Grade 11U Algebraic Challenge: Rational Expressions

1. (October 11/12) Simplify. Do not state restrictions.

$$\frac{(x+y)^2-4}{5x^3+5xy+10x} \div \frac{(x+y)(x+2)+2(x+2)}{(3x+1)^2-(2x-1)^2}$$

2. (October 15/16) Simplify. State restrictions. Note: You can only do this one if you know the factor theorem and/or long division or synthetic division.

$$\frac{2x^3-8x^2+5x-20}{5x^2-40x+80}$$

3. (October 17/18) Simplify. State restrictions.

$$\left[\frac{5m-10}{m^2-4}-\frac{2m-6}{m^2-7m+12}\right] \bullet \frac{3m^2-12m}{3m^2-26m+16}$$

4. (October 19/22) Simplify. State restrictions.

$$\frac{\frac{y-2}{y+2}-\frac{y+2}{y-2}}{\frac{1}{2}+\frac{1}{y-2}}$$

5. (October 23/24) Simplify. State restrictions.

$$\frac{\frac{3}{r^2}-\frac{2}{rs}-\frac{5}{s^2}}{\frac{5}{s^2}+\frac{14}{rs}+\frac{9}{r^2}}$$

Gr. 11 Challenge Questions

(1) $$\frac{(x+y)^2-4}{5x^3+5xy-10x} \div \frac{(x+y)(x+2)+2(x+2)}{(3x+1)^2-(2x-1)^2}$$

$$= \frac{(x+y-2)(x+y+2)}{5x(x^2+y+2)} \times \frac{(3x+1)^2-(2x-1)^2}{(x+y)(x+2)+2(x+2)}$$

$$= \frac{(x+y-2)\cancel{(x+y+2)}}{5x(x^2+y+2)} \times \frac{(3x+1+2x-1)[3x+1-(2x-1)]}{(x+2)\cancel{(x+y+2)}}$$

$$= \frac{x+y-2}{5x(x^2+y+2)} \times \frac{(5x)(x+2)}{(x+2)}$$

$$= \frac{x+y-2}{x^2+y+2}.$$

note: not asked to state restrictions for this one.

(2) $$\frac{2x^3-8x^2+5x-20}{5x^2-40x+80}$$

$$= \frac{2x^3-8x^2+5x-20}{5(x^2-8x+16)}$$

$$= \frac{2x^3-8x^2+5x-20}{5(x-4)^2}$$

$$= \frac{(2x^2+5)(x-4)}{5(x-4)^2}$$

$$= \frac{2x^2+5}{5(x-4)}, \quad x \neq 4.$$

* if it reduces, $x-4$ divides into $2x^3-8x^2+5x-20$

long division

$$\begin{array}{r} 2x^2 \quad +5 \\ x-4 \overline{\big) 2x^3-8x^2+5x-20} \\ \underline{2x^3-8x^2} \\ 0+5x-20 \\ \underline{5x-20} \\ 0 \end{array}$$

synthetic division.

coefficients of $2x^3-8x^2+5x-20$.

$$\begin{array}{r|rrrr} 4 & 2 & -8 & 5 & -20 \\ & \downarrow & 8 & 0 & 20 \\ \hline & 2 & 0 & 5 & 0 \end{array}$$

$(x-4)$ → 4 (with sign opposite to value in bracket).

$2x^3-8x^2+5x-20 = (x-4)(2x^2+5)$

(3) $$\left[\frac{5m-10}{m^2-4} - \frac{2m-6}{m^2-7m+12}\right] \cdot \frac{3m^2-12m}{3m^2-26m+16}$$

m 48
A -26
-2, -24
÷3

$$= \left[\frac{5(\cancel{m-2})}{(\cancel{m-2})(m+2)} - \frac{2(\cancel{m-3})}{(\cancel{m-3})(m-4)}\right] \cdot \frac{3m(m-4)}{(3m-2)(m-8)}$$

$$= \frac{5(m-4)-2(m+2)}{(m+2)(\cancel{m-4})} \cdot \frac{3m(\cancel{m-4})}{(3m-2)(m-8)}$$

$$= \frac{(5m-20-2m-4)(3m)}{(m+2)(3m-2)(m-8)}$$

$$= \frac{(3m-24)(3m)}{(m+2)(3m-2)(m-8)}$$

$$= \frac{3(\cancel{m-8})(3m)}{(m+2)(3m-2)(\cancel{m-8})}$$

$$= \frac{9m}{(m+2)(3m-2)} \qquad m \neq -2, \tfrac{2}{3}, 8, 2, 3, 4$$

(4) $$\frac{\frac{y-2}{y+2} - \frac{y+2}{y-2}}{\frac{1}{2} + \frac{1}{y-2}}$$ Is the same as $\left[\frac{y-2}{y+2} - \frac{y+2}{y-2}\right] \div \left[\frac{1}{2} + \frac{1}{y-2}\right]$

$$= \frac{(y-2)^2-(y+2)^2}{(y+2)(y-2)} \div \left[\frac{(y-2)+2}{2(y-2)}\right]$$

$$= \frac{y^2-4y+4-(y^2+4y+4)}{(y+2)(y-2)} \times \frac{2(y-2)}{y}$$

$$= \frac{-8\cancel{y}}{(y+2)(\cancel{y-2})} \times \frac{2(\cancel{y-2})}{\cancel{y}}$$

$$= \frac{-16}{y+2} \qquad y \neq \pm 2, 0$$

(5) $$\frac{\frac{3}{r^2} - \frac{2}{rs} - \frac{5}{s^2}}{\frac{5}{s^2} + \frac{14}{rs} + \frac{9}{r^2}}$$

$$= \left[\frac{3}{r^2} - \frac{2}{rs} - \frac{5}{s^2}\right] \div \left[\frac{5}{s^2} + \frac{14}{rs} + \frac{9}{r^2}\right]$$

$$= \frac{3s^2 - 2rs - 5r^2}{r^2s^2} \div \frac{5r^2 + 14rs + 9s^2}{r^2s^2}$$

$$= \frac{3s^2 - 2rs - 5r^2}{r^2s^2} \times \frac{r^2s^2}{5r^2 + 14rs + 9s^2}$$

M −15
A −2
−5, 3
÷3

$$= \frac{(3s - 5r)(s + r)\, r^2s^2}{r^2s^2(5r + 9)(r + s)}$$

M 45
A 14
9, 5
÷5

$$= \frac{(3s - 5r)}{5r + 9}, \quad r \neq 0,\ s \neq 0,\ r \neq \frac{-9}{5}, -s$$