt{45}}$$

$$\tan\theta = \frac{5}{4}$$

(b)



$$r^2 = 7^2 + (-4)^2$$
$$= 49 + 16$$
$$= 65$$
$$r = \sqrt{65}$$

$$\sin\theta = \frac{-4}{\sqrt{65}}$$

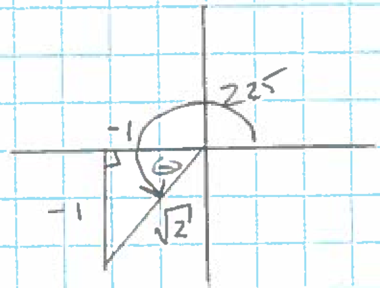
$$\cos\theta = \frac{7}{\sqrt{65}}$$

$$\tan\theta = -\frac{4}{7}$$

38.(a) $\tan 225°$

$\theta = 225 - 180°$
$= 45°$

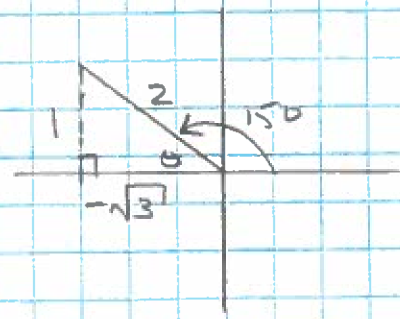
$\therefore \tan 225° = \frac{-1}{-1} = 1$



(b) $\cos 150°$

$\theta = 180 - 150°$
$= 30$

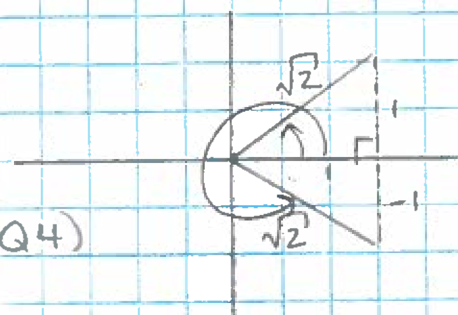
$\therefore \cos 150 = -\frac{\sqrt{3}}{2}$



39.(a) $\cos A = \frac{1}{\sqrt{2}}$

$A = 45°$ (in Q1)
or $A = 360 - 45°$ (in Q4)
$= 315°$

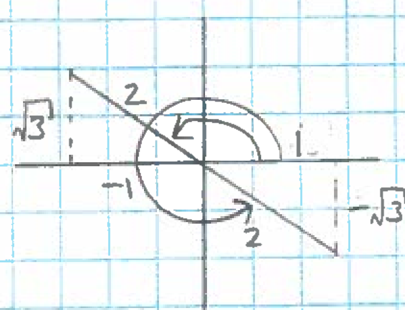
$\therefore A = 45°, 315°$



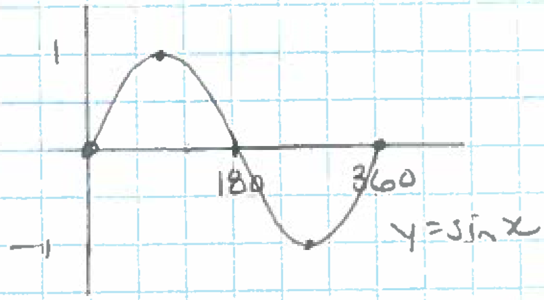
(b) $\tan A = -\sqrt{3}$

$A = 60°$ (in Q1)
$A = 180 - 60$ (in Q2)
$= 120°$
or $A = 360 - 60$ (in Q4)
$= 300$

$\therefore A = 120°, 300°$

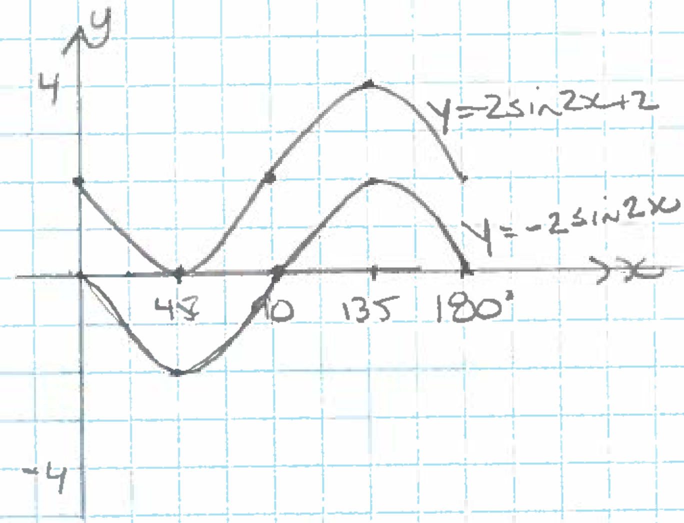


4.0: (a) $y = \sin x$



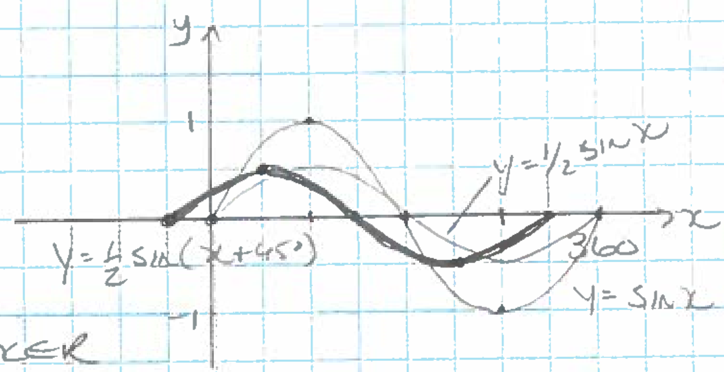
Domain: $0 \le x \le 360, x \in R$
Range: $-1 \le y \le 1, y \in R$
Amplitude: 1
Period: 360°
Phase Shift: none
Vertical Translation: none

(b) $y = -2\sin 2x + 2$



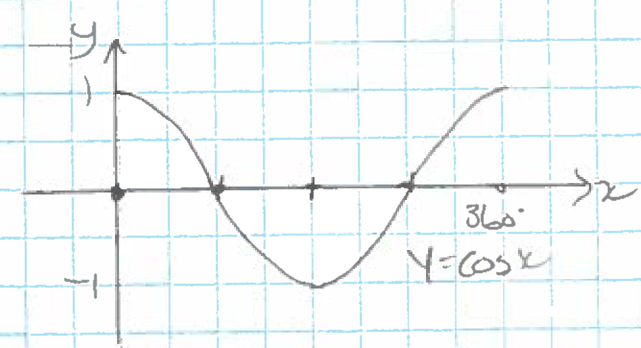
Period: $\frac{360}{2} = 180°$
Amplitude: 2
Phase Shift: none
Vertical translation: up 2
Domain: $0 \le x \le 180, x \in R$
Range: $0 \le y \le 4, y \in R$

(c) $y = \frac{1}{2}\sin(x + 45°)$



Period: 360°
Amplitude: ½
Phase Shift: left 45°
Vertical translation: none
Domain: $-45° \le x \le 315°, x \in R$
Range: $-½ \le y \le ½, y \in R$

(d) $y = \cos x$



Period: 360°
Amplitude: 1
Phase Shift: none
Vertical translation: none
Domain: $0 \le x \le 360°, x \in R$
Range: $-1 \le y \le 1, y \in R$

p. 27.

e) $y = 3\cos \frac{1}{3}x$

Period: $\frac{360}{1/3} = 360 \times 3 = 1080°$

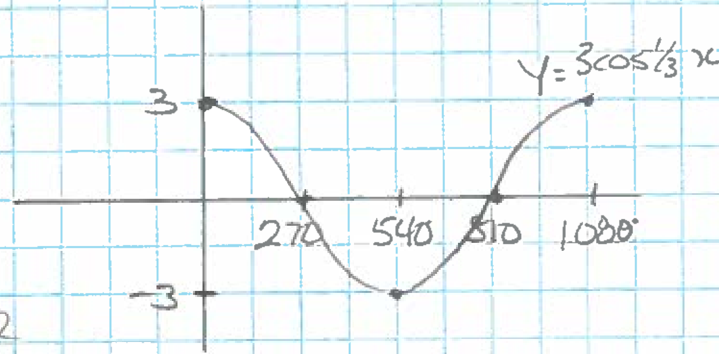
Amplitude: 3

Phase Shift: none

Vertical translation: none

Domain: $0 \le x \le 1080°, x \in R$

Range: $-3 \le y \le 3, y \in R$



(f) $y = 2\cos \frac{1}{2}(x - 180°) + 1$

Period: $\frac{360}{1/2} = 720°$

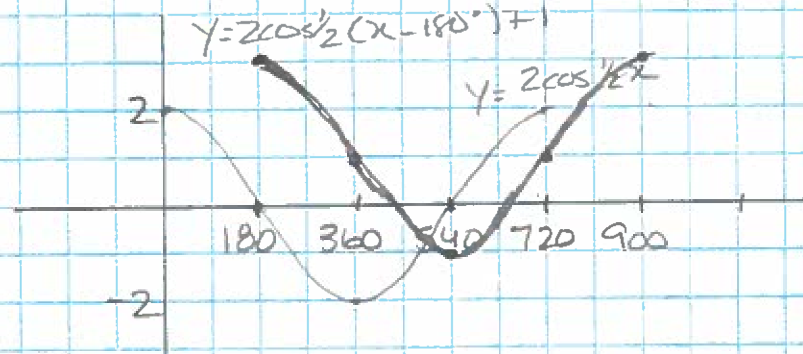
Amplitude: 2

Phase Shift: right 180°

Vertical translation: up 1

Domain: $180 \le x \le 900, x \in R$

Range: $-1 \le y \le 3, y \in R$



41. (a) $\frac{1 - \sin^2 x}{\cos x} = \cos x$

$LS = \frac{1 - \sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x}$ (PI)

$= \cos x$

$= RS$

(b) $1 + \tan^2 x = \frac{1}{\cos^2 x}$

$LS = 1 + \tan^2 x$

$= 1 + \frac{\sin^2 x}{\cos^2 x}$ (QI)

$= \frac{\cos^2 x + \sin^2 x}{\cos^3 x}$

$= \frac{1}{\cos^2 x}$ (PI)

(c) $\frac{1}{\sin} - \sin x = \frac{\cos x}{\tan x}$

$LS = \frac{1}{\sin x} - \sin x$

$= \frac{1 - \sin^2 x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$ (PI)

$RS = \frac{\cos x}{\tan x}$

$= \cos x \div \frac{\sin x}{\cos x}$ (QI)

$= \cos x \times \frac{\cos x}{\sin x} = \frac{\cos^2 x}{\sin x}$

(d) $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$

$LS = \left(1 - \dfrac{\sin^2 x}{\cos^2 x}\right) \div \left(1 + \dfrac{\sin^2 x}{\cos^2 x}\right)$

$= \left(\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}\right) \div \left(\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)$

$= \dfrac{\cos^2 x - \sin^2 x}{\cancel{\cos^2 x}} \times \left(\dfrac{\cancel{\cos^2 x}}{1}\right)$ (PI)

$= \cos^2 x - \sin^2 x$

$= RS.$

(e) $(1-\cos^2 x)(1+\tan^2 x) = \tan^2 x$

$LS = (1-\cos^2 x)(1+\tan^2 x)$

$= (\sin^2 x)\left(1 + \dfrac{\sin^2 x}{\cos^2 x}\right)$ (PI)

$= (\sin^2 x)\left(\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)$

$= (\sin^2 x)\left(\dfrac{1}{\cos^2 x}\right)$ (PI)

$= \dfrac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$ (QI)

$= RS$

(f) $(\sin x - \cos x)^2 = 1 - 2\sin x\cos x$

$LS = (\sin x - \cos x)^2$

$= (\sin x - \cos x)(\sin x - \cos x)$

$= \sin^2 x - \sin x\cos x - \sin x\cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) - 2\sin x\cos x$

$= 1 - 2\sin x\cos x$ (PI)

$= RS.$

(g) $(1+\cot^2 x)\tan^2 x = \sec^2 x$

$LS = (1+\cot^2 x)\tan^2 x$

$= \left(1 + \dfrac{1}{\tan^2 x}\right)(\tan^2 x)$ (RI)

$= \tan^2 x + \dfrac{\tan^2 x}{\tan^2 x}$

$= \tan^2 x + 1$

$= \dfrac{\sin^2 x}{\cos^2 x} + 1$ (QI)

$= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$ (PI)

$RS = \sec^2 x$

$= \dfrac{1}{\cos^2 x}$ (RI)

$\therefore LS = RS.$

(h) $\sin x \sec x = \tan x$

$LS = \sin x \sec x$

$= \sin x\left(\dfrac{1}{\cos x}\right)$ (RI)

$= \dfrac{\sin x}{\cos x}$

$= \tan x$ (QI)

$= RS$

(c) $\tan x(1+\cot x) = 1+\tan x$

$LS = \tan x(1+\cot x)$

$= \tan x + \tan x \cot x$

$= \tan x + \tan x\left(\dfrac{1}{\tan x}\right)$ (RI)

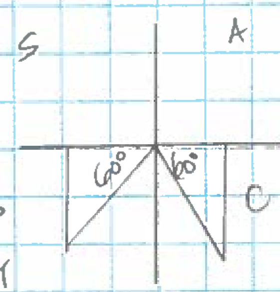
$= \tan x + 1$

$= RS$

42a)

$x = 240°$

OR $x = 300°$



42.(c) $2\sin x - 1 = 0$

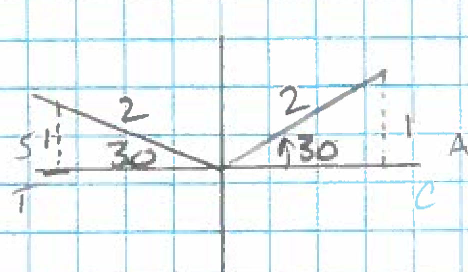
$2\sin x = 1$

$\sin x = \frac{1}{2}$

Q1 $\Rightarrow x = 30°$

Q2 $\Rightarrow x = 180 - 30$

$= 150$



$\therefore x = 30°, 150°$

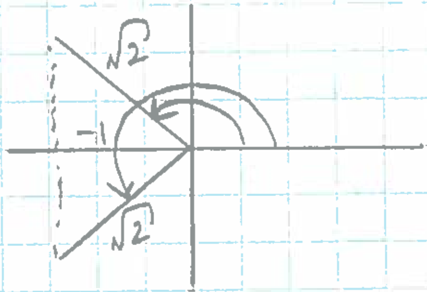
42b on next page

p.30

(b) $\sqrt{2}\cos x + 1 = 0$

$\sqrt{2}\cos x = -1$

$\cos x = \frac{-1}{\sqrt{2}}$



$Q1 \rightarrow x = 45°$

$Q2 \rightarrow x = 180 - 45 = 135°$

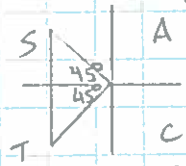
$Q3 \rightarrow x = 180 + 45 = 225°$

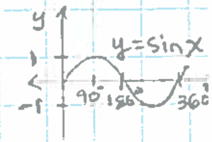
$\therefore\ x = 135°, 225°$

42.e)

$(\sqrt{2}\cos x + 1)(\sin x - 1) = 0$

$\cos x = -\frac{1}{\sqrt{2}}$ OR $\sin x = 1$



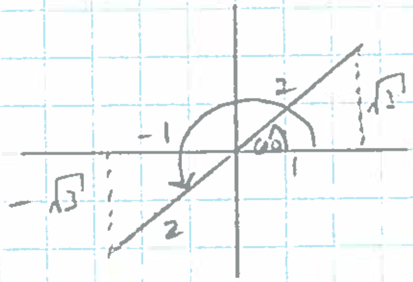


$x = 135°$ OR $225°$

$x = 90°$

$\therefore\ x = 90°, 135°$ OR $225°$

(d) $\tan x = \sqrt{3}$



$Q1 \rightarrow x = 60°$

$Q3 \rightarrow x = 180 + 60$

$= 240$

$\therefore\ x = 60°, 240°$

(h) $\cos^2 x - 1 = \sin^2 x$

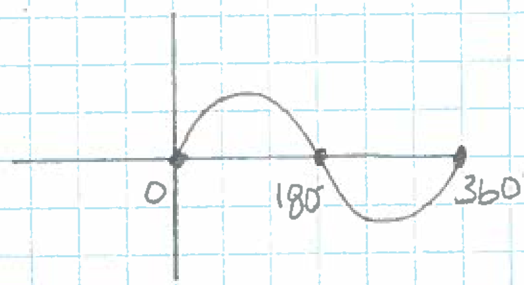
$(1 - \sin^2 x) - 1 = \sin^2 x$

$1 - \sin^2 x - 1 - \sin^2 x = 0$

$-2\sin^2 x = 0$

$\sin^2 x = 0$

$\sin x = 0$



$\therefore\ x = 0, 180°, 360°$

(f) $2\cos^2 x + 3\cos x = -1$

$2\cos^2 x + 3\cos x + 1 = 0$

1 2 1

2 1 1

may use quadratic formula

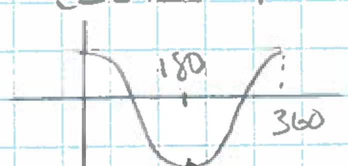
$((\cos x + 1)(2\cos x + 1) = 0$

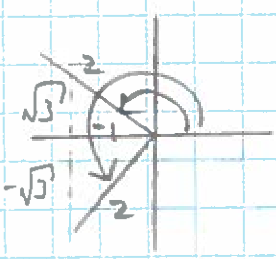
Either $\cos x + 1 = 0$ or $2\cos x + 1 = 0$

$\cos x = -1$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$





$Q1 \rightarrow x = 60$

$Q2 \rightarrow x = 180 - 60 = 120$

$Q3 \rightarrow x = 180 + 60 = 240$

$\therefore\ x = 180°, 120°, 240°$

(g) $\cos x + 1 = 2\sin^2 x$

p31

$\cos x + 1 = 2(1 - \cos^2 x)$ (PI)

$\cos x + 1 = 2 - 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

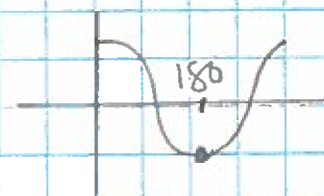
$(2\cos x - 1)(\cos x + 1) = 0$

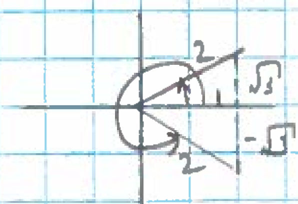
may use quadratic formula

Either $2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$2\cos x = 1$ $\qquad$ $\cos x = -1$

$\cos x = \frac{1}{2}$





Q1 → $x = 60°$

Q4 → $x = 360 - 60$

$= 300$

$\therefore x = 60°, 180°, 300°$

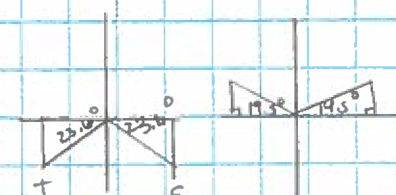
42(i) $15\sin^2 x + \sin x = 2$

$15\sin^2 x + \sin x - 2 = 0$

OR may you use quadratic formula

$(5\sin x + 2)(3\sin x - 1) = 0$

$\sin x = \frac{-2}{5}$ OR $\sin x = \frac{1}{3}$



$x = 203.6°$ OR $336.4°$

$x = 19.5°$ OR $160.5°$

$\therefore x = 19.5°, 160.5°, 203.6°$ OR $336.4°$

Unit 7

43. (a) $a = 3$, $d = +2$

$t_n = a + (n-1)d$

$= 3 + (n-1)(2)$

$= 3 + 2n - 2$

$= 2n + 1$

$\therefore t_n = 2n + 1$

$t_{30} = 2(30) + 1$

$= 61$

(b) $a = -4$, $d = +7$

$t_n = a + (n-1)d$

$= -4 + (n-1)(+7)$

$= -4 + 7n - 7$

$= 7n - 11$

$\therefore t_n = 7n - 11$

$t_{18} = 7(18) - 11$

$= 126 - 11$

$= 115$

44. (a) $a = 4$, $t_n = 169$, $d = 5$

$t_n = a + (n-1)d$

$169 = 4 + (n-1)(5)$

$169 = 4 + 5n - 5$

$169 = -1 + 5n$

$170 = 5n$

$n = 34$

$\therefore$ There are 34 terms

(b) $a = 19$, $d = -8$, $t_n = -229$

$t_n = a + (n-1)d$

$-229 = 19 + (n-1)(-8)$

$-229 = 19 - 8n + 8$

$-229 = 27 - 8n$

$-256 = -8n$

$n = 32$

$\therefore$ There are 32 terms

45. (a) $a=1991 \quad d=4$

$t_n = 1991 + (n-1)(4)$
$= 1991 + 4n - 4$
$= 4n + 1987$

$t_{35} = 4(35) + 1987$
$= 140 + 1987$
$= 2127$

∴ The 35^{th} tournament will be held in 2127

46. (a) $a=27 \quad r=\frac{1}{3}$

$t_n = ar^{n-1}$
$= 27\left(\frac{1}{3}\right)^{n-1}$
$= 27(3)^{-n+1}$
$= 27(3)^{1-n}$
$t_6 = 27(3)^{1-6}$
$= 27(3)^{-5}$
$= \frac{27}{3^5}$
$= \frac{27}{243}$
$= \frac{1}{9}$

(b) $a=1, \ d=-3$

$t_n = ar^{n-1}$
$= 1(-3)^{n-1}$
$= (-3)^{n-1}$

$t_7 = (-3)^{7-1}$
$= (-3)^6$
$= 729$

47, 48 - see pg 33***

49. (a) $a=1, \ r=2, \ t_n=1024$

$t_n = ar^{n-1}$
$1024 = 1(2)^{n-1}$
$2^{10} = 2^{n-1}$
$10 = n-1$
$n = 11$

Geometric

(b) $a=-5 \quad d=3 \quad t_n=133$

$t_n = a+(n-1)d$
$133 = -5 + (n-1)(3)$
$138 = 3(n-1)$
$\frac{138}{3} = (n-1)$
$46 = n-1$
$n = 47$

Arithmetic

if you try arithmetic

$a = 16384 \quad d = -12288 \quad t_n = 1$
$t_n = a+(n-1)(d)$
$1 = 16384 + (n-1)(-12288)$
$\frac{1 - 16384}{-12288} = \frac{(n-1)(-12288)}{-12288}$
$n-1 = 1.33$ so not arithmetic $\{n \in \mathbb{N}\}$

*really should have more terms to determine that it is Geometric.

(c) $a=16384 \quad r=\frac{1}{4} \quad t_n=1$

$t_n = ar^{n-1}$
$1 = 16384\left(\frac{1}{4}\right)^{n-1}$
$\frac{1}{16384} = \left(\frac{1}{4}\right)^{n-1}$
$16384 = 4^{n-1}$ take reciprocal of both sides.
$4^7 = 4^{n-1}$
$n-1 = 7$
$\boxed{n=8}$

Geometric

OR $a=1, \ t_n=16384, \ r=4$

$ar^{n-1} = t_n$
$1(4)^{n-1} = 16384$
$4^{n-1} = 4^7$
$n-1 = 7$
$n = 8$ Geometric.

47.(a) $t_n = ar^{n-1}$

$t_4 = 24$
$24 = ar^{4-1}$

$t_6 = 96$
$96 = ar^{6-1}$

$$\div \frac{96 = ar^5}{24 = ar^3}$$
$4 = r^2$
$r = \pm 2$

$\therefore a = 3, r = 2, t_n = 3(2)^{n-1}$
or $a = -3, r = -2, t_n = (-3)(-2)^{n-1}$

If $r = 2$, $24 = a(2)^3$
$24 = a(8)$
$a = 3$

If $r = -2$, $24 = a(-2)^3$
$24 = -8a$
$a = -3$

(b) $t_n = ar^{n-1}$

$t_2 = -6$
$-6 = ar^{2-1}$

$t_5 = -162$
$-162 = ar^{5-1}$

$$\div \frac{-162 = ar^4}{-6 = ar^1}$$
$27 = r^3$
$r = 3$

When $r = 3$, $-6 = a(3)$
$a = -2$
$\therefore a = -2, r = 3,$
$t_n = (-2)(3)^{n-1}$

48.(a) $t_1 = 3, t_2 = 3, t_n = t_{n-1} + t_{n-2}$

$t_3 = t_2 + t_1 = 3 + 3 = 6$
$t_4 = t_3 + t_2 = 6 + 3 = 9$
$t_5 = t_4 + t_3 = 9 + 6 = 15$

$\therefore 3, 3, 6, 9, 15$

(b) $f(1) = 8 \quad f(n) = 0.5f(n-1)$

$f(2) = 0.5f(1) = 0.5(8) = 4$
$f(3) = 0.5f(2) = 0.5(4) = 2$
$f(4) = 0.5f(3) = 0.5(2) = 1$
$f(5) = 0.5f(4) = 0.5(1) = \frac{1}{2}$

$\therefore 8, 4, 2, 1, \frac{1}{2}$

49. see page 32 ★★

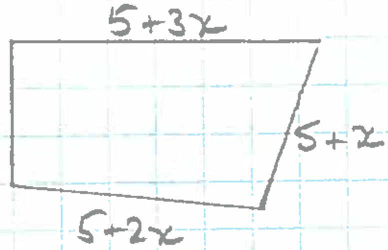
50. (a) $a = -20, d = 2, n = 25$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{25}{2}[2(-20) + (25-1)(2)]$
$= 12.5(-40 + 48)$
$= 12.5(8)$
$= 100$

use t_n formula to find n.

(b) $a = 1 \quad t_n = 20 \quad$ need n!
$20 = 1 + (n-1)(\frac{1}{4})$
$20 = 1 + (\frac{1}{4})n - \frac{1}{4}$
$20 = \frac{3}{4} + \frac{1}{4}n$
$\frac{77}{4} = \frac{1}{4}n$
$77 = n$

$S_{77} = \frac{77}{2}(1 + 20)$
$= \frac{77}{2}(21)$
$= 808.5$

51.



$38 = 5 + (5+x) + (5+2x) + (5+3x)$

$38 = 20 + 6x$

$18 = 6x$

$x = 3$

$\therefore$ 5, 8, 11, 14 cm are the lengths

52. (a) $a = 4 \quad r = -2$

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$S_{12} = \dfrac{4[(-2)^{12} - 1]}{-2-1}$

$= \dfrac{16380}{-3}$

$= -5460$

(b) $a = 3645 \quad r = -\frac{1}{3}$ need n!

$t_n = ar^{n-1}$

$5 = 3645\left(-\frac{1}{3}\right)^{n-1}$

$\dfrac{5}{3645} = \left(-\dfrac{1}{3}\right)^{n-1}$

$\dfrac{1}{729} = \left(-\dfrac{1}{3}\right)^{n-1}$

$\left(-\dfrac{1}{3}\right)^6 = \left(-\dfrac{1}{3}\right)^{n-1}$

* Can put the negative on the left to match bases because of the even exponent

$6 = n - 1$

$n = 7$

$S_7 = \dfrac{3645\left[\left(-\frac{1}{3}\right)^7 - 1\right]}{-\frac{1}{3} - 1}$

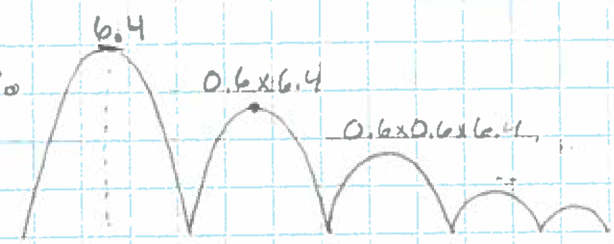
$= \left[3645\left[-\dfrac{1}{2187} - 1\right]\right] \div \left[-\dfrac{1}{3} - 1\right]$

$= \left[3645\left[-\dfrac{2188}{2187}\right]\right] \times \left[-\dfrac{3}{4}\right]$

$= -\dfrac{7975260}{2187} \times \left(-\dfrac{3}{4}\right)$

$= 2735$

53.



$2(6.4) + 2(6.4)(0.6) + 2(6.4)(0.6)^2 + \dots$

$a = 2(6.4) \quad r = 0.6 \quad n = 5$

$S_n = \dfrac{2(6.4)[0.6^5 - 1]}{0.6 - 1}$

$= \dfrac{12.8(-0.92224)}{-0.4}$

$\doteq 29.5$

$\therefore$ The ball has travelled about 29.5 m vertically.

Unit 8

54(a) $P = 2200$
$i = \frac{0.12}{12} = 0.01$
$n = 5 \times 12 = 60$
$A = P(1+i)^n$
$= 2200(1.01)^{60}$
$= 3996.73$

(b) $P = 12600$
$i = 0.0675/4 = 0.016875$
$n = 4 \times 4 = 16$
$A = P(1+i)^n$
$= 12600(1.016875)^{16}$
$= 16468.41$

55. $P = 4500$
$i = \frac{0.0525}{4} = 0.013125$
$n = 3 \times 4 = 12$

$A = 4500(1.013125)^{12}$
$= 5262.22$

∴ He will have \$5262.22

56.(a) $A = 9000$
$i = 0.056/2 = 0.028$
$n = 5 \times 2 = 10$
$PV = \frac{A}{(1+i)^n}$
$= \frac{9000}{(1.028)^{10}}$
$= 9000(1.028)^{-10}$
$= 6828.28$

(b) $A = 250{,}000$
$i = 0.0875/4 = 0.021875$
$n = 1 \times 4 = 4$
$PV = \frac{250000}{(1.021875)^4}$
$= 250000(1.021875)^{-4}$
$= 229270.85$

57. (a) $R = 200$
$i = 0.029/12$
$n = 8$
$A = \frac{R[(1+i)^n - 1]}{i}$
$= \frac{200[(1+0.029 \div 12)^8 - 1]}{(0.029 \div 12)}$
$\doteq 1613.60$

(b) $R = 400$
$i = 0.029/12$
$n = 8$
$A = \frac{R[(1+i)^n - 1]}{i}$
$= \frac{400[(1.002416667)^8 - 1]}{0.002416667}$
$\doteq 3222.20$

58. $A = 5000$
$i = 0.071/4$
$n = 3 \times 4 = 12$

$PV = A(1+i)^{-n}$
$= 5000(1.01775)^{-12}$
$= 4048.34$

∴ They need to deposit \$4048.34

59(a) $R = 4000$
$i = 0.07/2 = 0.035$
$n = 5 \times 2 = 10$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$
$$= \frac{4000[1-(1.035)^{-10}]}{0.035}$$
$$= 33266.42$$

∴ He will need to invest \$33266.42

(b) $I = \text{Total Withdrawls} - P$
$I = (4000 \times 10) - 33266.4$
$I = \$6733.58$
∴ \$6733.58 in interest is earned over the life of the loan

60. $P = 50000$
$i = 0.055$
$n = 25$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$
$$50000 = \frac{R[1-(1.055)^{-25}]}{0.055}$$
$$2750 = R(0.737766296)$$
$$R = 3727.47$$

∴ Each scholarship is worth \$3727.47

61. $R = 10000$
$i = 0.064$
$n = 10$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$
$$= \frac{10000(1-1.064^{-10})}{0.064}$$
$$= 72\,225.92$$

∴ The present value of her winnings is \$72 225.12

62(a) $P = 13500$
$i = \frac{0.039}{12} = 0.00325$
$n = 4 \times 12 = 48$

$$P = \frac{R[1-(1+i)^{-n}]}{i}$$
$$R = \frac{Pi}{[1-(1+i)^{-n}]}$$
$$R = \frac{13500(0.00325)}{[1-(1.00325)^{-48}]}$$
$$R = \$304.21$$

∴ Mrs. Behnke's monthly payments are \$304.21

(b) $\text{Extra} = (48 \times 321.89) - (48 \times 304.21)$
$= 15450.72 - 14602.08$
$= \$848.64$

∴ She would pay \$848.64 extra if she didn't say anything!