(a)

Day	Population
0	20
1	80
2	320
3	1280
4	5120
5	20480

First Differences
60
240
960
3840
15360

Second Differences
180
720
2880
11520

ratio of consecutive first differences is 4.

ratio of consecutive second differences is 4.

(b) This is an exponential relationship.
Since $y_n = 4 \times y_{n-1}$.

(c)

(d) Yes, I expect the pattern to continue for any finite difference.

(e)

Third Differences
540
2160
8640

Fourth Differences
1620
6480

ratio of consecutive terms is 4

ratio of consecutive terms is 4.

(f) Yes, the ratio of consecutive terms of all finite differences is equal to the rate at which the population is growing.

Unit 5 day 1.

2(a)

Day	Number of students who learn the rumour that day	Total Number of students who know the rumour
0	5	5
1	10	15
2	20	35

* note: each person tells only two people total!

(i) 10 students were told on the first day
(ii) 20 " " " " " second day

(b) without developing a "formula" and using it (the expected approach).

3	40	75
4	80	155
5	160	315
6	320	635
7	640	1275 ←
8	1280	2555

∴ sometime on the eigth day 1400 students will know.

(c) This is an example of exponential growth. As the number of days increases by one, the number of students increase by a factor of 2. (Multiply by 2 to get from one y-value to the next).

3. Everyday Deal $1, 2, 3, 4, 5, \ldots, n$
Square Deal $1, 4, 9, 16, 25, \ldots, n^2$
Double Deal $1, 2, 4, 8, 16, \ldots, 2^{n-1}$

when $n=14$
14
$14^2 = 196$
$2^{14-1} = 2^{13} = 8192$

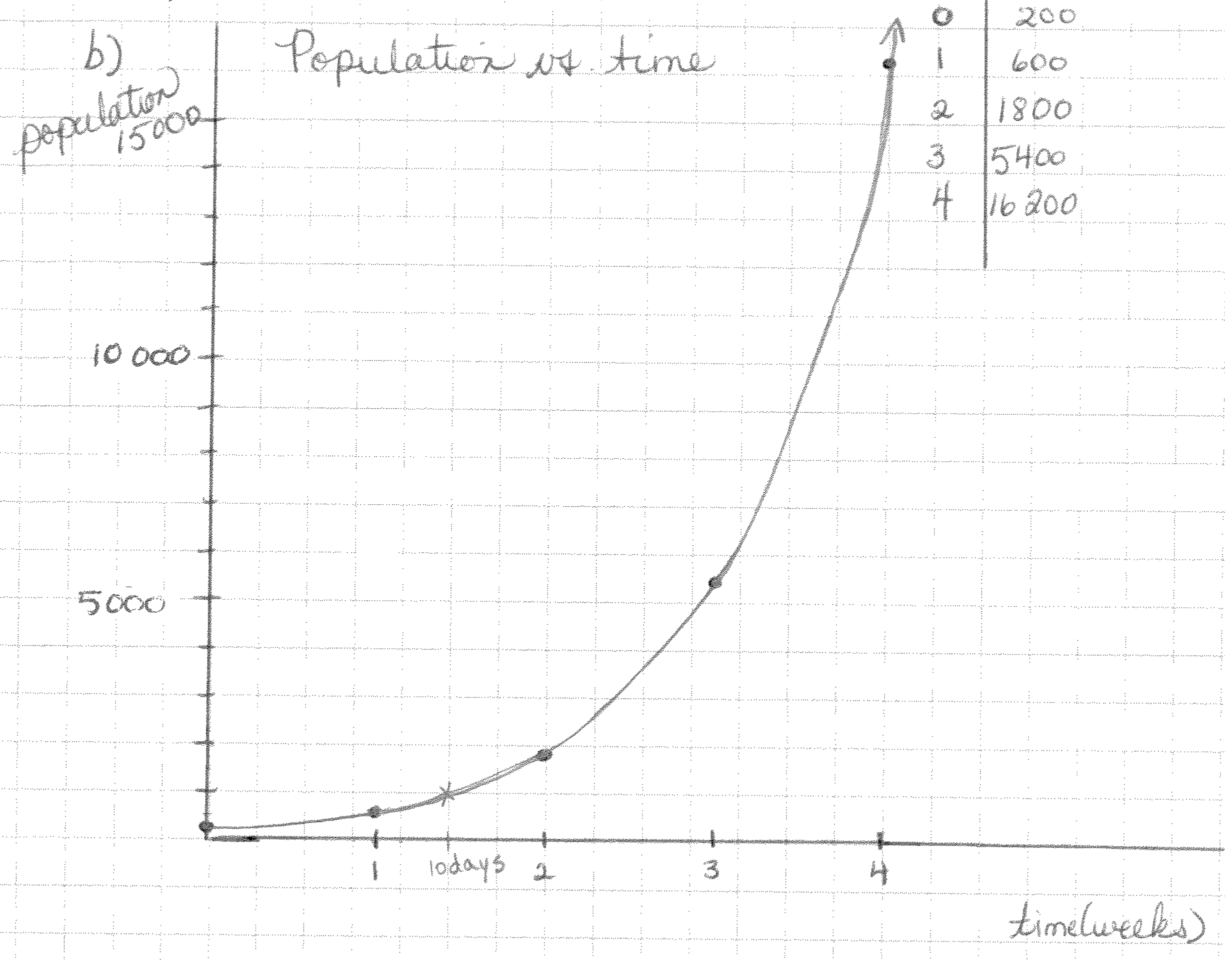
∴ the double deal is certainly the best choice!

4. $P_0 = 200$, Population triples every week

a) $p = 200 \times 3^t$, t - time in weeks, p - population

t	p
0	200
1	600
2	1800
3	5400
4	16 200

b)



c) See x on $\boxed{\text{graph}}$ → after 10 days, we expect the population to be 1000.

$\boxed{\text{equation}}$ $p = 200 \times 3^{\frac{10}{7}}$

$= 960.797$

∴ I would expect 960 bacteria after 10 days.

* graph is quicker to use but equation is much more accurate.

(d) $p = 200 \times 3^{13}$
$= 318\,864\,600$

3 months = $\frac{1}{4}$ of year
$\frac{1}{4} \times 52$ weeks
= 13 weeks.

∴ after 3 months (13 weeks) the population would be about 318 864 600 — impossible to find on a graph drawn by hand. Absolutely prefer equation!

Using $P = P_0 \times N^t$

(A) $P_0 = 500$, $N = 2$, t in days

$P_A = 500 \times 2^t$

(B) $P_0 = 50$, $N = 3$, t in days

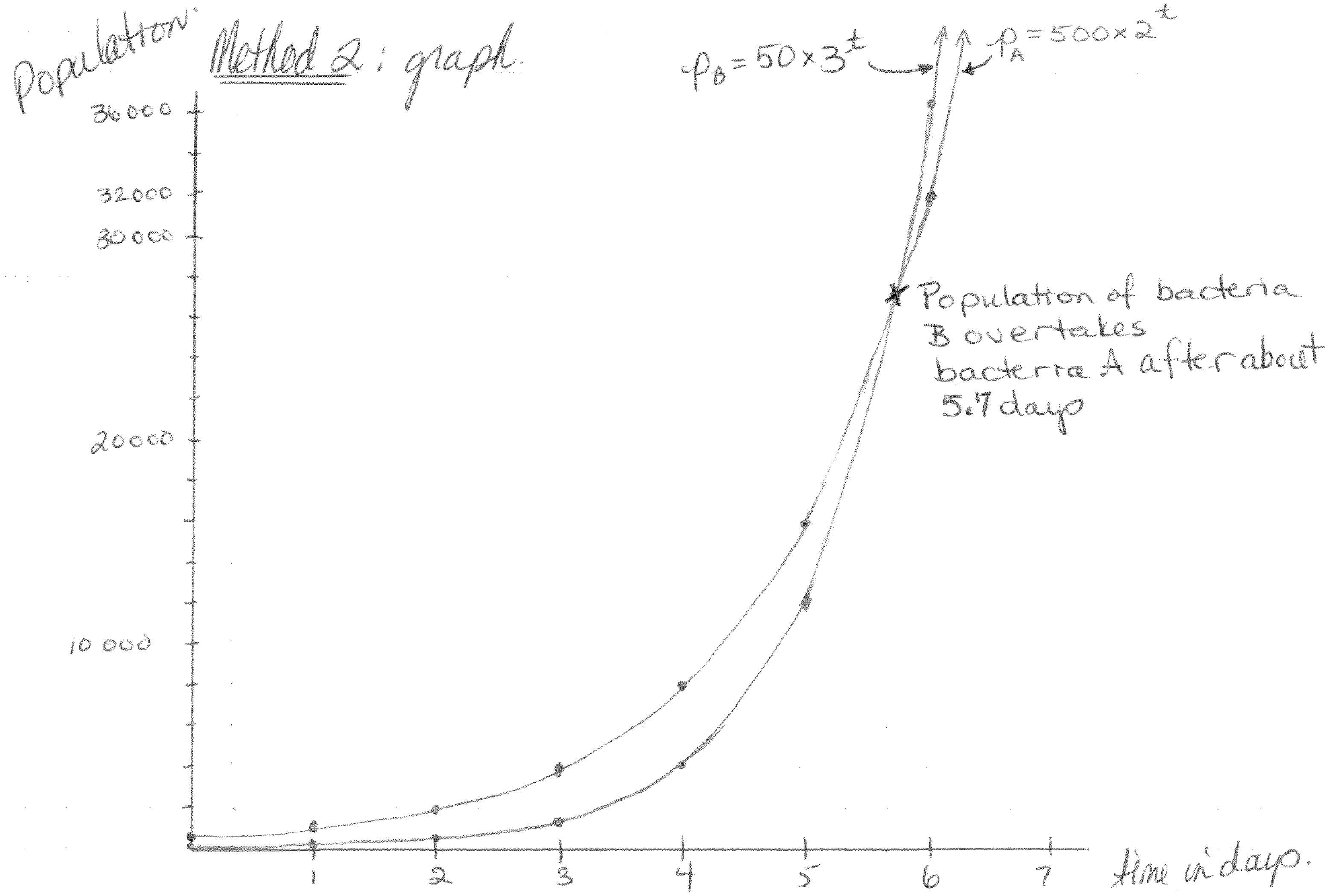
$P_B = 50 \times 3^t$

Method 1 : table

t	P_A	P_B
0	500	50
1	1000	150
2	2000	450
3	4000	1350
4	8000	4050
5	16000	12150
6	32000	36450

On the sixth day the population of bacteria B will overtake the population of bacteria A. At the end of the sixth day, there will be 32000 bacteria A and 36450 bacteria B.

Method 2 : graph.



<u>Method 3</u>: equating equations – requires knowledge of logarithms (Something to look forward to next year ☺).

5(b)

t (days)	$P_A = 500 \times 2^{\frac{t}{2}}$	P_B
0	500	50
1	707	150
2	1000	450
3	1414	1350
4	2000 2 *	4050
5		

Sometime on the 4th day B would overtake A if A's doubling period was every two days.

6. $P = 1 \times 2^{\frac{t}{3}}$, t in days.

$t = 30$ days.

$P = 1 \times 2^{\frac{30}{3}}$
$= 1 \times 2^{10}$
$= 1024$

$\therefore$ C is the correct answer.