Unit 5 Day 7 Review

1a) \( y = 27 \left( \frac{1}{3} \right)^x \)
    
    \[
    y = 3^3 (3^{-x}) = 27 \\
    \frac{3^{-x}}{3} = 3^{-x} \\
    = 3^{-x}(x-3) \\
    = 3^{-(x-3)} \\
    \]

begin with \( f(x) = 3^x \)

1. reflection in y-axis,
2. shift right 3.

b) (i) \( D = \mathbb{R} \times \mathbb{R}^+ \)

(ii) \( R = \{ y \in \mathbb{R} \mid y > 0 \} \)

(iii) no x-intercepts.

\[ y \text{-int at } 3^{-(0-3)} = 3^3 = 27 \]

(iv) graph is always decreasing.

\[ \text{decreasing on } x \in \mathbb{R} \]

\[ \text{increasing } y \]

(v) asymptote is \( y = 0 \).

2. given \((0,10)\) \((1,40)\) \((2,160)\)

y-int is \(a\) so \( a = 10 \)

\[
\begin{array}{c|c|c}
 x & y & \text{ratio is common factor} \\
 \hline
 0 & 10 & 10 \\
 1 & 40 & 40 \quad 120 \\
 2 & 160 & 4 \quad \text{so } b = 4 \\
\end{array}
\]

\[ y = 10 \left( 4^x \right) \]
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3. \( y = 2^{x-3} + 4 \)
   \( f \) begins with \( f(x) = 2^x \)
   1. right 3
   2. up 4

   (i) \( D = \{ x \in \mathbb{R} \} \)

   (ii) \( R = \{ y \in \mathbb{R} | y > 4 \} \)

   (iii) Asymptote is given as \( y = 4 \)

4. \( y = 5^x \)
   (a) \( y = 2(5^x) \) 
      vertical stretch factor 2
   (b) \( y = 5^{-2x} \) 
      horizontal compression factor \( \frac{1}{2} \)
   (c) \( y = -5^{-x} \) 
      1. reflection in x-axis
      2. reflection in y-axis
   (d) \( y = 5^{-5x-10} \) 
      1. reflection in y-axis
      2. horizontal compression factor \( \frac{1}{5} \)
      3. shift left 2.
Unit 5 Review

(a) | n | h | first diff | second diff | third diff |
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<td>&gt;24</td>
<td>&gt; 5</td>
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<tr>
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<td>&gt;19</td>
<td>&gt; 5</td>
<td>&gt; -2</td>
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<td>43</td>
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The trend is not linear, not quadratic, and the curve appears exponential.

(b) The curve appears to be exponential.

- \( \frac{76}{100} = 0.76 \)
- \( \frac{57}{76} = 0.75 \)
- \( \frac{43}{57} = 0.75 \)
- \( \frac{32}{43} = 0.74 \)
- \( \frac{24}{32} = 0.75 \)

The ratio of consecutive \( y \)-values have a common ratio around 0.75.

(c) Using TI-83

Exponential Regression

\[ y = 100 \times 0.7354 \times (0.7512)^x \]

\( r = -0.999947569 \)

\( r^2 = 0.999895 \)

(d) (i) According to the mathematical model, there is an asymptote at \( h = 0 \) which means that although the height of the bouncing ball will get very close to zero metres, the ball will never stop bouncing.

(ii) In real-life, the ball would stop bouncing.

(e) Other factors may come into play; the exponential model does not allow us to extrapolate too far beyond the data collected.