

**Warm Up: QUIZ on Surface Area and Volume of Prisms, Pyramids, Cylinders and Surface Area of Cones.**

### Volume of Cones

A cylinder is a three dimensional solid with identical parallel circular bases. The lateral surface is curved and extends from one base to the other base.

The volume of a cylinder is the same as a prism:  $V = A_{\text{base}} \times \text{height}$   
or  $V =$

Similar to the relationship between the pyramid and the prism, the volume of a cone is one third the volume of a prism with the same radius and height.



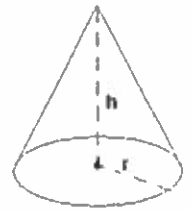
Volume of a cone  $= \frac{1}{3} A_{\text{base}} \times \text{height}$

or,  $V_{\text{cone}} = \frac{1}{3} \pi r^2 h$  or  $\frac{\pi r^2 h}{3}$

**Example 1:** Calculate the volume of a cone with radius 3 mm and height 7 mm.

$$\begin{aligned} V &= \frac{\pi r^2 h}{3} \\ V &= \frac{\pi (3)^2 (7)}{3} \\ &= 21\pi \\ &= 65.973... \end{aligned}$$

$\approx 66 \text{ mm}^3$



**Example 2:**

a) If the height of a cone is tripled, does this triple the volume? Explain.

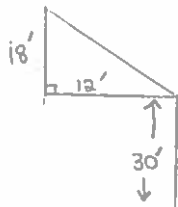
$$V_{\text{cone}} = \frac{\pi r^2 h}{3} \leftarrow \times 3$$

Multiplying the right side of the equation by 3 will result in triple the value on the left side.

b) If the radius of a cone is tripled, does this triple the volume? Explain.

$$V_{\text{cone}} = \frac{\pi r^2 h}{3} \leftarrow \times 3 \text{ means } V \text{ will be } 3^2 = 9 \text{ times larger}$$

**Example 3:** A grain bin has a radius of 12 ft and a height of 48 ft. How much grain will the farmer need to order to fill the bin? (Note: 1 kg of grain fills 1 ft<sup>3</sup> of space. Also, assume grain (oats) is ordered in tonnes (1 metric ton = 1000kg).) (Note: the cone portion has a height of 18 feet)



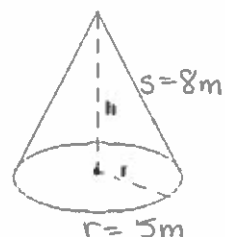
$$\begin{aligned}
 V_{\text{total}} &= V_{\text{cylinder}} + V_{\text{cone}} \\
 &= \pi r^2 h + \frac{\pi r^2 h}{3} \\
 &= \pi (12)^2 (30) + \frac{\pi (12)^2 (18)}{3} \\
 &= 13571.68 + 2714.33 \\
 &= 16286 \text{ ft}^3 \\
 &= 16286 \text{ kg} \\
 &= 16.2 \text{ tonnes}
 \end{aligned}$$

$\therefore$  the farmer must order 16.2 tonnes of grain.

**Example 4:** A conical pile of sand has a base diameter of 10 m and a slant height of 8 m. Determine the volume of the sand in the pile, to the nearest cubic metre.

$$\begin{aligned}
 V &= \frac{\pi r^2 h}{3} \\
 &= \frac{\pi (5)^2 (\sqrt{39})}{3} \\
 &= 163.49366... \\
 &\approx 163 \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 h^2 &= s^2 - r^2 \\
 h^2 &= 8^2 - 5^2 \\
 h^2 &= 39 \\
 h &= \sqrt{39}, h > 0
 \end{aligned}$$



$\therefore$  the volume of sand is about 163 m<sup>3</sup>.

**Example 5:** A fountain firework is packaged in a conical container. Its volume is 210 m<sup>3</sup>. Its diameter is 8 cm. What is the height of the fountain firework, to the nearest tenth of a centimeter?



$$\begin{aligned}
 V &= \frac{\pi r^2 h}{3} \\
 210 &= \frac{\pi (4)^2 h}{3} \\
 210 \times \frac{3}{16\pi} &= h \\
 h &= 12.533... \\
 h &\approx 12.5
 \end{aligned}$$

$\therefore$  the height is 12.5 cm.