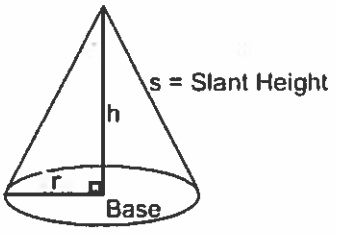
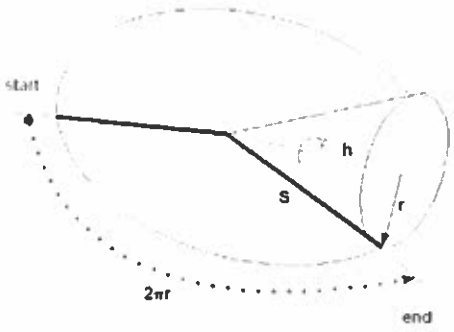
MPM 1DI Unit 9 lesson 3 (8.4) Name:_____

Surface Area of Cones

A *cone* is a three dimensional solid with a circular base. The lateral surface is curved and extends from the base to a point called the vertex.



Developing a formula for surface area of a cone:

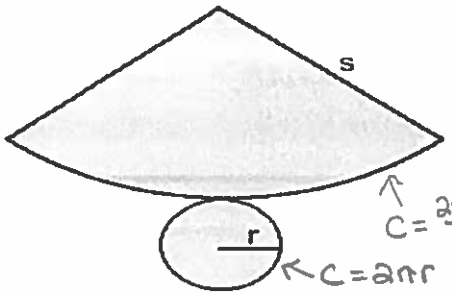


The lateral surface is a circle-sector. This sector is some fraction (one n^{th}) of a circle with radius s.

$$A_{sector} = \frac{\pi s^2}{n}$$

The circumference of the sector is one n^{th} of the circumference of the whole circle with radius s.

$$C_{sector} = \frac{2\pi s}{n}$$



Since the circumference of the sector wraps around the circumference of the base (which is a circle with radius r), $C_{sector} = C_{base\ circle}$

$\frac{2\pi s}{n} = 2\pi r$ ↓ ÷2π

$\frac{s}{n} = r$

Substituting this into $A_{sector} = \frac{\pi s^2}{n}$, we get $A_{lateral\ side} = \frac{\pi s \cdot s}{n} = \pi s(r) = \pi r s$

So, the formula for **Surface area of a cone** is:

$A_{total} = A_{base} + A_{lateral\ side}$
$= \pi r^2 + \pi r s$

Example 1: Calculate the surface area of a waffle cone (before it is filled with ice cream) with height 4.2 cm and radius 1.8 cm.

* note: waffle cone has no "circular base"

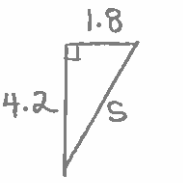
$SA = \pi r s$ ← just the lateral surface

$= \pi (1.8)(\sqrt{20.88})$

$= 25.8397...$

$\doteq 25.8$

∴ the surface area is $25.8\ cm^2$



$s^2 = 1.8^2 + 4.2^2$

$s^2 = 20.88$

$s = \sqrt{20.88}, s > 0$

 Name:_____

Example 2: The slant height of a cone is tripled. Does this triple the surface area of the cone? Explain.

$SA_{cone} = \pi r^2 + \pi r s$ ↑ ×3

the lateral surface area will be tripled but the full surface area will not be since the area of the base is not tripled.

Example 3: A cone is formed from a circle with a 90° sector removed. Another cone is formed from a semicircle with the same radius. How do the two cones differ? How are they the same?

Both cones have the same slant height.

taller with smaller circular base

shorter with larger circular base

Example 4: The lateral area of a cone with slant height 14 cm is 132 cm².

a) Find the radius of the cone, to the nearest cm.

$\pi r s = SA_{\text{lateral surface}}$

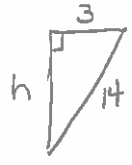
$\pi r(14) = 132$

$r = \frac{132}{14\pi}$

$r = 3.0012\ldots$

$r \doteq 3$ cm

b) Find the height of the cone, to the nearest .tenth of a cm.



$h^2 = 14^2 - 3^2$

$h^2 = 187$

$h = \sqrt{187}$, $h > 0$

$h = 13.67\ldots$

$h \doteq 13.7$ cm

Example 5: An old construction pylon needs to be painted. The base the pylon sits on is 20cm by 20 cm by 1.5 cm, the radius of the cone is 8 cm and the height of the pylon is 31 cm. If only the part that shows is to be painted, find the surface area to be painted. to the nearest hundredth.

$SA = \left(A_{4\ \square's} + A_{\square} - A_{\bigcirc}\right) + \left(A_{\text{lateral surface}}\right)$

$= 4(20 \times 1.5) + 20(20) - \pi(8)^2 + \pi(8)(\sqrt{934.25})$

$= 120 + 400 - 201.061 + 768.194$

$\doteq 1087.13$

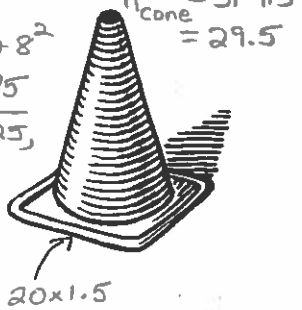
$\therefore$ 1087.13cm² needs to be painted.

$h_{cone} = 31 - 1.5 = 29.5$

$s^2 = 29.5^2 + 8^2$

$s^2 = 934.25$

$s = \sqrt{934.25}$, $s > 0$



* Assumption: cone comes to a point (it is not a