MPM1DI Unit 9 Lesson 8 (9.5/9.6) Name: Complete Lesson

Date______________

Optimization of a Cylinder

Investigation A: How can you compare the volumes of cylinders with the same surface area?

Many products come in cylinders. Your task is to design a cylindrical juice can that uses no more than 375 cm^2 of aluminum. The can should have the greatest capacity possible.

1. To investigate the volume of the cylinder as its radius changes, you will need an expression for the height in terms of the radius, given that the surface area is 375 cm^2.

$SA = 2\pi r^2 + 2\pi rh$

$2\pi rh = SA - 2\pi r^2$

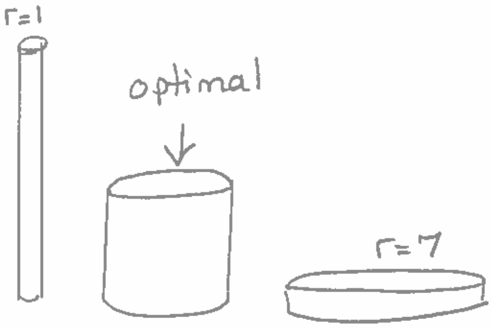
$h = \dfrac{SA - 2\pi r^2}{2\pi r}$

$h = \dfrac{375 - 2\pi r^2}{2\pi r}$

2. Complete the table below by calculating the height and volume of each cylinder. $V = \pi r^2 h$

d	Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
2	1	58.7	184	375
4	2	27.8	349	375
6	3	16.9	478	375
8	4	10.9	548	375
10	5	6.9	542	375
12	6	3.9	441	375
14	7	1.5	231	375

$\pi r^2 h$



3. **REFLECT: Summarize your Findings**

a) What is the maximum volume for the cans in your table? And what are the radius and height of the can with the volume? Max. volume is 548 cm^3 when $r=4$ ($d=8$), $h=10.9$

b) What relationship do you notice between the radius and height? closest to $d=h$

c) Do these dimensions give the optimal volume for the surface area of 375 cm^2? How could you extend your investigation to determine the dimensions of a can with a volume greater than the value in the table? How can you solve for the dimensions algebraically?

No. Include decimal radii

Set $h=2r$

substitute $h=2r$

$\boxed{A_{total} = 2\pi r^2 + 2\pi r(2r)}$

$A_{total} = 2\pi r^2 + 4\pi r^2$

$A_{total} = 6\pi r^2$

Isolate r in $SA = 6\pi r^2$

$\dfrac{A_{total}}{6\pi} = \dfrac{6\pi r^2}{6\pi}$

$\sqrt{\dfrac{A_{TOTAL}}{6\pi}} = \sqrt{r^2}$, $r>0$

$r = \sqrt{[A_{TOTAL} \div (6\pi)]}$

Example 1 Maximize the Volume of a Cylinder

a) Determine the dimensions of the cylinder with maximum volume that can be made with 600 cm^2 of aluminum. Round the dimensions to the nearest hundredth of a centimetre.

SA

$r = \sqrt{\dfrac{600}{6\pi}}$

quick way if you remember $r = \sqrt{SA \div 6\pi}$

$r \doteq 5.64$

$h \doteq 11.28$ (recall: $h=2r$)

longer way if you forget

$2\pi r^2 + 2\pi r(2r) = 600$

$2\pi r^2 + 4\pi r^2 = 600$

$6\pi r^2 = 600$

$r^2 = \dfrac{600}{6\pi}$

$r = \sqrt{\dfrac{100}{\pi}}$, $r>0$

$r \doteq 5.64$, $h = 2r$

so $h \doteq 11.28$

$\therefore$ the optimal cylinder has $r = 5.64$cm, $h = 11.28$cm

b) What is the volume of this cylinder, to the nearest cubic centimetre?

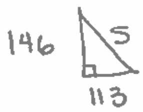
$V = \pi r^2 h$

$V = \pi(5.64)^2(11.28)$

$V = 1127$ cm^3

4. **Pyramid of Khafre in Egypt is a square based pyramid that reaches a height of 146 m and its square base has side lengths of 226 m.**

a) Calculate the surface area of the lateral faces of this pyramid. Round your answer to one decimal place. **[3 marks]**



$s^2 = 146^2 + 113^2$

$s^2 = 34085$

$s = \sqrt{34085}$

$s \doteq 184.6212339$ ✓

$A_{\text{lateral faces}} = A_{4\Delta\text{'s}}$

$= 2bs$ ✓

$= 2(226)(\sqrt{34085})$

$= 83448.79771\ldots$

$\doteq 83448.8$ ✓

$\therefore$ the area of the lateral faces is $83448.8\ m^2$.

b) What is the volume of this pyramid? [2 marks]

$V = \frac{b^2h}{3}$ ✓

$V = 226^2(146) \div 3$

$= 2\ 485\ 698.667$ ✓

$\doteq 2\ 485\ 698.7$

$\therefore$ the volume of the pyramid is $2485698.7\ m^2$

5. **The surface area of sphere A is double the surface area of sphere B. The radius of sphere B is 5 cm.**

a) **What is the surface area of each sphere? [3 marks]** $A_{\text{sphere}} = 4\pi r^2$

$A_{\text{sphere B}} = 4\pi(5)^2$ ✓

$= 100\pi$

$\doteq 314.2$ ✓

$A_{\text{sphere A}} = 2A_{\text{sphere B}}$

$= 200\pi$

$\doteq 628.3$ ✓

$\therefore$ the surface area of sphere A is 628.3cm^2, sphere B is 314.2 cm^2.

b) **What is the radius of sphere A? [3 marks]**

Option 1:

doubling A means

$2(4\pi r^2)$

$= 4\pi(2r^2)$

$= 4\pi(\sqrt{2}r)^2$

So, sphere A radius is $\sqrt{2}(5)$

$= 7.07\ldots$

$\boxed{\doteq 7.1\text{cm}}$

Option 2:

$4\pi r^2 = 200\pi$ ✓

$r^2 = 50$ ✓

$r = \sqrt{50}, r > 0$

$r \doteq 7.1\text{ cm}$ ✓

6. **Two square-based prisms both have a volume of 729 cm^3. Prism A has a base area of 9 cm^2, prism B has a base area of 81 cm^2.**

a) **Find the height of each prism. [4 marks]** $\text{Prism} = b^2h$ or $A_{\text{base}} \times h$

Prism A

$b^2h = 729$ ✓

$9h = 729$

$h = 81$ ✓

Prism B

$b^2h = 729$ ✓

$81h = 729$

$h = 9$ ✓

$\therefore$ Prism A has a height of 81cm, Prism B has a height of 9cm

b) **Is either of these prisms optimal (the minimum surface area) for the given volume? Explain. [2 marks]**

Prism B is optimal ✓ It is a cube ✓ 9cm × 9cm × 9cm.