 Name:

Date

Optimization of a Square Based Prism

Investigation A: How can you compare the surface areas of square-based prisms with the same volume?

1. Use 16 interlocking cubes to build as many different square-based prisms as possible with a volume of 16 cubic units.
2. Calculate the surface area of each prism. Record your results in a table.

Length	Width	Height	Volume (u^3)	Surface Area (u^2)
4	4	1	16	$2(16)+4(4)$ $=48$
1	1	16	16	$2(1)+4(16)$ $=66$
2	2	4	16	$2(4)+4(8)$ $=40$

3. What are the dimensions of the square-based prism that has the minimum, or optimal, surface area?

2 units x 2 units x 4 units

4. Describe the shape of this prism compared to the other prisms.

closest to a cube

5. Predict the dimensions of the square-based prism with minimum surface area if you use:

a) 27 cubes $3\times3\times3$ b) 64 cubes $4\times4\times4$ c) 125 cubes $5\times5\times5$

6. **REFLECT:** Summarize your findings.

a) Do any relationships exist between the length, width, and height of a square-based prism with minimum surface area for a given volume? $l=w=h$

b) What is the ideal shape for minimizing the surface area of a square-based prism when given a fixed volume? a cube

c) How can you predict the dimensions of a square-based prism with minimum surface area if you know the volume? * take the cubed root $V=x^3$ so, $x=\sqrt[3]{V}$

EX. 1. Cardboard Box Dimensions.

a) The Pop-a-Lot popcorn company ships kernels of popcorn to movie theatres in large cardboard boxes with a volume of 500,000 cm^3. Determine the dimensions of the square-based prism box, to the nearest tenth of a centimeter, the will require the least amount of cardboard.

~~$x=\sqrt[3]{500}$~~

$V=500000$, $V=x^3$

$x^3=500\,000$

so, $x=\sqrt[3]{500\,000}$

$x=79.4$ cm

$\therefore$ a cube with sidelengths 79.4 cm is optimal.

b) Find the amount of cardboard required to make this box, to the nearest tenth of a square centimetre. Describe any assumptions you have made.

$A_{TOTAL}=A_{6\,squares}$

$=6x^2$

$=6(79.4)^2$

$=37\,826.16$

$\doteq 37\,826.2$

Assume: no extra needed for seams

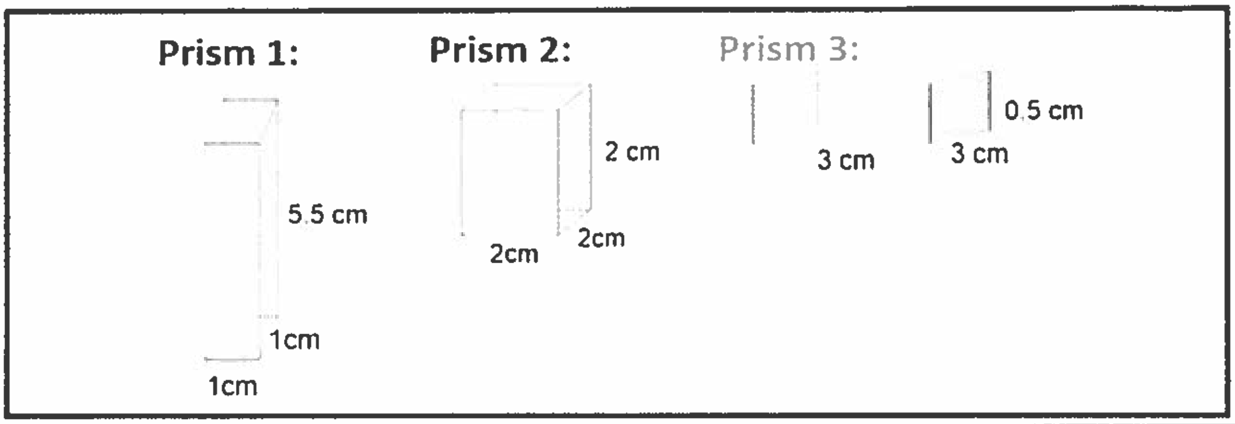
no waste

etc.

$\therefore$ the surface area is $37\,826.2\ cm^2$

Investigation B: How can you compare the volumes of square-based prisms with the same surface area?

1. Each of the square-based prisms below has a surface area of 24 cm^2. Calculate the area of the base and the volume of each prism. Record your data in the table.



Prism Number	Side length of base (cm)	Area of base (cm^2)	Surface area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	24	5.5	5.5
2	2	2	24	2	8
3	3	3	24	0.5	4.5

2. What are the dimensions of the square-based prism that has the maximum, or optimal, volume?

$2 \times 2 \times 2$

3. Describe the shape of this prism compared to the other prisms. a cube

4. Predict the dimensions of the square-based prism with maximum volume if the surface area is 54 cm^2.

$6x^2 = 54$
$x^2 = 9$ → $x = 3, x > 0$

recall: $A_{total\ cube} = 6x^2$

5. **REFLECT:** Summarize your findings.

a) Do any relationships exist between the length, width, and height of a square-based prism with maximum volume for a given surface area? $l = w = h$

b) What is the ideal shape for maximizing the volume of a square-based prism when given a fixed surface area? a cube

c) How can you predict the dimensions of a square-based prism with maximum volume if you know the surface area? $A_{total} = 6x^2$, solve for x.

EX. 2. Maximize the Volume of a Square-Based Prism

a) Determine the dimensions of the square-based prism with maximum volume that can be formed using 5400 cm^2 of cardboard.

$A_{total} = A_{6\ squares}$
$5400 = 6x^2$ →

$6x^2 = 5400$
$x^2 = 900$
$x = \sqrt{900}, x > 0$
$x = 30$ cm

∴ the cube should have side lengths 30cm to maximize volume.

b) What is the volume of the prism?

$V = x^3$
$V = 30^3$
$V = 27000 cm^3$

∴ the max. volume is $27000 cm^3$

Pg 495 #2, 3, 5a, 7 & Pg 501 #2, 3, 6, 7