

Optimization of a Cylinder**Investigation A: How can you compare the volumes of cylinders with the same surface area?**

Many products come in cylinders. Your task is to design a cylindrical juice can that uses no more than 375 cm^2 of aluminum. The can should have the greatest capacity possible.

- To investigate the volume of the cylinder as its radius changes, you will need an expression for the height in terms of the radius, given that the surface area is 375 cm^2 .
- Complete the table below by calculating the height and volume of each cylinder.

| Radius (cm) | Height (cm) | Volume (cm^3) | Surface Area (cm^2) |
|-------------|-------------|--------------------------|--------------------------------|
| 1 | | | 375 |
| 2 | | | 375 |
| 3 | | | 375 |
| 4 | | | 375 |
| 5 | | | 375 |
| 6 | | | 375 |
| 7 | | | 375 |

3. REFLECT: Summarize your Findings

- What is the maximum volume for the cans in your table? And what are the radius and height of the can with the volume?
- What relationship do you notice between the radius and height?
- Do these dimensions give the optimal volume for the surface area of 375 cm^2 ? How could you extend your investigation to determine the dimensions of a can with a volume greater than the value in the table? How can you solve for the dimensions algebraically?

Example 1 Maximize the Volume of a Cylinder

- Determine the dimensions of the cylinder with maximum volume that can be made with 600 cm^2 of aluminum. Round the dimensions to the nearest hundredth of a centimetre.

- What is the volume of this cylinder, to the nearest cubic centimetre?

Investigation B: How can you compare the surface areas of cylinders with the same volume?

Your task is to design a cylindrical juice can that must hold 1000mL of juice.

1. To investigate the surface area of the cylinder as its radius changes, you will need an expression for the height in terms of the radius, given that the volume is 1000mL.

2. Complete the table.

| Radius (cm) | Height (cm) | Volume ($1\text{cm}^3=1\text{mL}$) | Surface Area (cm^2) |
|-------------|-------------|--------------------------------------|--------------------------------|
| 1 | | 1000 | |
| 2 | | 1000 | |
| 3 | | 1000 | |
| 4 | | 1000 | |
| 5 | | 1000 | |
| 6 | | 1000 | |
| 7 | | 1000 | |

REFLECT: Summarize your Findings

- a) Describe the dimensions of the cylinder with the least surface area. Are these dimensions the optimal ones?
- b) Given a specific volume, how could you determine the optimal dimensions of a cylinder algebraically?

Example 2 Minimize the Surface Area of a Cylinder

- a) Determine the least amount of aluminum required to construct a cylindrical container with a 4L capacity, to the nearest tenth of a square centimetre.
- b) Describe any assumptions you made.