Unit 2 Review

Pgs 86-89 #13-33

13a) $5x^2-4x-2+(8x^2+3x-3)$
$= 5x^2-4x-2+8x^2+3x-3$
$= 13x^2-x-5$

b) $2x^2-6xy+7y^2+(4x^2+3xy-11y^2)$
$= 6x^2-3xy-4y^2$

14a) $7y^2+4y-7-(9y^2+3y-3)$
$= 7y^2+4y-7-9y^2-3y+3$
$= -2y^2+y-4$

b) $3m^2+mn-7n^2-(5m^2+3mn-8n^2)$
$= 3m^2+mn-7n^2-5m^2-3mn+8n^2$
$= -2m^2-2mn+n^2$

15 a) $4(x+5)+3(x-7)$
$= 4x+20+3x-21$
$= 7x-1$

b) $6(3s-4t)-(7s-t)+5$
$= 18s-24t-7s+t+5$
$= 11s-23t+5$

c) $2x(x+3)-x(3x+8)$
$= 2x^2+6x-3x^2-8x$
$= -x^2-2x$

d) $3y(y-2)+2y(3y+4)-4y(2y-3)$
$= 3y^2-6y+6y^2+8y-8y^2+12y$
$= 9y^2-8y^2+14y$
$= y^2+14y$

16a) $3[4-2(y-3)]+4[3(2-y)-5]$
$= 3[4-2y+6]+4[6-3y-5]$
$= 3[10-2y]+4[1-3y]$
$= 30-6y+4-12y$
$= 34-18y$

OR

$= -18y+34$

b) $2x[2-x(x-1)]-[3-x(x+20)]$
$= 2x[2-x^2+x]-[3-x^2-20x]$
$= 4x-2x^3+2x^2-3+x^2+20x$
$= -2x^3+3x^2+24x-3$

17 a) $(y-8)(y-9)$
$= y^2 - 9y - 8y + 72$
$= y^2 - 17y + 72$

b) $2(7-3x)(4+x)$
$= 2(28 + 7x - 12x - 3x^2)$
$= 56 - 10x - 6x^2$
$= -6x^2 - 10x + 56$

c) $3(3x-1)^2$
$= 3(9x^2 - 6x + 1)$
$= 27x^2 - 18x + 3$

Don't forget to "Have a Blast!"

d) $(4x+3y)(2x-5y)$
$= 8x^2 - 20xy + 6xy - 15y^2$
$= 8x^2 - 14xy - 15y^2$

18 a) $(m-4)(m+4) + (m-3)^2$
$= m^2 - 16 + m^2 - 6m + 9$
$= 2m^2 - 6m - 7$

b) $(x+6)^2 - (x+4)(x-7)$
$= x^2 + 12x + 36 - (x^2 - 3x - 28)$
$= x^2 + 12x + 36 - x^2 + 3x + 28$
$= 15x + 64$

Don't forget to "Have a Blast!"

c) $3(4y+1)^2 + 2(3y-4)(2y-3)$
$= 3(16y^2 + 8y + 1) + 2(6y^2 - 9y - 8y + 12)$
$= 48y^2 + 24y + 3 + 12y^2 - 34y + 24$
$= 60y^2 - 10y + 27$

d) $2(3x-2y)(x+3y) - 2(2x-y)^2$
$= 2(3x^2 + 9xy - 2xy - 6y^2) - 2(4x^2 - 4xy + y^2)$
$= 6x^2 + 14xy - 12y^2 - 8x^2 + 8xy - 2y^2$
$= -2x^2 + 22xy - 14y^2$

19 a) $(x-3)(x^2 - 3x + 2)$
$= x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$
$= x^3 - 6x^2 + 11x - 6$

b) $(2t+1)(3t^2 - t - 1)$
$= 6t^3 - 2t^2 - 2t + 3t^2 - t - 1$
$= 6t^3 + t^2 - 3t - 1$

c) $(x^2 + 2x + 3)(x^2 - x - 1)$
$= x^4 - x^3 - x^2 + 2x^3 - 2x^2 - 2x + 3x^2 - 3x - 3$
$= x^4 + x^3 - 5x - 3$

d) $(3z^2 - 2z + 1)(2z^2 + 2z - 3)$
$= 6z^4 + 6z^3 - 9z^2 - 4z^3 - 4z^2 + 6z + 2z^2 + 2z - 3$
$= 6z^4 + 2z^3 - 11z^2 + 8z - 3$

20.

New length is $(2x+1+2) = 2x+3$ units
New width is $(x-1+2) = x+1$ units

Increase in Area = ~~Old~~ New area − ~~New~~ Old Area
$= (2x+3)(x+1) - (2x+1)(x-1)$
$= 2x^2+2x+3x+3-(2x^2-2x+x-1)$
$= 2x^2+5x+3-2x^2+x+1$
$= 6x+4$

∴ the increase in area is $6x+4$ units.

21 a) $\frac{3x}{3x+9}$
$= \frac{3x}{3(x+3)}$
$= \frac{x}{x+3}, x \neq -3$

b) $\frac{8y^2-10xy}{4y}$
$= \frac{2y(4y-5x)}{4y}$
$= \frac{4y-5x}{2}, y \neq 0$

c) $\frac{5x-5y}{7x-7y}$
$= \frac{5(x-y)}{7(x-y)}$
$= \frac{5}{7}, x \neq y$

d) $\frac{6x-10}{5-3x}$
$= \frac{2(3x-5)}{-1(3x-5)}$
$= -2, x \neq \frac{5}{3}$

e) $\frac{3w}{3w^2-12w}$
$= \frac{3w}{3w(w-4)}$
$= \frac{1}{w-4}, w \neq 0, 4$

f) $\frac{3m^2-3m}{4m^2-4m}$
$= \frac{3m(m-1)}{4m(m-1)}$
$= \frac{3}{4}, m \neq 0, 1.$

g) $\frac{t-2}{t^2-3t+2}$
$= \frac{t-2}{(t-1)(t-2)}$
$= \frac{1}{t-1}, t \neq 1, 2$

h) $\frac{2a^2-7a-15}{a-5}$
$= \frac{(2a+3)(a-5)}{a-5}$
$= 2a+3, a \neq 5$

M − 30
A − 7
3, −10
÷2

i) $\frac{6n^2-7n-3}{12n^2+7n+1}$
$= \frac{(2n-3)(3n+1)}{(4n+1)(3n+1)}$
$= \frac{2n-3}{4n+1}, n \neq -\frac{1}{4}, -\frac{1}{3}$

÷3 ÷2
−9, 2
M −18
A −7

M 12
A 7
3, 4
÷3 ÷4

j) $\frac{y^2-9}{y^2+y-12}$
$= \frac{(y+3)(y-3)}{(y+4)(y-3)}$
$= \frac{y+3}{y+4}, y \neq 3, -4$

22a) $\text{length} = \dfrac{\text{area}}{\text{width}}$

$= \dfrac{2x^2+4x+2}{x+1}$

$= \cancel{\dfrac{(2x\quad)(x}{x+1}}$

$= \dfrac{2(x^2+2x+1)}{x+1}$

$= \dfrac{2(x+1)^2}{x+1}$

$= 2(x+1)$, $x \neq -1 \rightarrow$ in fact $x > -1$ (dimensions must be positive).

b) length : width

$= 2(x+1) : (x+1)$

$= 2:1$

23. a) $\dfrac{\cancel{5}^{1}x^3}{\cancel{2}_{1}y} \times \dfrac{\cancel{8}^{4}y}{\cancel{15}_{3}x^2}$

$= \dfrac{4x}{3}$, $x, y \neq 0$.

b) $\dfrac{-4a^3}{3b} \div \dfrac{2a}{3b^2}$

$= \dfrac{-\cancel{4}^{-2}a^3}{\cancel{3}b} \times \dfrac{\cancel{3}^{1}b^2}{\cancel{2}_{1}a}$

$= -2a^2b$, $a, b \neq 0$.

c) $\dfrac{\cancel{3}^{1}a^2b}{\cancel{-}4xy} \times \dfrac{\cancel{-}5x^2y}{\cancel{6}_{2}ab^2}$

$= \dfrac{5ax}{8b}$, $x, y, a, b \neq 0$.

d) $\dfrac{b^2}{8x^3y} \div \dfrac{3b}{4xy}$

$= \dfrac{b^2}{\cancel{8}_{2}x^3y} \times \dfrac{\cancel{4}^{1}xy}{3b}$

$= \dfrac{b}{6x^2}$, $x, y, b \neq 0$

e) $\dfrac{3x-3}{2x+2} \times \dfrac{5x+5}{6x-6}$

$= \dfrac{\cancel{3}^{1}(x-1)}{2(x+1)} \times \dfrac{5(x+1)}{\cancel{6}_{2}(x-1)}$

$= \dfrac{5}{4}$, $x \neq \pm 1$

f) $\dfrac{4m+8}{3n-3} \div \dfrac{2m+6}{7n-7}$

$= \dfrac{\cancel{4}^{2}(m+2)}{3(\cancel{n-1})} \times \dfrac{7(\cancel{n-1})}{\cancel{2}(m+3)}$

$= \dfrac{14(m+2)}{3(m+3)}$, ~~$m \neq 1$,~~ $m \neq -3$, $n \neq 1$

g) $\dfrac{t^2+4t+4}{t-2} \div \dfrac{3t+6}{t^2-5t+6}$

$= \dfrac{(t+2)^2}{t-2} \div \dfrac{3(t+2)}{(t-2)(t-3)}$

$= \dfrac{(t+2)^{\cancel{2}}}{\cancel{(t-2)}} \times \dfrac{\cancel{(t-2)}^{1}(t-3)}{3\cancel{(t+2)}}$

$= \dfrac{(t+2)(t-3)}{3}$, $t \neq \pm 2, 3$.

23 h) $\dfrac{2x^2-5x-3}{2x^2-5x+2} \times \dfrac{2x^2+3x-2}{x^2-4x+3}$

$= \dfrac{(2x+1)\cancel{(x-3)}}{(x-2)\cancel{(2x-1)}} \times \dfrac{(x+2)\cancel{(2x-1)}}{\cancel{(x-3)}(x-1)}$

$= \dfrac{(2x+1)(x+2)}{(x-2)(x-1)}, \quad x \neq 1, 2, 3, \frac{1}{2}$

i) $\dfrac{6y^2-5y+1}{12y^2-5\cancel{x}-2} \div \dfrac{3y^2-4y+1}{4y^2+3y-1}$

type

$= \dfrac{6y^2-5y+1}{12y^2-5y-2} \times \dfrac{4y^2+3y-1}{3y^2-4y+1}$

$= \dfrac{\cancel{(3y-1)}(2y-1)}{(4y+1)(3y-2)} \times \dfrac{(y+1)\cancel{(4y-1)}}{\cancel{(3y-1)}(y-1)}$

$= \dfrac{(2y-1)(y+1)\cancel{(4y-1)}}{(4y+1)(3y-2)(y-1)} \quad y \neq \frac{-1}{4}, \frac{2}{3}, \frac{1}{3}, \frac{-1}{3}, 1$

24.

a) width of B $= \dfrac{\text{Area of B}}{\text{length of B}}$

$= \dfrac{2t^2-3t+1}{2t-1}$

$= \dfrac{(t-1)(2t-1)}{2t-1}$

$= t-1, \quad t > \frac{1}{2}$

b) width of C $= \dfrac{\text{Area of C}}{\text{length of C}}$

$= \dfrac{3t^2-2t-1}{3t+1}$

$= \dfrac{(3t+1)(t-1)}{3t+1}$

$= t-1, \quad t > 1$

c) Rectangle A is a square.
[length, width are both $(t-1)$]

Pg. 88-89 #25-33

25 a) $\frac{5}{x} + \frac{1}{x} - \frac{8}{x}$

$= \frac{-2}{x}, x \neq 0$

b) $\frac{2m+1}{m-2} + \frac{3m-5}{m-2}$

$= \frac{5m-4}{m-2}, m \neq 2$

c) $\frac{4z-3}{z^2} - \frac{3z-1}{z^2}$

$= \frac{4z-3-3z+1}{z^2}$

$= \frac{z-2}{z^2}, z \neq 0$

d) $\frac{2t}{3} - \frac{3t}{4} + \frac{t}{6}$

$= \frac{8t-9t+2t}{12}$

$= \frac{t}{12}$ (no restrictions on t).

e) $\frac{4x+1}{5} + \frac{2x-1}{4}$

$= \frac{16x+4+10x-5}{20}$

$= \frac{26x-1}{20}$

f) $\frac{2a-3b}{6} - \frac{3a-2b}{4}$

$= \frac{2(2a-3b)-3(3a-2b)}{12}$

$= \frac{4a-6b-9a+6b}{12}$

$= \frac{-5a}{12}$

g) $\frac{4}{2y-5} + \frac{2}{5-2y}$

$= \frac{4}{2y-5} - \frac{2}{2y-5}$

$= \frac{2}{2y-5}, y \neq \frac{5}{2}$

h) $\frac{x^2+5}{x^2-4} - \frac{x^2-2}{4-x^2}$

$= \frac{x^2+5+(x^2-2)}{x^2-4}$

$= \frac{2x^2+3}{x^2-4}, x \neq \pm 2$

← note: $-\frac{x^2-2}{4-x^2}$

$= -\frac{x^2-2}{-(x^2-4)}$

$= \frac{x^2-2}{x^2-4}$

i) $\frac{2x+5}{x^2+3x+2} - \frac{x+4}{x^2+3x+2}$

$= \frac{2x+5}{(x+2)(x+1)} - \frac{x+4}{(x+1)(x+2)}$

$= \frac{x+1}{(x+2)(x+1)}$

$= \frac{1}{(x+2)}, x \neq -1, -2$

* only need to factor denominator for restrictions and to check whether fraction can be reduced.

26. $\frac{9x+1}{4} - \frac{x+1}{2} - \frac{2x-1}{2}$

$= \frac{9x+1}{4} - \frac{2x+2}{4} - \frac{4x-2}{4}$

notice $-(4x-2)$

$= \frac{9x+1-2x-2-4x+2}{4}$

$= \frac{3x+1}{4}$

$\therefore$ the third side length is $\frac{3x+1}{4}$ units.

27. a) $\frac{2}{y} + \frac{4}{y^2} - \frac{1}{y}$

$= \frac{2y+4-y}{y^2}$

$= \frac{y+4}{y^2}, y \neq 0.$

b) $\frac{4}{x^2} - \frac{5}{xy} + \frac{2}{y^2}$

$= \frac{4y^2-5xy+2x^2}{x^2y^2}, x,y \neq 0$

c) $\frac{a}{2a-2} + \frac{2}{3a-3}$

$= \frac{a}{2(a-1)} + \frac{2}{3(a-1)}$

$= \frac{3a+4}{6(a-1)}, a \neq 1$

d) $\frac{2}{x+3} - \frac{4}{x+1}$

$= \frac{2(x+1)-4(x+3)}{(x+3)(x+1)}$

$= \frac{2x+2-4x-12}{(x+3)(x+1)}$

$= \frac{-2x-10}{(x+3)(x+1)}, x \neq -3, -1.$

e) $\frac{2}{t^2+3t+2} - \frac{1}{t^2+t-2}$

$= \frac{2}{(t+2)(t+1)} - \frac{1}{(t+2)(t-1)}$

$= \frac{2(t-1)-1(t+1)}{(t+2)(t+1)(t-1)}$

$= \frac{2t-2-t-1}{(t+2)(t+1)(t-1)}$

$= \frac{t-3}{(t+2)(t+1)(t-1)}, t \neq \pm 1, -2$

f) $\frac{x+1}{3x^2+4x+1} + \frac{2x-1}{3x^2-5x-2}$

better to reduce first

$= \frac{x+1}{(3x+1)(x+1)} + \frac{2x-1}{(3x+1)(x-2)}$

$= \frac{(x+1)(x-2)+(2x-1)(x+1)}{(3x+1)(x+1)(x-2)}$

$= \frac{x^2-x-2+2x^2+x-1}{(3x+1)(x+1)(x-2)}$

$= \frac{3x^2-3}{(3x+1)(x+1)(x-2)}$

$= \frac{3(x-1)}{(3x+1)(x-2)}$

$x \neq -\frac{1}{3}, -1, 2$

Pgs 88-89 # 26-27.

27 f)

$$\frac{x+1}{3x^2+4x+1} + \frac{2x-1}{3x^2-5x-2}$$

$$= \frac{x+1}{(3x+1)(x+1)} + \frac{2x-1}{(3x+1)(x-2)}$$

$$= \frac{1}{3x+1} + \frac{2x-1}{(3x+1)(x-2)}$$

$$= \frac{1(x-2)+2x-1}{(3x+1)(x-2)}$$

$$= \frac{x-2+2x-1}{(3x+1)(x-2)}$$

$$= \frac{3x-3}{(3x+1)(x-2)}$$

$$= \frac{3(x-1)}{(3x+1)(x-2)}, \quad x \neq -\tfrac{1}{3}, 2, -1$$