MCR3UI

Unit 4 lesson 4

Exploring Properties of Exponential Functions

Investigation:

1. Complete the following tables.

i)

x	y=x
0	0
1	1
2	2
3	3
4	4
5	5
6	6

ii)

x	y=2x
0	0
1	2
2	4
3	6
4	8
5	10
6	12

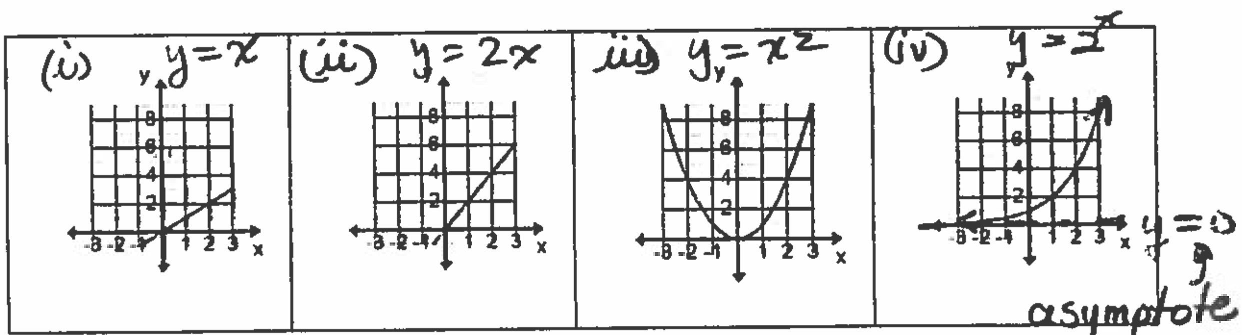
iii)

x	$y=x^2$
0	0
1	1
2	4
3	9
4	16
5	25
6	36

iv)

x	$y=2^x$
0	1
1	2
2	4
3	8
4	16
5	32
6	64

2. Which pattern is growing:
   a) Fastest? $y=2^x$
   b) Slowest? $y=x$



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3. Complete the First and second differences.

x	y=x	First Differences	Second Differences
0	0		
1	1	1-0 1	
2	2	2-1 1	0
3	3	3-2 1	0
4	4	4-3 1	0
5	5	5-4 1	0
6	6	1	0

x	y=2x	First Differences	Second Differences
0	0		
1	2	2-0 2	
2	4	4-2 2	0
3	6	2	0
4	8	2	0
5	10	2	0
6	12	2	0

x	$y=x^2$	First Differences	Second Differences
0	0		
1	1	1-0 1	
2	4	4-1 3	3-1 2
3	9	5	5-3 2
4	16	7	2
5	25	9	2
6	36	11	2

x	$y=2^x$	First Differences	Second Differences
0	1		
1	2	1	
2	4	2	2
3	8	4	4
4	16	8	8
5	32	16	16
6	64	32	common ratio is 2.
		common ratio is 2	

4. What do you notice about the finite differences?

line → first differences all the same, 2nd diff all 0.

parabola → second diff. all the same.

exponential → ~~finite differences always~~ differences always the ratio of consecutive finite differences equals the same common factor.

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$\left(\frac{1}{2}\right)^x$ $\quad -\frac{1}{4} \div -\frac{1}{2}$ $\quad -\frac{1}{4} \times -2 = \frac{1}{2}$

5. Complete the following tables.

i)

x	$y=3^x$	First Differences	Second Differences
0	1		
		2	
1	3		4
		6	
2	9		12
		18	
3	27		36
		54	
4	81	C.R. 3.	CR 3.

ii)

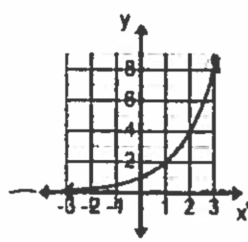
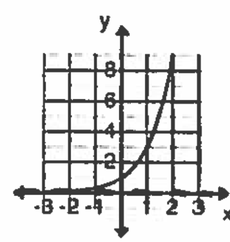
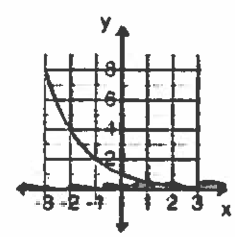
x	$y=0.5^x$	First Differences	Second Differences
0	1		
		$-\frac{1}{2}$	
1	$\frac{1}{2}$		$\frac{1}{4}$
		$-\frac{1}{4}$	
2	$\frac{1}{4}$		$\frac{1}{8}$
		$-\frac{1}{8}$	
3	$\frac{1}{8}$		$\frac{1}{16}$
		$-\frac{1}{16}$	
4	$\frac{1}{16}$	C.R. $\frac{1}{2}$	C.R. $\frac{1}{2}$

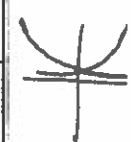
6. How do $y = 3^x$ and $y = 0.5^x$ compare with $y=2^x$?

The base of the exponential function is the same as the common ratio (only because $\Delta x=1$)

$y=3^x$ grows fastest $\quad y=0.5^x$ is decreasing.

7. Complete the following chart.

	$y=2^x$	$y=3^x$	$y=0.5^x$
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R} \mid y>0\}$	$\{y \in \mathbb{R} \mid y>0\}$	$\{y \in \mathbb{R} \mid y>0\}$
x-intercepts?	none	none	none
y-intercept	$y=1$	$y=1$	$y=1$
Interval of increase	$x \in \mathbb{R}$	$x \in \mathbb{R}$	—
Interval of decrease	—	—	$x \in \mathbb{R}$
Description of graph	always increasing	always increasing • grows faster than $y=2^x$	always decreasing • reflection of $y=2^x$ in the y-axis NOTE $2^{-x} = \left(\frac{1}{2}\right)^x$
Sketch of graph	 $y=0$	 $y=0$	 $y=0$
Asymptotes ?	$y=0$	$y=0$	$y=0$



* for intervals always use on which x-values this is happening.

* Interval of increase means 'for what x-values is the graph increasing'.

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8. Sam has $0.50 in his wallet. His mom told him that if he consistently does all of his chores, she will give him double the amount that is in his wallet the first day and each day following, she will give him double the amount she gave him on the previous day.

a) Assuming Sam does his chores consistently, how much money will his mom give him on the fourth day?

1st	2nd	3rd	4th
$1	$2	$4	$8

She will give him $8 on the fourth day.

$$M = 2^{d-1}$$

where M is the amount of money mom gives sam that day, d is the day number

b) Sam is saving up to buy a new $300 graphics card for his computer. On what day can he buy his graphics card?

50¢ + $1 → 1st	$1.50	5th	$15^{50} + 16 = 31^{50}$
$1^{50} + 2$ → 2nd	$3.^{50}$	6th	$31^{50} + 32 = 63^{50}$
$3^{50} + 4$ → 3rd	7^{50}	7th	$63^{50} + 64 = 127^{50}$
$7^{50} + 8$ → 4th	15^{50}	8th	$127^{50} + 128 = 255^{50}$
		9th	$255^{50} + 256 = 511^{50}$

∴ he can buy the graphics card on the 9th day.

Properties of Exponential Functions:

- For b>1: As the independent variable increases by a constant amount, the dependent variable increases by a common factor.
  (As the independent variable increases by one, the dependent variable increases by a common factor equal to the base of the exponential function.)
- For 0<b<1: As the independent variable increases by a constant amount, the dependent variable decreases by a common factor.
  (As the independent variable increases by one, the dependent variable decreases by a common factor equal to the base of the exponential function.)
- The ratio of consecutive finite differences is a constant.
- The graph increases at an increasing rate for bases greater than one. (The slope of the graph keeps getting steeper as x increases for bases greater than one.)
- $b^0 = 1$, for all $b \in \mathbb{R}$, $b \neq 0$

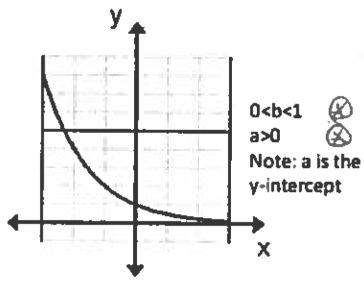
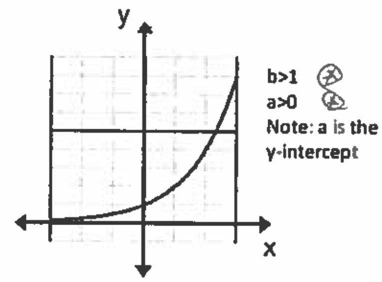
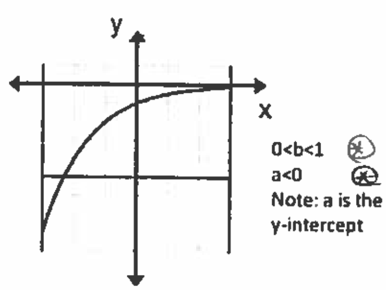
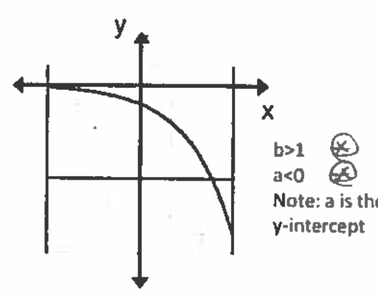
Unit 4 lesson 5

Determining the Equation of an Exponential Function

1. Complete the chart to compare the effect of changing the value of a in $y=a(2^x)$.

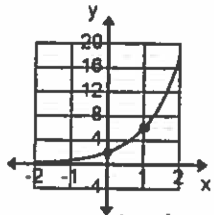
	$f(x)=2^x$	$y=3(2^x)$	$y=0.5(2^x)$	$y=-(2^x)$	$y=-3(2^x)$
Domain	$\{x\in\mathbb{R}\}$	$\{x\in\mathbb{R}\}$	$\{x\in\mathbb{R}\}$	$\{x\in\mathbb{R}\}$	$\{x\in\mathbb{R}\}$
Range	$\{y\in\mathbb{R}\mid y>0\}$	$\{y\in\mathbb{R}\mid y>0\}$	$\{y\in\mathbb{R}\mid y>0\}$	$\{y\in\mathbb{R}\mid y<0\}$	$\{y\in\mathbb{R}\mid y<0\}$
y-intercept	$y=1$	$y=3$	$y=0.5$	$y=-1$	$y=-3$
asymptote	$y=0$	$y=0$	$y=0$	$y=0$	$y=0$
Inc./dec.	increasing	increasing	increasing	decreasing	decreasing

2. Summary: Exponential Equations of the form $y=ab^x$

$0<b<1$	$b>1$
	
Decreasing on $x\epsilon R$ $D=\{x\epsilon R\}$ $R=\{y\epsilon R\mid y>0\}$ Horizontal Asymptote: $y=0$ y-intercept: $y=a$	Increasing on $x\epsilon R$ $D=\{x\epsilon R\}$ $R=\{y\epsilon R\mid y>0\}$ Horizontal Asymptote: $y=0$ y-intercept: $y=a$
	
Increasing on $x\epsilon R$ $D=\{x\epsilon R\}$ $R=\{y\epsilon R\mid y<0\}$ Horizontal Asymptote: $y=0$ y-intercept: $y=a$	Decreasing on $x\epsilon R$ $D=\{x\epsilon R\}$ $R=\{y\epsilon R\mid y<0\}$ Horizontal Asymptote: $y=0$ y-intercept: $y=a$

3. Determine the exponential equation in the form $y=ab^x$, for the given graphs.

a)



$(0, 2), (1, 6)$

$\boxed{a=2}$

x	y
0	2
1	6
2	18

+1, +1; ×3, ×3

so $b=3$

or

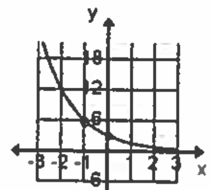
$y = 2b^x$

$6 = 2b^1$

$3 = b$

$\therefore y = 2(3^x)$

b)



$(-1, 6), (0, 3)$

$\boxed{a=3}$

x	y
-1	6
0	3

+1; $\times\frac{1}{2}$

so $b=\frac{1}{2}$

or

$y = 3b^x$

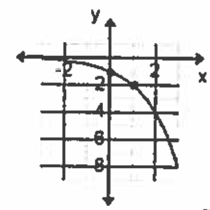
$6 = 3b^{-1}$

$2 = b^{-1}$

$\frac{1}{2} = b$

$\therefore y = 3(\frac{1}{2})^x$

c)



$(0, -1)$ $(1, -2)$

$\boxed{a=-1}$

or

$y = -b^x$

$-2 = -(b)^1$

$-2 = -b$

$2 = b$

$\therefore y = -2^x$ — same as $(-1)(2^x)$ $= -(2^x)$ $\neq (-2)^x$

x	y
0	-1
1	-2

+1; ×2

so $b=2$

4. Write an Exponential Function given its properties.

A radioactive sample has a half-life of 3 days. The initial sample is 200 mg.

a) Write a function to relate the amount remaining, in milligrams, to the time, in days.

initially $P = 200$ $\Big) \times\frac{1}{2}$ (one half-life interval)

after 3 days $P = 100$ $\Big) \times\frac{1}{2}$ (another half-life interval)

after 6 days $P = 50$

⋮

number of half-life intervals	time (days)	Population
0	0	200
1	3	100
2	6	50
3	9	25

$$P = 200\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

b) Restrict the domain of the function so that the mathematical model fits the situation it is describing.

$D = \{t \in \mathbb{R} \mid t \geq 0\}$.

Translations and Reflections of Exponential Functions

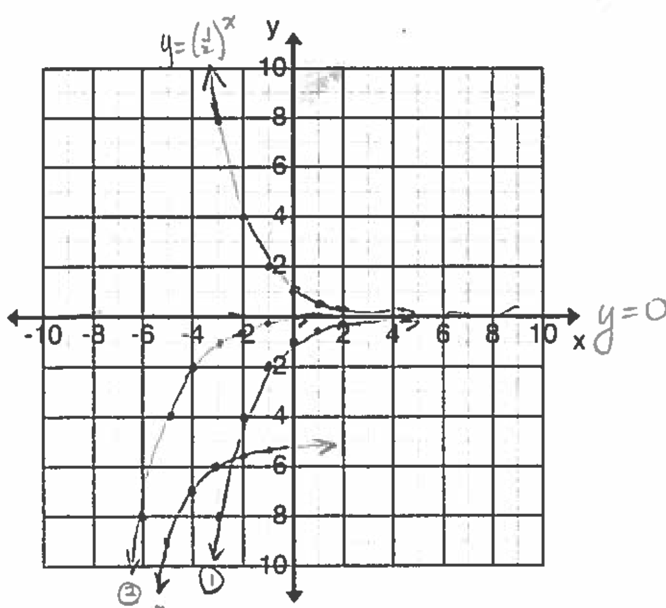
1. Each function given below is a translation and/or reflection of the exponential function $f(x)=3^x$. For each of these transformations, write the equation as a transformation of $f(x)=3^x$ in function notation. Then, describe how $f(x)=3^x$ should be shifted and/or reflected to obtain the new graph of the transformed function.

Function	$y=3^x+1$	$y=3^{x-2}$	$y=3^{(x+4)}$
Function Notation	$y=f(x)+1$	$y=f(x-2)$	$y=f(x+4)$
Description of Transformation	shift up 1	shift right 2	shift left 4

Function	$y=-3^x$	$y=3^{-x}$	$y=-3^{x+3}-1$
Function Notation	$y=-f(x)$	$y=f(-x)$	$y=-f(x+3)-1$
Description of Transformation	reflect in x-axis	reflect in y-axis	reflect in x-axis, shift left 3, down 1

2. Draw the graph of $f(x)=\left(\frac{1}{2}\right)^x$ and the transformation $y=-f(x+3)-5$. What is the equation of the transformed function?

① reflect in x-axis
② shift left 3
③ shift down 5



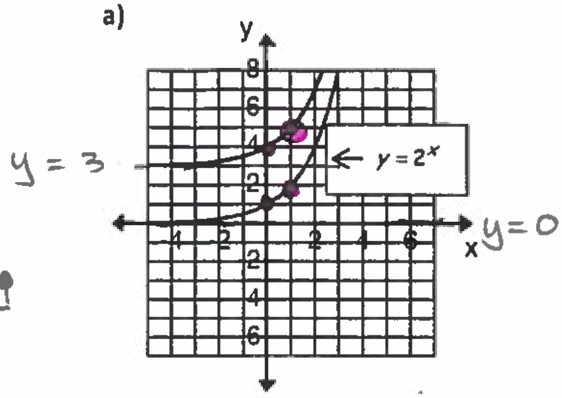
③ $y=-\left(\frac{1}{2}\right)^{x+3}-5$

⊛ Spend time helping them graph $y=\left(\frac{1}{2}\right)^x$ ⊛

x	$y=\left(\frac{1}{2}\right)^x$
-2	$\left(\frac{1}{2}\right)^{-2}=2^2=4$
-1	$\left(\frac{1}{2}\right)^{-1}=2^1=2$
0	$\left(\frac{1}{2}\right)^0=1$
1	$\left(\frac{1}{2}\right)^1=\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2=\frac{1}{2^2}=\frac{1}{4}$

3. Given the original graph $y = 2^x$ and each of the following four transformations, Describe each of the transformations and write the new equation.

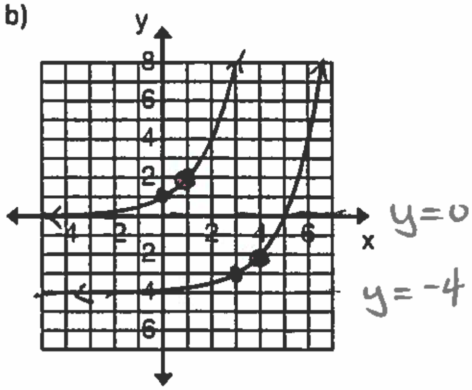
a)



Description: up 3

New Equation: $y = 2^x + 3$

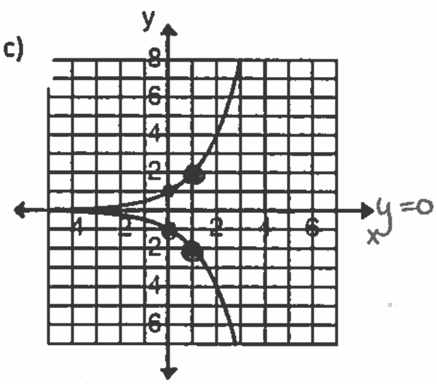
b)



Description: right 3
down 4

New Equation: $y = 2^{(x-3)} - 4$

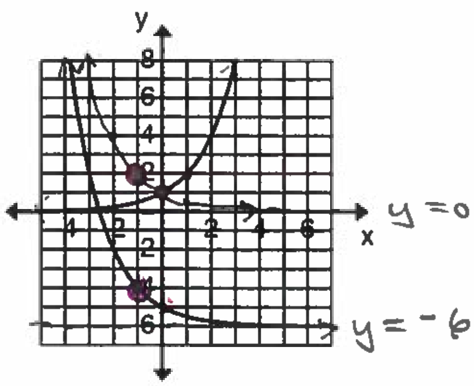
c)



Description: reflect in x-axis

New Equation: $y = -2^x$

d)



Description: reflect in y-axis
shifted down 6

New Equation: $y = 2^{-x} - 6$

General Equation of Exponential Functions:

$$y = ab^{k(x-d)} + c$$

$a < 0 \Rightarrow$ reflection in x-axis

$k < 0 \Rightarrow$ reflection in y-axis

$d \Rightarrow$ shift right/left ($d > 0 \Rightarrow$ right, $d < 0 \Rightarrow$ left)

$c \Rightarrow$ shift up/down ($c > 0 \Rightarrow$ up, $c < 0 \Rightarrow$ down)

* Asymptote y = c ALWAYS

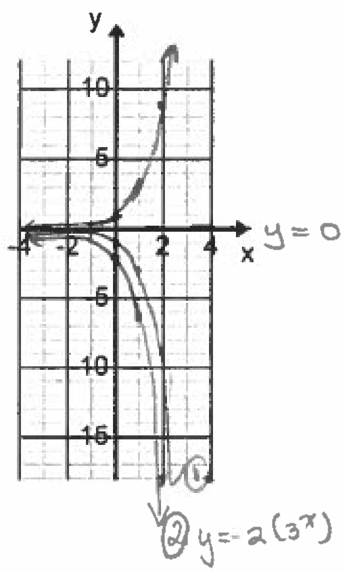
* y-int is not y = a if the graph is shifted.

Transformations of Exponential Functions

1. Take up test on Transformations of Functions. Today's work is just an extension of transformations of functions to Exponential Functions.

2. a) Graph $f(x) = 3^x$ on the grid below.

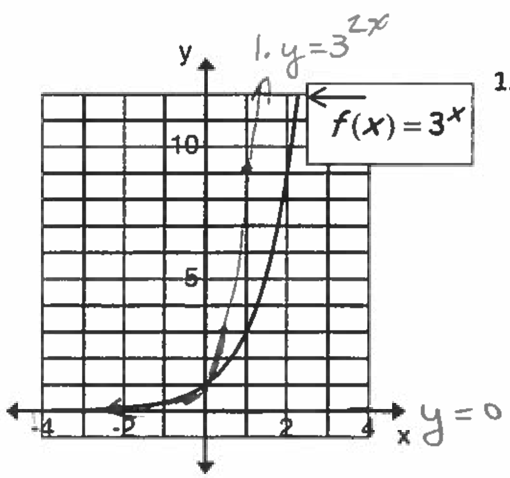
b) On the same set of axes, graph $y = -2(3^x)$. Describe this transformation.



1. reflection in x-axis
2. vertical stretch factor 2.

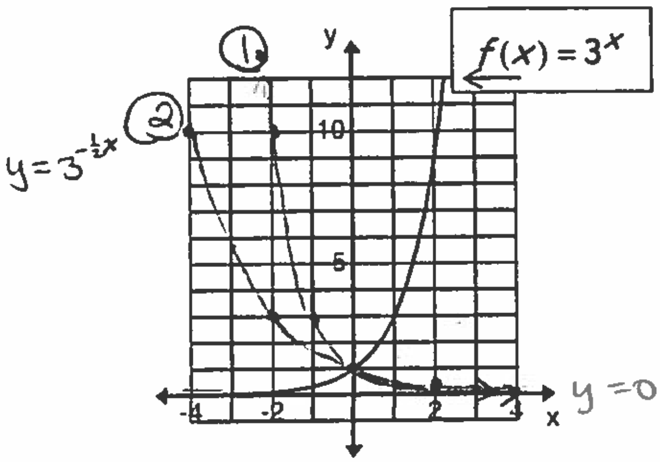
x	$y=3^x$
-2	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

3. Given $f(x) = 3^x$, graph $y = 3^{2x}$ and describe the transformation.



1. Horizontal Compression factor $\frac{1}{2}$

4. Given $f(x) = 3^x$, graph $y = 3^{-\frac{1}{2}x}$ and describe the transformation.



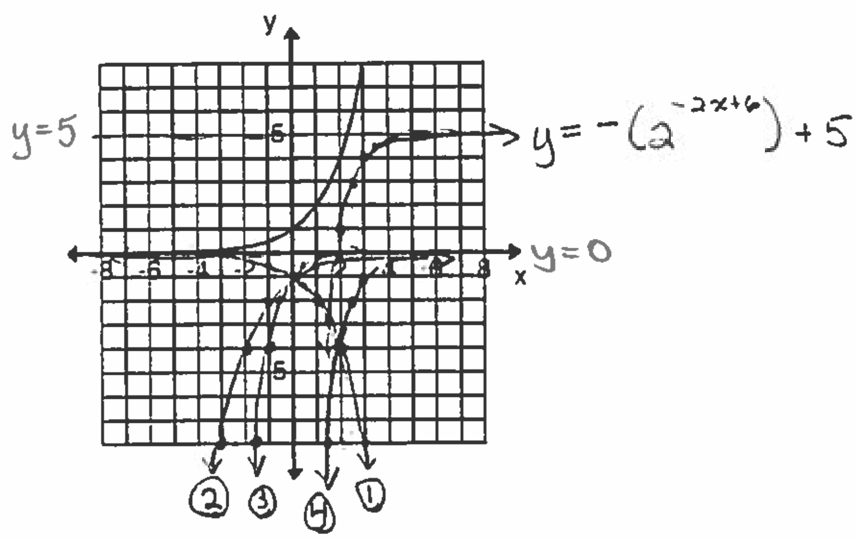
1. reflect in y-axis
2. Horizontal Stretch factor 2.

5. a) Identify the transformations of $f(x) = 2^x$ that will produce the graph of

$y = -f(-2x + 6) + 5$ $\qquad$ $y = -f[-2(x-3)] + 5$

1. reflect in x-axis
2. " " y-axis.
3. horizontal compression factor $\frac{1}{2}$
4. shift right 3
5. Shift up 5

b) Graph the transformation. Label the final graph with its equation.



6. Match each transformation with the corresponding equation, using $f(x)=10^x$ as the base. Not all transformations will match an equation.

Transformation	Equation
a) Horizontal stretch factor 3 ___ $y=10^{\frac{1}{3}x}$	A $y=10^x+3$ b
b) Shift 3 units up A	B $y=10^{x+3}$ c
c) Shift 3 units left B	C $y=-10^x$ g
d) Vertical compression factor $\frac{1}{3}$ G	D $y=10^x-3$ h
e) Vertical stretch factor 3 ___ $y=3(10^x)$	E $y=10^{3x}$ i
f) Shift 3 units right ___ $y=10^{x-3}$	F $y=10^{-x}$ reflect in y-axis
g) Reflect in x-axis C	G $y=\left(\frac{1}{3}\right)10^x$ d.
h) Shift 3 units down D	
i) Horizontal compression factor $\frac{1}{3}$ E	

General Equation of an Exponential Function:

$$y=ab^{k(x-d)}+c$$

$a<0 \Rightarrow$ reflection in x-axis
$k<0 \Rightarrow$ reflection in y-axis

$|a|<1 \Rightarrow$ Vertical Compression factor $|a|$
$|a|>1 \Rightarrow$ Vertical Stretch factor $|a|$

$|k|<1 \Rightarrow$ Horizontal Stretch factor $\frac{1}{|k|}$

$|k|>1 \Rightarrow$ Horizontal Compression factor $\frac{1}{|k|}$

$d \Rightarrow$ shift right/left ($d>0 \Rightarrow$ right, $d<0 \Rightarrow$ left)
$c \Rightarrow$ shift up/down ($c>0 \Rightarrow$ up, $c<0 \Rightarrow$ down)

Unit 4: Exponents and Exponential Functions

Day 8: Application - Growth and Decay Problems

Today we will...

1. Create equations for real-world problems
2. Solve exponential equations

In general, for exponential growth / decay problems:

$$f(x) = a(b)^x$$

where,

$f(x)$ is the final value (y-value, population at time = x)

a is the initial value

b is the growth factor (if it is > 1) or the decay factor (if it is 0 < b < 1)

x is the number of growth or decay periods

Important Notes:

If a growth rate is given as a percent, then the base of the power in the equation (b) can be obtained by adding or subtracting the rate (as a decimal) from 1.

ex. A growth rate of 18% involves multiplying repeatedly by 1.18

Also, the units for the growth and decay rate and for the number of growth and decay periods must be the same!

ex. For a *monthly* interest rate of 0.05%, the growth period needs to be in *months* too!

Growth Problem

1. In 1996, Alberta had a population of 35 000. Recently, it has experienced a large population increase due to the discovery of one of the world's largest oil deposits. Alberta's population has grown at an annual rate of approximately 8%. The algebraic model for this case is $P(n) = P_0(1+r)^n$

a.) What is the initial population? $P_0 = 35\,000$

b.) What is the growth rate, r? $r = 8\% = 0.08 \Rightarrow b = 108\% = 1.08$

c.) Write an algebraic model for this situation using the above information.

$$P(n) = 35000(1.08)^n$$

where $P(n)$ is the population of Alberta as a function of n, the number of years since 1996.

d.) Use the equation to determine the current population. updated values will be used each year

$$\begin{array}{r} 2015 \\ -1996 \\ \hline 19 \end{array}$$

$P(19) = 35000(1.08)^{19}$

$= 151\,049.5371$

$\doteq 151\,049$

$\therefore$ the population would be 151 049.

e.) How long ~~will~~ did it take for the population to triple at this growth rate?

$P(n) = 105000$

35000×3

$105\,000 = 35000(1.08)^n$

$\frac{105\,000}{35000} = 1.08^n$

$3 = 1.08^n$

n	1.08^n
3	1.259...
13	2.72
15	3.17
14	2.94
14.3	3.005

$\therefore n = 14.3$

$\therefore$ it would take about 14.3 years for the population to triple. (or about 14 years 4 mos.)

100% − 50%
= 50%

0.5

Decay Problems

2. A 200g sample of radioactive polonium-210 has a half-life of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount. The mass of polonium in grams, that remains after t days can be modeled by the equation:

$$M(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$$

a. What is the rate of decay? 50% or $\frac{1}{2}$ or 0.5

b. Why is the exponent $\frac{t}{138}$?

$\frac{t}{138}$ is the number of "half-life intervals".

The number of half-life periods is the time divided by the time it takes the sample to halve.

c. Determine the mass that remains after 5 years.

$t = 5$ years (leap year)
$= 5 \times 365 + 1$
$= 1825 + 1$
$= 1826$

$M(1826) = 200(0.5)^{\frac{1826}{138}}$
$\doteq 0.02$

d. How long does it take for this 200g sample to decay to 110g?

$110 = 200(0.5)^{t/138}$

$\frac{110}{200} = 0.5^{\frac{t}{138}}$

$0.55 = 0.5^{\frac{t}{138}}$

$t = 119$

t	$0.5^{\frac{t}{138}}$
120	0.547
119	0.550

∴ it will take 119 days for the sample size to be 110g.

$$f(x) = a(b)^x$$

3. A new car costs $24,000. It *loses* 18% of its value each year after it is purchased. This is called depreciation. * decay

a.) What is the rate of depreciation? 18% = 0.18 . use 100% − 18% = 82% or 0.82 (82% remains)

b.) Write an equation that models the decay/decline of the investment.

$$V(t) = 24000(0.82)^t$$

where $V(t)$ is the value of the car as a function of time in years since the car was purchased.

c.) Use the equation to determine the value of the automobile after 6 *years.*

$$V(6) = 24000(0.82)^6$$
$$\doteq 7296.16$$

d.) Use the equation to determine the value of the automobile after 30 *months.*

$t = 30$ months

$t = \frac{30}{12}$ years

$t = 2.5$ years

$$V = 24000(0.82)^{2.5}$$
$$\doteq 14613.22$$

$\therefore$ after 30 months, the car is worth $14613.22.

Chromebooks.
+ E-video cart

Unit 4: Exponents and Exponential Functions

Day 9 - Exponential Functions and Regression

Today we will...

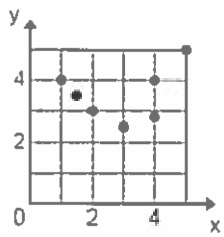
- use technology to determine a model that best fits given data

MCR 3UI - U4 - D9 - Exponential Regression

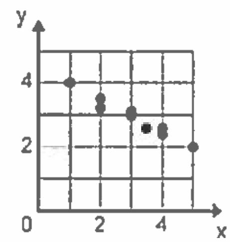
Choosing a Function to Model a Set of Data

1. What type of function do you believe would best model the data below?

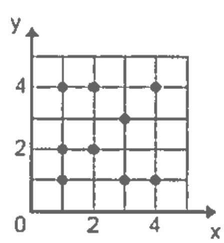
a)



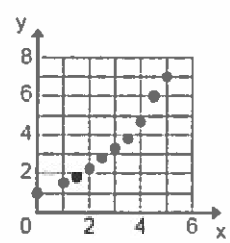
b)



c)



d)



MCR 3UI - U4 - D9 - Exponential Regression

Determining an Equation Using Regression

Given the following points, determine the equation of the function:

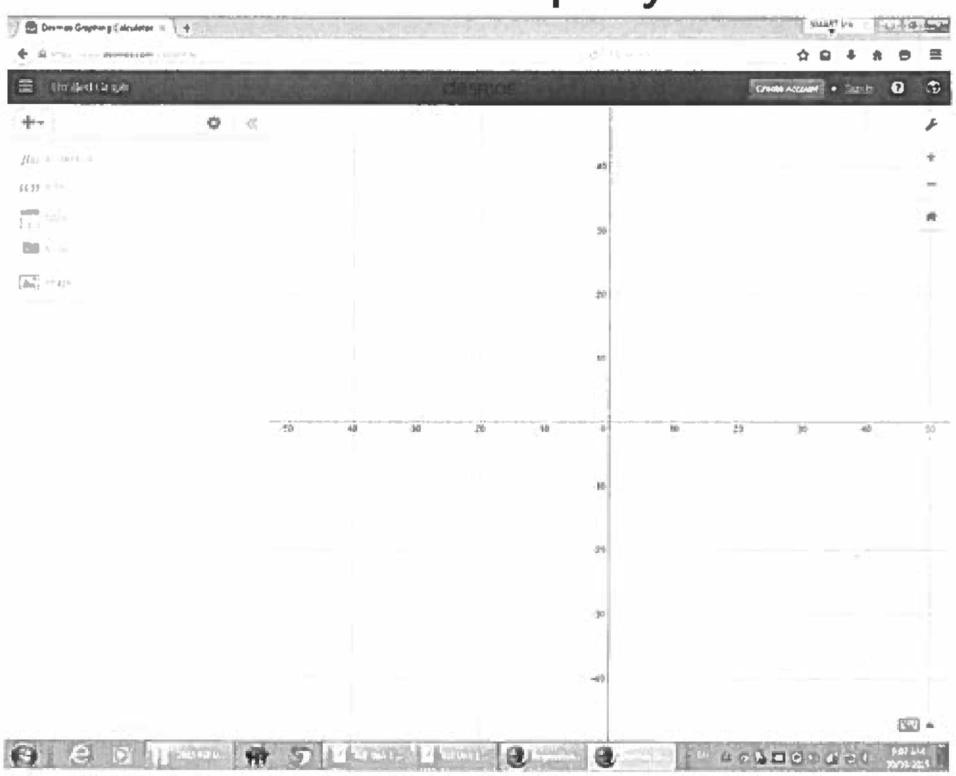
x	y
0	10
2	5.6
4	3.2
6	1.8
8	1
10	0.6
12	0.3

**Need to use REGRESSION

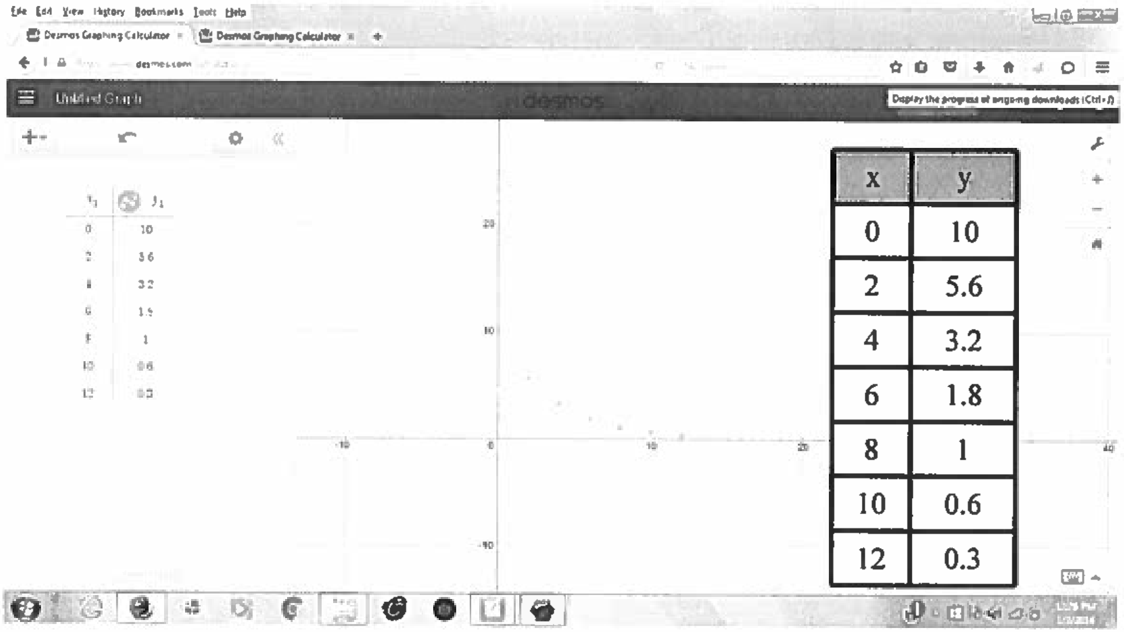
There are software programs to do this for us...

And DESMOS works pretty nicely...

Steps: 1. Add a table to input your data:



2. Enter the data in the table:



x	y
0	10
2	5.6
4	3.2
6	1.8
8	1
10	0.6
12	0.3

3. Add a new equation that is the 'general form' for the type of function we think this would be:

ie. Linear $y_1 \sim mx_1 + b$

Quadratic $y_1 \sim a(x_1 - h)^2 + k$

Exponential $y_1 \sim ab^{k(x_1 - h)} + c$

* type y1 * the computer will write it as y_1

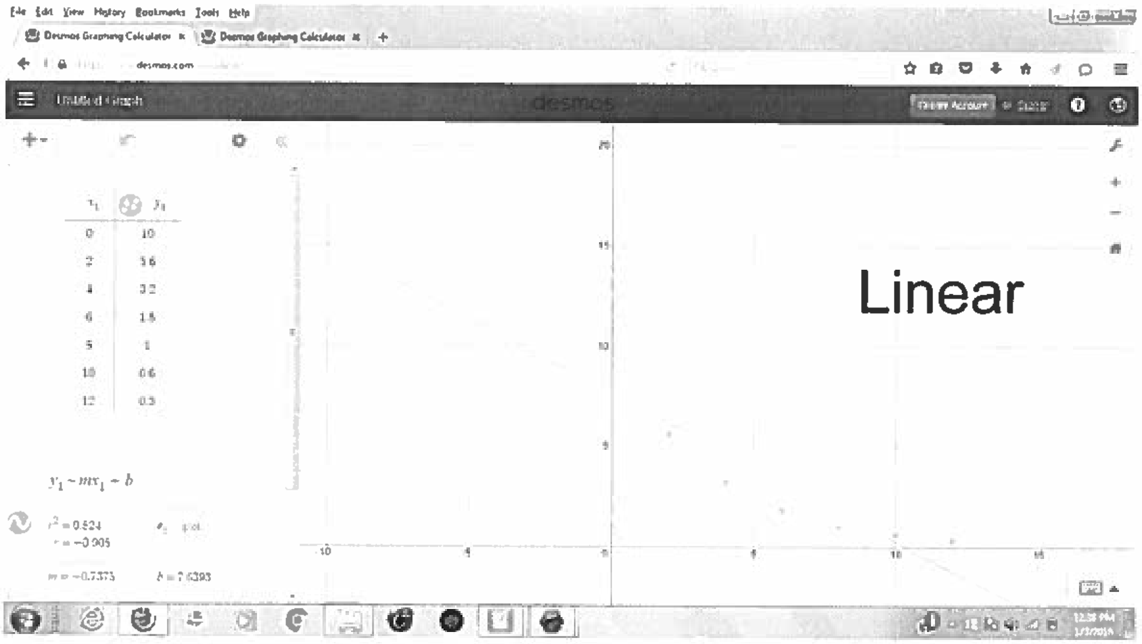
except use:

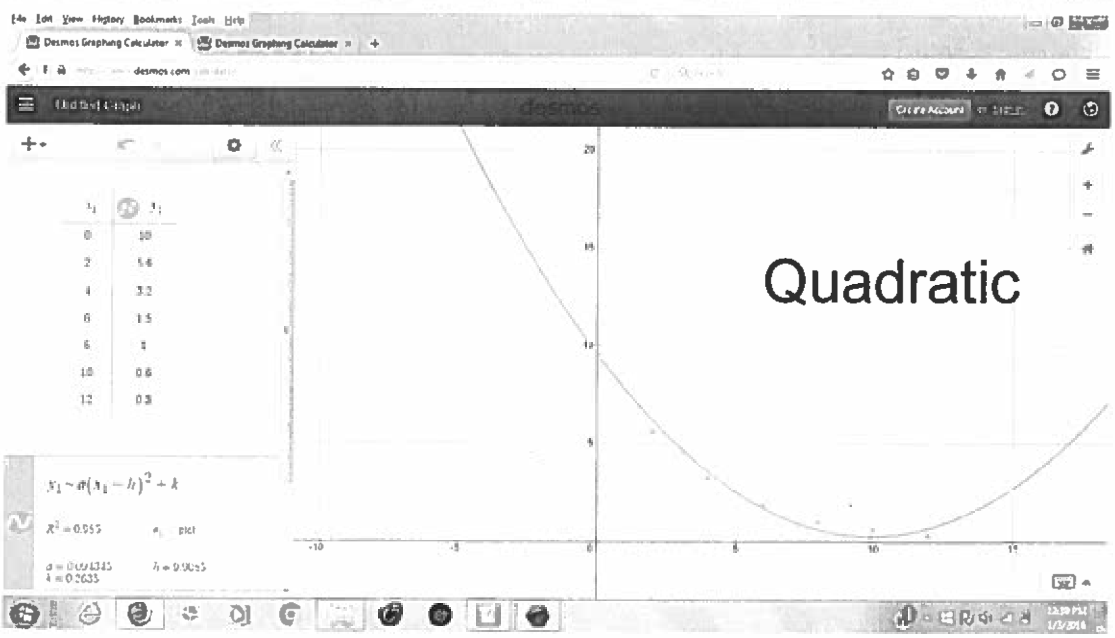
y_1 in place of the y (so that Desmos knows to use the data we entered in y_1 in our table)

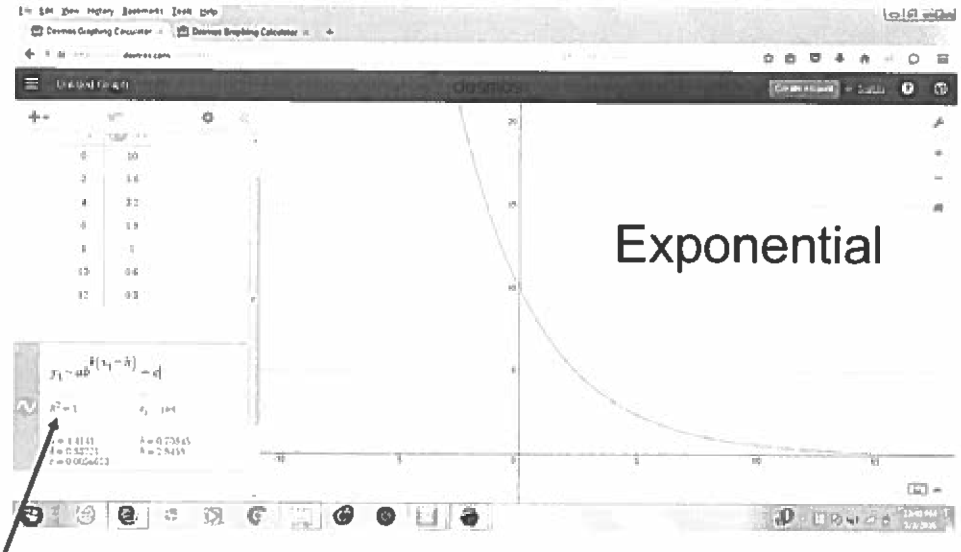
x_1 in place of the x (so that Desmos knows to use the data we entered in x_1 in our table)

~ instead of the equal sign (so Desmos knows the model doesn't have to be a perfect fit)

MCR 3UI - U4 - D9 - Exponential Regression







Note:

How well the function models the data can actually be quantified using ...

R^2, the Coefficient of Determination (or Correlation Coefficient) - which is a measure of how well the regression line/ curve represents the data

0 = function does not model the data at all

0.5 = function is a poor model of the data

(50% of the time the model will make a correct prediction)

1 = function is a perfect model of the data

(100% of the time the model will make a correct prediction)

So.... the closer your R^2 value is to 1, the better the function models the data!

Looking at all three functions, the third (exponential) models the data the best with an R^2 value of 1.

Using the information given, VOILA.... You have your equation!!!

$$y=4.41(0.71)^{0.83(x - 2.85)} + 0.0057$$

Now, use your equation to determine the height of the ball after 2.25 seconds. ($x \to$ seconds, $y \to$ metres

Cursor on curve at $x = 2.25$, ordered pair should show

Today's Practice Questions:

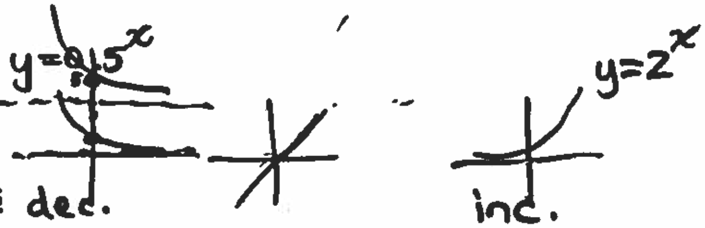
Duotang - Day 5 # 1 - 3

Day 6 # 1 - 3 (whatever you can get to in class)

$y=0.5^x+4$

Unit 5 Review $y=4$

$y=0$



Exponential Functions

1. For a large x-value, put the following functions in order of increasing y-values: $y = 2^x$, $y = x$, $y = 3^x$, $y = 0.5^x$, $y = x^2$

$y=0.5^x$, $y=x$, $y=x^2$, $y=2^x$, $y=3^x$

2. Which graph grows faster on the Domain $\{x \in R \mid x \geq 0\}$: $y = 0.5^x$, $y = 2^x$, $y = 3^x$?

3. Complete the following statements: First Difference column tells us

If all first differences are the same the function is linear (a line).

The Second Difference column tells us

If all second differences are the same the function is quadratic (a parabola)

4. For an exponential function, do the finite difference columns tell us anything that you cannot discover from the y-column of the table of values? For gr. 12 - yes.

For gr. 11 - No (The ratio of consecutive y-values gives us the same information as the ratio of consecutive finite differences.)

5. Given an Exponential Function, be able to state
   a) whether the function is increasing or decreasing on $\{x \in R\}$
   b) the domain $\{x \in R\}$ $\qquad y=a(b^{k(x-d)})+c$
   c) the range $\{y \in R \mid y > c\}$ or $\{y \in R \mid y < c\}$
   d) the asymptote $y = c$
   e) the intercepts
      * there are no x-intercepts unless the graph is shifted up or down
      * the y-intercept is only $y = a$ if the graph has not been shifted

6. Given the y-intercept and another point on an exponential graph, find the equation of the exponential function. of the form $y=ab^x$.

ex. a) (0, 3) and (1, 15) $\qquad$ b) (0, -2) and (-1, -5)

a)

$y=ab^x$

$a=3$ (y-int is the a-value if there has been no shifts).

$y=3b^x$

$15=3(b^1)$

$5=b$

x	y
0	3
1	15

$+1$, $\times 5$; $b=5$

$\therefore y=3(5^x)$

b)

$a=-2$

$y=-2b^x$

$-5=-2\cdot b^{-1}$

$\frac{-5}{-2}=\frac{-2}{-2}b^{-1}$

$\frac{5}{2}=b^{-1}$

$\frac{5}{2}=\frac{1}{b}$

$\frac{2}{5}=b$

x	y
-1	-5
0	-2

$+1$, $\times\frac{2}{5}$

$\frac{y_2}{y_1}$

$\therefore y=-2\left(\frac{2}{5}\right)^x$

7. Given the original exponential graph and a graph that has been translated and/or reflected, you need to be able to find the equation of the translated/reflected exponential function (see lesson ~~3~~ 6 example 3).

$h = \frac{1}{2}$

8. Penicillin V has a half-life of 30 minutes. The initial dosage to an adult is 512 mg.

   a) Write a function to relate the amount of penicillin V remaining, in mg, to the time, in hours.

   $\frac{t}{h}$ $\frac{t}{\frac{1}{2}}$ ← half-life every $\frac{1}{2}$ hour.

   $P(t) = 512\left(\frac{1}{2}\right)^{2t}$ $\qquad y = a\,b^{k(x-d)} + c$

   If half-life is every 2 hours → $\frac{t}{2}$ $= t \times \frac{2}{1}$

   b) The function is a transformation of $y = 2^x$. Write function in part (a) as a power of 2.

   $P(t) = 2^9(2^{-1})^{2t} = 2^9(2^{-2t}) = 2^{9-2t} = 2^{-2t+9} = 2^{-2(t-4.5)}$

   c) Describe the transformations required to map $y = 2^x$ onto the simplified function in part (b).

      1. reflection in y-axis
      2. horizontal compression factor $\frac{1}{2}$
      3. shift right 4.5 units.

9. Be able to describe the transformations that map an original function onto a given equation and then graph each function using transformations.
10. Be able to draw a scatter plot and identify an appropriate model for that scatter plot.
11. Be able to determine an appropriate function to model exponential growth data. (Given either a table of values or a scatter plot) in the form $y = ab^x$

+ more review of exponential growth/decay
percent increase/decrease