# Polynomials & Equations

**Ex. 1** Expand & Simplify

a) $4(x-2)-6(3x-4)$
$= 4x-8-18x+24$
$= -14x+16$

**Ex. 2** Simplify, Solve & Check.

a) $10(x-3)-2(x+1)=-8$
$10x-30-2x-2=-8$
$8x-32=-8$
$8x=-8+32$
$8x=24$
$x=3$

check

| LS | RS |
|---|---|
| $10(3-3)-2(3+1)$ | $-8$ |
| $=10(0)-2(4)$ | |
| $=-8$ ✓ | |

b) $\frac{y+4}{3}=\frac{y+5}{2}+2$ * FRACTION! FIND the LCD! LCD=6.

$6\left(\frac{y+4}{3}\right)=6\left(\frac{y+5}{2}+\frac{2}{1}\right)$

$\frac{6}{3}(y+4)=\frac{6}{2}(y+5)+6(2)$

$2(y+4)=3(y+5)+12$

$2y+8=3y+15+12$
$2y+8=3y+27$
$2y-3y=27-8$
$-y=19$
$y=-19$

check

| LS | RS |
|---|---|
| $\frac{-19+4}{3}$ | $\frac{-19+5}{2}+2$ |
| $=\frac{-15}{3}$ | $=\frac{-14}{2}+2$ |
| $=-5$ ✓ | $=-7+2$ |
| | $=-5$ |

# Factoring

1. Common Factoring

a) $6x^4+2x^2-4x^3$
$=2x^2(3x^2+1-2x)$

2. Factor Completely Trinomials of the form $x^2+bx+c$

a) $x^2-8x+15$ looking for two numbers that multiply to 15 and add to $-8$

$= (x-3)(x-5)$ $-3, -5$

Factor + Solve

b) $10x^2-30x-100=0$

$10(x^2-3x-10)=0$

$(x-5)(x+2)=0$

$x=5$ OR $x=-2$

3. Difference of Squares.

Factor.

$t^2-4$ ← Two terms, "−" in middle, both terms are "perfect squares".

$=(t-2)(t+2)$

4. Trinomials of the form $ax^2+bx+c$, $a \neq 1$.

Factor Completely.

a) $6x^2-7x-3$

A. "Decomposition Method"

Find two numbers that multiply to $6(-3) = -18$ and add to $-7$

$-9, 2$

"decompose the middle term"

$= \underbrace{6x^2-9x}+\underbrace{2x-3}$

$= 3x(2x-3)+(2x-3)$ factor by grouping

$= (2x-3)(3x+1)$ $(2x-3)$ is a common factor

B. "Australian Method"

Find the two numbers as in part (a)

$-9, 2$

put them at end of brackets

$\frac{(6x\quad)(6x\quad)}{6}$

$= \frac{(6x-9)(6x+2)}{6}$

$= \frac{3(2x-3)(2)(3x+1)}{6}$ common factor

$= \frac{6(2x-3)(3x+1)}{6}$

$= (2x-3)(3x+1)$

## C. "Chart Method" "Cross Products"

| list all factors of 6 | + reversed | | | list all factors of 3 |
|---|---|---|---|---|
| 1 | 2 | 6 | 3 | 1 |
| 6 | 3 | 1 | 2 | 3 |

now use X's to determine which pairs could sum to −7 with the correct signs.

| | | | |
|---|---|---|---|
| 1 1 | 2 1 | 6 1 | 3 1 |
| 6 3 | 3 3 | 1 3 | 2 3 |
| 3, 6 | 6, 3 | 18, 1 | 9, 2 |
| no! | no! | no! | yes! |

$-9+2=-7$

$3(-3)=-9$

so

3 1

2 −3

$\boxed{(3x+1)(2x-3)}$

## D. "Box Method"

↳ similar to decomposition.

multiply to −18, add to −7

−9, 2

* most academic students prefer decomposition to box-factoring

| | | |
|---|---|---|
| | $6x^2$ | $-3$ |
| | $6x^2$ | $-9x$ |
| | $2x$ | $-3$ |

place the "decomposition" of $-7x$ in these two spaces.

CF of $6x^2, 2x$ — CF of $-9x, -3$

* common factor every pair vertically and horizontally.

| | | |
|---|---|---|
| | $2x$ | $-3$ |
| $3x$ | $6x^2$ | $-9x$ |
| 1 | $2x$ | $-3$ |

$3x$ is a common factor of $6x^2, -9x$ →

1 is CF of $2x, -3$ →

take factors from side and top →

$\boxed{(2x-3)(3x+1)}$

4.

b) $9x^2-6x+1$

A. "Decomposition Method"

$$9x^2-6x+1$$
$$= \underline{9x^2-3x}\underline{-3x+1}$$
$$= 3x(\underline{3x-1})-1(\underline{3x-1})$$
$$= (3x-1)(3x-1)$$
$$= (3x-1)^2$$

Product 9
Sum $-6$
$-3, -3$

B. "Australian Method"

$$9x^2-6x+1$$
$$= \frac{(9x-3)(9x-3)}{9}$$
$$= \frac{3(3x-1)(3)(3x-1)}{9}$$
$$= (3x-1)^2$$

m 9, Add $-6$
$-3, -3$

C. "Chart Method"

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 9 | | 1 |
| 9 | 3 | 1 | | 1 |

add to $-6$

1, 9 no!

3, 3 yes!

$-3, -3$

| | |
|---|---|
| 3 | $-1$ |
| 3 | $-1$ |

$$(3x-1)(3x-1)$$
$$= (3x-1)^2$$

5. Recognize the Perfect Square

$9x^2 - 6x + 1$

Perfect Square  Perfect square

$= \left(\sqrt{9x^2} - \sqrt{1}\right)^2$

$= (3x - 1)^2$

then double check that "Twice the product" = middle term.

$(3x)(-1) \times 2$

$= -6x$ which is the middle term. ✓

6. Factor by Grouping

a) $x^2 - 6x + 9 - y^2$

$= (x^2 - 6x + 9) - y^2$

$= (x-3)^2 - y^2$

$= (x-3-y)(x-3+y)$

b) $6x^3 - 9x^2 + 2x - 3$

$= 3x^2(2x-3) + (2x-3)$

$= (2x-3)(3x^2+1)$