Unit 1

Circumference of a circle $C = 2\pi r$, $2r = d$

Area $A_{circle} = \pi r^2$ 

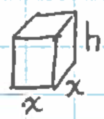
$A_{square} = x^2$ 

$A_{rectangle} = lw$ 

$A_{trapezoid} = \frac{(a+b)h}{2}$ 

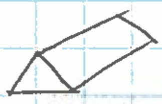
$A_{triangle} = \frac{bh}{2}$ 

Surface Area – calculate the area of each shape on the surface of the 3-D figure then find the sum.

E.g. 

$A_{\text{square-base prism}} = A_{2\,squares} + A_{4\,rectangles}$

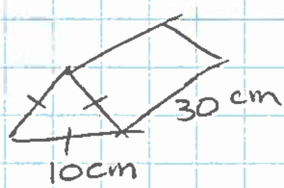
$= 2(x^2) + 4(xh)$

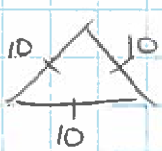
 $A_{\text{triangular prism}} = A_{2\,triangles} + A_{3\,rectangles}$

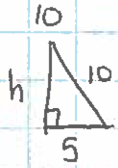
 $A_{cylinder} = 2\pi r^2 + 2\pi r h$

$Volume_{prism} = A_{base} \times height$

$Volume_{cylinder} = \pi r^2 h$

Example 





$h^2 = 10^2 - 5^2$

$h^2 = 100 - 25$

$h = \sqrt{75}$

$h \doteq 8.66\text{ cm}$

$V = A_{base} \times height$

$= \frac{bh}{2} \times H$

$= \frac{10(8.66)}{2} \times 30$

$= 1299\text{ cm}^3$

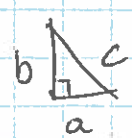
$A_{surface} = A_{2\Delta's} + A_{3\square}$

$= 2\left(\frac{10(8.66)}{2}\right) + 3(10 \times 30)$

$= 86.6 + 900$

$= 986.6\text{ cm}^2$

Pythagorean Theorem $a^2 + b^2 = c^2$

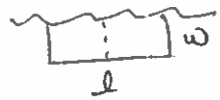
$b^2 = c^2 - a^2$ 

→

Optimizing Perimeter/Area

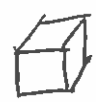
- "Fencing 4-sides" 

  Optimal is a square

- "Fencing 3-sides" 

  Optimal $l = 2w$

Optimizing Surface Area/Volume

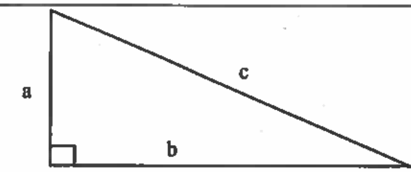
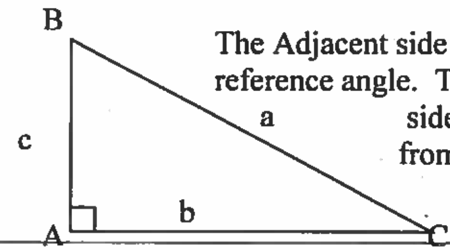
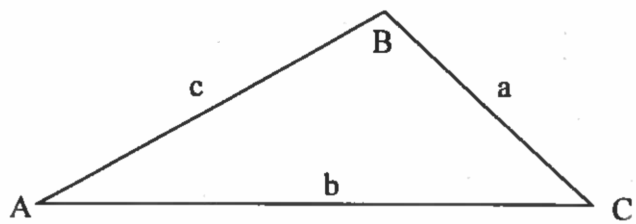
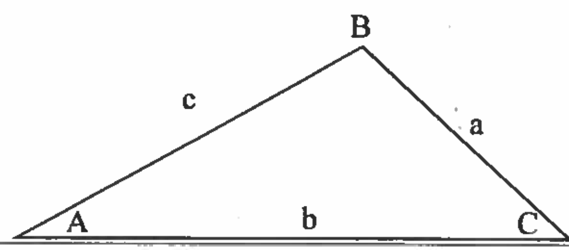
- square-based prism 

  Optimal is a cube

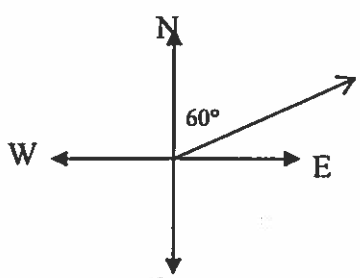
- cylinder 

  Optimal is when $h = d$ OR $h = 2r$.

MAP 4CI Trigonometry Reference Sheet

Formula	Picture	When to use	
Pythagorean $\boxed{a^2 + b^2 = c^2}$		Right angle triangle - given 2 sides	- asked to find third side
Trig Ratios SOHCAHTOA $sin\theta = \frac{O}{H}, cos\theta = \frac{A}{H}, tan\theta = \frac{O}{A}$ In standard position, $r = \sqrt{x^2 + y^2}$ $sin\theta = \frac{y}{r}, cos\theta = \frac{x}{r}, tan\theta = \frac{y}{x}$	 The Adjacent side is beside the reference angle. The Opposite side is across ~~the~~ from the reference angle.	Right angle triangle - given two sides - given one side and an angle	- asked to find angle - asked to find side
Sine Law $\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$		No right angle - given two angles and one opposite side - given two sides and one opposite angle	- asked to find other opposite side - asked to find other opposite angle
Cosine Law $\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ or $\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$		No right angle - given two sides & a contained angle - given three sides	- calculate the third side - can calculate angle

Angle of elevation is always measured UP from the HORIZONTAL. Angle of depression always measured DOWN from the HORIZONTAL.



Bearing 060° is the same as N60°E
Bearing is measured clockwise from North.
So a bearing of 200° is the same as S20°W.

MAP 4CI : Review

Day 1: Unit 1 - Trig

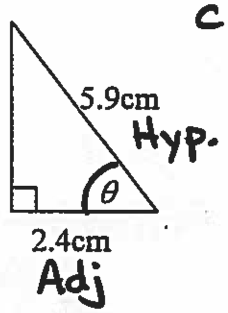
Sample Problems

* BE SURE CALCULATOR IS IN DEGREE MODE

SOHCAHTOA

1) Find the unknown in each triangle.

a. CAH

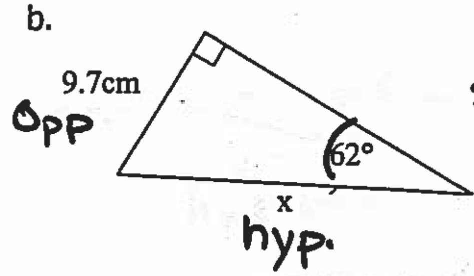


$$\cos\theta = \frac{2.4}{5.9}$$

$$\theta = \cos^{-1}(2.4 \div 5.9)$$

$$\boxed{\theta \doteq 66.0^\circ}$$

b. SOH



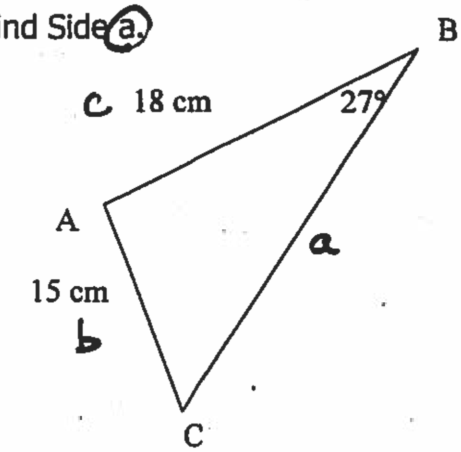
$$\frac{\sin 62^\circ}{1} \times \frac{9.7}{x}$$

$$(\sin 62^\circ)x = 9.7$$

$$x = \frac{9.7}{\sin 62^\circ}$$

$$\boxed{x \doteq 11.0\text{ cm}}$$

3) Find Side a.



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{18} = \frac{\sin 27^\circ}{15}$$

$$\sin C = \frac{18 \times \sin 27^\circ}{15}$$

$$C = \sin^{-1}(0.544788\ldots)$$

$$\angle C \doteq 33.0^\circ$$

So, $\angle A = 180^\circ - 33.0^\circ - 27^\circ$

$A = 120^\circ$

Now, use sine law OR cosine law to find side a

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 120^\circ} = \frac{15}{\sin 27^\circ}$$

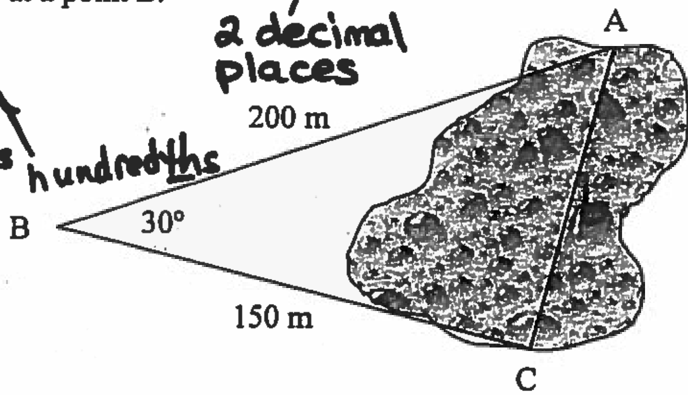
$a = 15 \times \sin 120^\circ \div \sin 27^\circ$

$a \doteq 28.6$ cm

4) Find the width, to the nearest hundredth, of a small lake if the lengths 200 m and 150 m contain an angle of 30° at a point B.

2 decimal places

tenths hundredths



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 150^2 + 200^2 - 2(150)(200)\cos 30^\circ$

$\sqrt{b^2} = \sqrt{10538.47577}$

$b \doteq 102.66$ m

Determine all possible values for angle, θ, given

a) $\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = 45^\circ}$

b) $\sin\theta = 0.866$

$\theta = \sin^{-1}(0.866)$

$\boxed{\theta \doteq 60^\circ}$

or $\boxed{\theta \doteq 120^\circ}$

c) $\tan\theta = -2.5$

$\theta \doteq -68^\circ$ x

$\theta = 180^\circ - 68^\circ$

$\boxed{\theta = 112^\circ}$

Answers:

1. a. 66°, **b.** 11.0cm

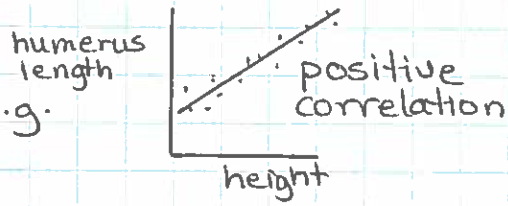
2. 3.7 cm

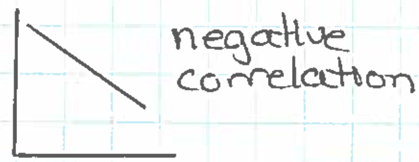
3. 28.6 cm

4. 102.66m

MAP4C1 Unit 3 Two-Variable Statistics

E.g.

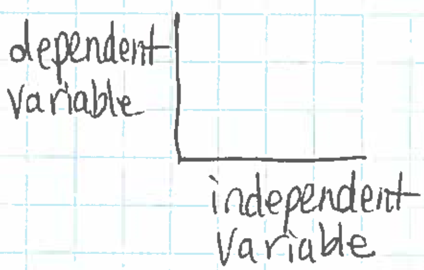




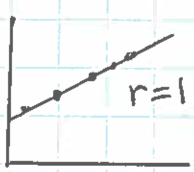
Relationship between variables

As <u>indep. variable</u> increases, the <u>dep. variable</u> $\left\{ \begin{array}{l} \text{increases} \\ \text{decreases} \end{array} \right.$

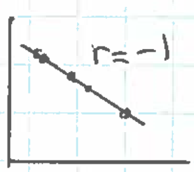
E.g. As the height increases, the humerus length increases.



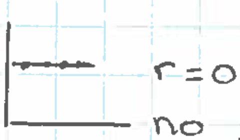
Correlation Coefficient

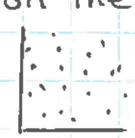


positive correlation
every point on the line.



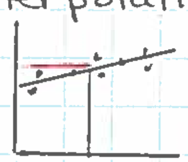
negative correlation
every point on the line.



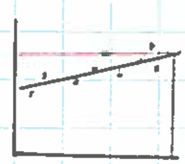


no correlation.

Interpolation



Extrapolation



Unit 4 Data Management

Quartiles – Q3 same as 75% percentile

Percentiles

Percentile Rank $p = \frac{(L + 0.5E)}{n} \times 100$

where p is the percentile rank
L is the number of scores less than the value
E is the number of scores equal to the value
n is the number of pieces of data.

To find score in the p^{th} percentile,
Calculate $n \times p \div 100$
↳ bump that up to the next whole number
Count up that score number from the lowest score.

E.g.

Data
12
17
17
18
25
27
29

What number is in the 80^{th} percentile?
$n \times p \div 100$, $n = 7$, $p = 80$
$7 \times 80 \div 100$
$= 5.6 \rightarrow$ bump up to 6
The 6^{th} lowest piece of data is 27
∴ 27 is in the 80^{th} percentile.

25 has what percentile rank →

$P = \frac{4 + 0.5(1)}{7} \times 100$

$P = \frac{4.5}{7} \times 100$

$P \doteq 64$ ∴ 25 is in the 64^{th} percentile

Percent Change $= \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100\%$

Statistical Indices → base value set at 100%. All other values are written as a percentage of base value.
→ used to make it easier to compare other values to the base value.

Accurate to within ☐ percentage points 19 times out of 20.
E.g. 10% of students like exams – accurate to within 3 percentage points 19 times out of 20.
Means: if the study were repeated several times, 95% of the time between 7% and 13% of students would say they like exams.

- BIAS
- CRITICAL ANALYSIS ① Is there bias in sample? ② Is the author an independent researcher ③ What is the data source? ④ Data still relevent?

Unit 5 - Graphical Models

- Given graph, be able to determine if rate of change (slope) is increasing, decreasing, constant and not zero, or zero.
  (RATE of CHANGE is the SLOPE)

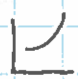
- Calculate the slope / rate of change

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- First Differences, meaning
- Second Differences, meaning
- Ratio Column, meaning

x	y	Ratio (use y-values)
1	12	
2	24	$24 \div 12 = 2$
3	48	$48 \div 24 = 2$
4	96	$96 \div 48 = 2$

- because ratio column is all the same, graph is exponential
- because > 1, graph is exponential growth



- rate of change (slope) is increasing (graph is getting steeper).

1. **Multiplication Law:** $x^m \times x^n = x^{m+n}$

When multiplying powers with the same base, keep the base the same and add the exponents.

Ex 1. $x^3 \times x^2$ (Note: $x^3 \times x^2 = x \cdot x \cdot x \cdot x \cdot x$) Ex. 2 $2^3 \times 2^4$

$= x^{3+2}$

$= x^5$

$= 2^{3+4}$

$= 2^7$

$= 128$

2. **Division Law:** $x^m \div x^n = x^{m-n}$

When dividing powers with the same base, keep the base the same and subtract the exponents.

Ex. 1 $x^5 \div x^2$

$= x^{5-2}$

$= x^3$

Ex. 2 $2^4 \div 2^3$

$= 2^{4-3}$

$= 2^1$

Note: $2^4 \div 2^3$

$= \dfrac{2^4}{2^3}$

$= \dfrac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$

$= 2$

3. **Power of a Power Law:** $(x^m)^n = x^{m \times n}$

If a power is raised to an exponent, multiply the exponents.

Ex. $(x^3)^2 = x^{3\times 2}$

$= x^6$

NOTE: $(x^3)^2 = (x^3)(x^3)$

$= x^{3+3}$

$= x^{3\times 2}$

$= x^6$

4. **Power of a Product Law:** $(x \cdot y)^m = x^m y^m$

If a Product is raised to an exponent, distribute the exponent to each factor in the base.
NOTE: This rule does NOT apply to the power of a sum or difference!

Ex. 1 $(x \cdot y)^5 = x^5 y^5$

Ex. 2 $(3x^5y^3)^2$

$= (3)^2(x^5)^2(y^3)^2$

$= 9x^{10}y^6$

5. **Power of a Quotient Law:** $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

If a Quotient is raised to an exponent, distribute the exponent to every factor in the numerator and denominator.

Ex. 1 $\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$
$= \frac{x^2}{y^2}$

Ex. 2 $\left(\frac{2}{3}\right)^2$
$= \frac{2^2}{3^2}$
$= \frac{4}{9}$

Ex. 3 $\left(\frac{2x^3}{3y^2}\right)^3 = \frac{(2)^3(x^3)^3}{(3)^3(y^2)^3}$
$= \left(\frac{8x^9}{27y^6}\right)$

6. **Zero Exponents:** $x^0 = 1$

Any power with an exponent of zero is equal to one.

Ex. 1 $(-2)^0 = 1$

Ex. 2 $-2^0 = -(2^0)$
$= -1$

Ex. 3 $(-237x^3y^7)^0 = 1$

Proof:

$3^2 \div 3^2$	$3^2 \div 3^2$
$= \frac{3 \times 3}{3 \times 3}$	$= 3^{2-2}$
$= \frac{9}{9}$	$= 3^0$
$= 1$	So, $3^0 = 1$

7. **Negative Exponents:** $x^{-m} = \frac{1}{x^m}$

A negative in the exponent of a power means to 'flip the base' or 'take the reciprocal'. A negative exponent has nothing to do with the sign of the number.

Ex. 1 $x^{-2} = \frac{1}{x^2}$

Ex. 2 $4^{-2} = \frac{1}{4^2}$

Ex. 3 $\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$
$= \frac{5^3}{4^3}$
$= \frac{125}{64}$

Ex. 4 $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2$
$= 3^2$
$= 9$

Ex. 4 Simplify first, then evaluate using x = 2.

$(x^{-3})(x^2)(x^5)$
$= x^{-3+2+5}$
$= x^4$

When x = 2,
$= 2^4$
$= 16$

8. Powers of the form: $x^{\frac{1}{n}}$

The exponent $\frac{1}{n}$ means to take the n[th] root. i.e. $x^{\frac{1}{n}} = \sqrt[n]{x}$

Ex 1. $x^{\frac{1}{2}}$ $= \sqrt[2]{x}$ $= \sqrt{x}$

Ex. 2 $x^{\frac{1}{3}}$ $= \sqrt[3]{x}$

Ex. 3 $x^{\frac{1}{12}}$ $= \sqrt[12]{x}$

Ex. 4 $81^{\frac{1}{2}}$ $= \sqrt{81}$ $= 9$

must write as a radical first then evaluate →

Ex. 5 $(-27)^{\frac{1}{3}}$ $= \sqrt[3]{-27}$ $= -3$

Ex. 6 $(-64)^{\frac{1}{4}}$ $= \sqrt[4]{-64}$ = not possible

Ex 7. $64^{1/3}$ $= \sqrt[3]{64}$ $= 4$

Ex 8. $64^{1/6}$ $= \sqrt[6]{64}$ $= 2$

(You may not take the even root of a negative number)

+ students may need help using their calculators to calculate 4th root.

9. Powers of the form: $x^{\frac{m}{n}}$

The exponent $\frac{m}{n}$ means to take the n[th] root and raise the answer to an exponent m.

i.e. $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{\left(x^m\right)}$

Ex 1. $x^{\frac{3}{4}}$ $= \sqrt[4]{x^3}$ Or $= \left(\sqrt[4]{x}\right)^3$

Ex. 2 $x^{\frac{2}{3}}$ $= \sqrt[3]{x^2}$ Or $= \left(\sqrt[3]{x}\right)^2$

Ex. 3 $81^{\frac{3}{4}}$ $= (\sqrt[4]{81})^3$ $= (3)^3$ $= 27$

Ex. 4 $(-125)^{\frac{2}{3}}$ $= \left(\sqrt[3]{-125}\right)^2$ $= (-5)^2$ $= 25$

Day 3 : Unit 6 : Algebraic Models

Algebraic Models

A. Simplifying / Evaluating Exponents

1. Simplify (remember – no negative exponents)

a. $\frac{y^{-1}}{y^{-2}}$

$= y^{-1-(-2)}$

$= y^{-1+2}$

$= y$

b. $x^{-1}(x^{-3})^{-2}x^{-7}$

$= x^{-1} \cdot x^{6} \cdot x^{-7}$

$= x^{-1+6+(-7)}$

$= x^{-2}$

$= \frac{1}{x^2}$

c. $\frac{t}{v}\left(\frac{v}{t}\right)^{-3} v^4$

$= \frac{t}{v}\left(\frac{t}{v}\right)^3 v^4$

$= \frac{t}{v} \times \frac{t^3}{v^3} \times v^4$

$= \frac{t^4 \cancel{v^4}}{\cancel{v^4}} = t^4$

2. Convert to Radical Form $x^{\frac{4}{3}}$

$= \left(x^{\frac{1}{3}}\right)^4$

$= \left(\sqrt[3]{x}\right)^4$

3. Convert to Exponent Form $\sqrt[4]{\frac{1}{x^3}}$

$= \left(\frac{1}{x^3}\right)^{\frac{1}{4}}$

$= \left(x^{-3}\right)^{\frac{1}{4}}$

$= x^{-\frac{3}{4}}$

4. Evaluate

a. $16^{\frac{1}{2}}$

$= \sqrt{16}$

$= 4$

b. $16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

c. $(-27)^{\frac{1}{3}}$

$= \sqrt[3]{-27}$

$= -3$

d. $\left(\frac{1}{9}\right)^{\frac{3}{2}}$

$= \left(\sqrt{\frac{1}{9}}\right)^3$

$= \left(\frac{1}{3}\right)^3$

$= \frac{1}{27}$

B. Exponential Equations

5. Solve for the unknown. Express with a common base, if possible. Otherwise use systematic trial.

a. $4^x = 8$

$2^{2x} = 2^3$

since bases are common exponents are equal

$2x = 3$

$x = \frac{3}{2}$

b. $81^{\frac{x}{2}} = 243^{x+1}$

$(3^4)^{\frac{x}{2}} = (3^5)^{x+1}$

$4\left(\frac{x}{2}\right) = 5(x+1)$

$2x = 5x + 5$

$2x - 5x = 5$

$-3x = 5$

$x = -\frac{5}{3}$

c. $4^x = 40$

try $x = 3$ $\quad 4^3 = 64$

try $x = 2.5$ $\quad 4^{2.5} = 32$

try $x = 2.7$ $\quad 4^{2.7} \doteq 42.2$

try $x = 2.66$ $\quad 4^{2.66} \doteq 39.9$

$\therefore x = 2.66$

HW: Pg. 495 #25, 26bdf, 27acd, 29, 30ab, 32, 33

✓ Answers Pg. 571

Unit 7 Finance.

Simple Interest

$I = Prt$

$I \rightarrow$ interest (\$)
$P \rightarrow$ principal (present value) (\$).
$r \rightarrow$ annual interest (%)
$t \rightarrow$ time (years)

Compound Interest

$A = P(1+i)^n$

$P = A(1+i)^{-n}$

$A \rightarrow$ Amount (future value) \$
$P \rightarrow$ Principal (\$)
$i \rightarrow$ interest rate per compounding period as a decimal

- calculate percents

Ex. Putting 12% of pay into savings.
If pay is \$900 savings is

$0.12 \times 900 = \$108.$

bi-weekly = 26 times/year
weekly = 52 times/year
monthly = 12 times/year.