

# Simplifying Rational Expressions

→ FRACTIONS → with variables.

Simplifying rational expressions is like simplifying (reducing) rational numbers (fractions).

e.g.  $\frac{4}{20}$  We can reduce both the numerator and denominator by a factor of 4.  
 $= \frac{1}{5}$  (the GCF - greatest common factor of 4 and 20 is 4)

Now consider,  $\frac{4x}{20x^2}$  \*same idea... reduce by a factor of  $4x$   
 ( $4x$  is the GCF of  $4x$  and  $20x^2$ )

\* This fraction has a variable in the denominator. Since we can never divide by zero, we must "restrict" the variable.

$20x^2$  cannot equal zero so,  $x \neq 0$

The "restriction" is  $x \neq 0$ .

Example. Express the following in simplest form. State any restrictions on the variables)

$$\begin{aligned} \text{a) } & \frac{6a^2 + 9a}{12a} \\ & = \frac{3a(2a+3)}{12a} \\ & = \frac{\cancel{3a}(2a+3)}{\cancel{12a}^4} \\ & = \frac{2a+3}{4} \end{aligned}$$

\* We can only reduce by dividing out FACTORS

note:  $\frac{5}{8} = \frac{2+3}{8}$

we cannot reduce the 2 over 8 because 2 is NOT a factor in the numerator.

If the numerator and/or the denominator are/is not a monomial, you MUST FACTOR.

$a \neq 0$

$$\begin{aligned} \text{b) } & \frac{-6x^2y^3}{-18x^3y} \\ & = \frac{\cancel{-6x^2y^3}^1}{\cancel{-18x^3y}^3x} \\ & = \frac{y^2}{3x}, \quad x \neq 0, y \neq 0 \end{aligned}$$

\* numerator and denominator are both monomials so no need to factor.

GCF is  $-6x^2y$  so reduce top and bottom by a factor of  $-6x^2y$

\* use exponent rules

$$\begin{aligned}
 c) \quad & \frac{7x^2 - 21x}{14x^3 - 42x^2} \\
 &= \frac{7x(x-3)}{14x^2(x-3)} \\
 &= \frac{\cancel{7x}(x-3)}{14x^{\cancel{2}}(x-3)} \\
 &= \frac{1}{2x}, \quad x \neq 0, x \neq 3
 \end{aligned}$$

\* Do restrictions from the first line of work where the denominator is factored.

Each variable in a monomial factor in the denominator  $\neq 0$ .

$14x^2 \neq 0$        $x-3 \neq 0$       solve for restriction like you solve an equation.  
 $x^2 \neq 0$        $x \neq 3$   
 $x \neq 0$

$$\begin{aligned}
 d) \quad & \frac{a^2 + a}{a^2 + 2a + 1} \\
 &= \frac{a(a+1)}{(a+1)(a+1)} \\
 &= \frac{a}{a+1}, \quad a+1 \neq 0 \\
 & \quad \quad \quad a \neq -1
 \end{aligned}$$

note: a can equal zero because it is okay to have a factor of zero in the NUMERATOR.

$$\begin{aligned}
 e) \quad & \frac{y-2}{2-y} \quad * \text{ note } \dots \text{ both have the same terms with opposite signs.} \\
 &= \frac{-2+y}{2-y} \\
 &= \frac{-1(\cancel{2-y})}{(\cancel{2-y})} \\
 &= -1, \quad y \neq 2
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{3x^2 + 5x + 2}{9x^2 - 4} \\
 &= \frac{(\cancel{3x+2})(x+1)}{(3x-2)(\cancel{3x+2})} \\
 &= \frac{x+1}{3x-2}, \quad x \neq \pm \frac{2}{3}
 \end{aligned}$$

\* rough work for factoring complex trinomial is done at the side ... not in the fraction.

$$\begin{aligned}
 g) \quad & \frac{3m+15}{4m+20} \\
 &= \frac{3(m+5)}{4(m+5)} \\
 &= \frac{3}{4}, \quad m \neq -5
 \end{aligned}$$