Volume and Surface Area of Prisms and Pyramids

Polyhedron: A three-dimensional object with faces that are polygons.

Prism:

A prism is a three-dimensional solid (a polyhedron). The top and bottom (the bases) are parallel, identical polygons. The lateral faces are rectangles; they meet the bases at right angles. A prism are named by the shape of its bases, for example, rectangular prism, triangular prism, square-based prism.

Volume of any Prism:

$V = A_{base} \times height$

Surface Area:

$A_{total} = 2 \times A_{base} + A_{rectangles}$

Pyramid:

A pyramid is a three-dimensional solid (a polyhedron) with a polygon-shaped base. The remaining sides are triangles that come to a point at the top.

https://www.youtube.com/watch?v=qXC8uzy_HFw

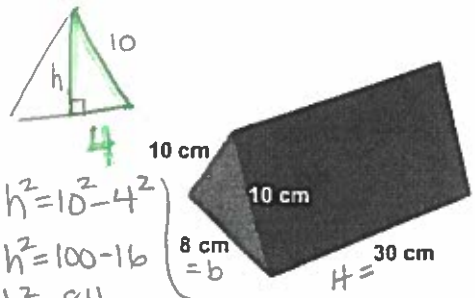
Volume of any Pyramid:

$V = \frac{1}{3}(A_{base} \times height)$ OR $\frac{A_{base} \times Height}{3}$

Surface Area:

$A_{total} = A_{base} + A_{triangles}$

Example 1: Calculate the volume and the surface area of the following triangular-based prism.



$h^2 = 10^2 - 4^2$

$h^2 = 100 - 16$

$h^2 = 84$

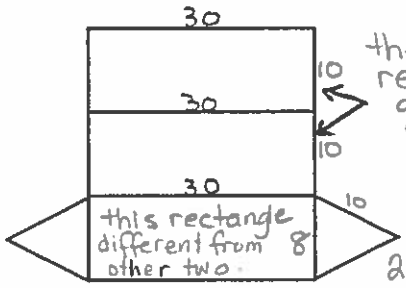
$h = \sqrt{84}, h > 0$

$V = A_{base} \times Height$

$= \frac{bh}{2} \times H$

$= \frac{8(\sqrt{84})}{2} \times 30$

$\doteq 1099.8$



$SA = A_{rectangles} + 2A_{triangle}$

$= 2 \times 10(30) + 8(30) + 2\frac{bh}{2}$

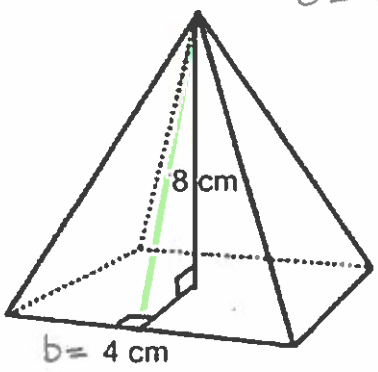
$= 600 + 240 + 8(\sqrt{84})$

$\doteq 840 + 806.5$

$= 1646.5$

$\therefore$ the volume is $1099.8 cm^3$, the surface area is $1646.5 cm^2$

Example 2: Calculate the volume of the following square-based pyramid.



$b = 4\text{ cm}$ height of pyramid $= 8\text{ cm}$

$V = A_{base} \times H$

$= b^2 \times H$

$= (4)^2 \times 8$

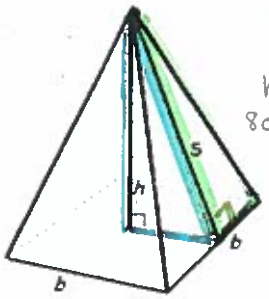
$= 128$

recall: $A_{square} = b^2$

$\therefore$ the pyramid has a volume of 128 cm^3.

Example 3: Calculate the surface area of the square-based pyramid in example 2.

* need the "slant height" – it is the height of the triangles

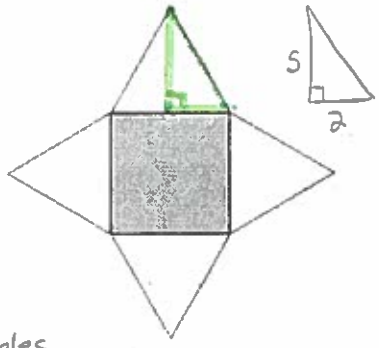


$s^2 = 8^2 + 2^2$

$s^2 = 64 + 4$

$s^2 = 68$

$s = \sqrt{68}, s > 0$



$SA = A_{\text{square base}} + A_{\text{4 triangles}}$

$= b^2 + 4\frac{bs}{2}$

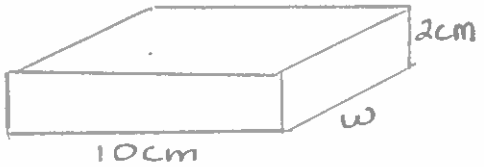
$= b^2 + 2bs$

$= 4^2 + 2(4)(\sqrt{68})$

$= 81.96\ldots$

$SA \doteq 82.0\text{ cm}^2$

Example 4: A box of chocolates has a volume of 80 cm³. If its length is 10 cm and its height is 2 cm, what is its width?



$V = lwh$, $V = 80$, $l = 10$, $w = ?$, $h = 2$

$80 = 10w(2)$

$80 = 20w$

$\frac{80}{20} = \frac{20w}{20}$

$w = 4\text{ cm}$

$\therefore$ the width of the box is 4cm