**Solutions:**

1. The graph of \( y = f(x) \) is shown. List the transformations and sketch the following functions.
   a) \( y = f(x) + 3 \) - vertical shift up 3 units
   b) \( y = -0.5f(x) - 1 \) - vertical compression by factor of 0.5
   - reflection in the x-axis
   - shifted down 1
   c) \( y = f\left(-\frac{1}{2}x\right) \) - horizontal stretch by a factor of 2
   - reflection in the y-axis
   d) \( y = -2f(-x) \) - vertical stretch by a factor of 2
   - reflection in the x-axis
   - reflection in the y-axis

2. Describe how the graph of each of the following functions can be obtained from the graph of \( y = f(x) \).
   a) \( y = f(x - 2) - 3 \) - horizontal shift right 2 units
   - vertical shift down 3 units
   b) \( y = -f(x + 5) - 1 \) - reflection in the x-axis
   - horizontal shift left 5 units
   - vertical shift down 1 unit
   c) \( y = 4f\left(-\frac{1}{5}x\right) + 5 \) - vertical stretch by a factor of 4
   - horizontal stretch by a factor of 5
   - reflection in the y-axis
   - vertical shift up 5 units
   d) \( y = 2f(-2x + 2) - 4 \) - vertical stretch by a factor of 2
   - horizontal compression by a factor of \( \frac{1}{2} \)
   - reflection in the y-axis
   - horizontal shift right 1 unit
   - vertical shift down 4 units

3. The graph of \( y = x^3 \) is expanded vertically by a factor of 2, translated 3 units to the left, and translated 4 units upward. Write the equation of the transformed function, and state its domain and range.
   \[ y = 2(x + 3)^2 + 4 \quad \text{D:} \{x \in \mathbb{R}\} \quad \text{R:} \{y \in \mathbb{R}\mid y \geq 4\} \]
4. The graph of \( f(x) = \frac{1}{x} \) is compressed horizontally by a factor of \( \frac{1}{2} \), reflected in the \( x \)-axis, and translated 4 units to the left. Write the new image equation, and state its domain and range.

\[
y = -\frac{1}{2(x+4)} \quad \text{or} \quad y = -\frac{1}{2(x+8)}
\]

\( D: \{ x \in \mathbb{R} | x \neq -4 \} \ \ R: \{ y \in \mathbb{R} | y \neq 0 \} \)

5. a) Given \( f(x) = x^2 - 4x + 5 \), write equations for \( -f(x) \) and \( f(-x) \).

\[
-f(x) = -(x^2 - 4x + 5) \quad \quad f(-x) = (-x)^2 - 4(-x) + 5 \\
-f(x) = -x^2 + 4x - 5 \quad \quad f(-x) = x^2 + 4x + 5
\]

b) Sketch the three graphs on the same set of axes.

***To graph the original, complete the square to determine the vertex or use a table of values***

![Graphs of f(x), -f(x), and f(-x)](image)

c) Determine any points that are invariant for each reflected function.

For \(-f(x)\) ➞ none

For \(f(-x)\) ➞ \((0, 5)\)

***From the equation \(f(-x)\), we can solve for the invariant point by substituting \(x=0\) into the equation to solve for \(y\), because we know that if there is only a reflection, the invariant point will lie on the line of reflection***

d) State the domain and range of each of the three functions.

\( f(x) \Rightarrow \quad D: \{ x \in \mathbb{R} \} \quad R: \{ y \in \mathbb{R} | y \geq 1 \} \)

\( -f(x) \Rightarrow \quad D: \{ x \in \mathbb{R} \} \quad R: \{ y \in \mathbb{R} | y \leq -1 \} \)

\( f(-x) \Rightarrow \quad D: \{ x \in \mathbb{R} \} \quad R: \{ y \in \mathbb{R} | y \geq 1 \} \)

6. The point \((4, -5)\) lies on the graph of \(h(x)\). Determine the new coordinate of the transformed function, \(y = 3h(-2x - 6) + 1\).

***Don’t forget to factor the -2 out to see the true translation***

(we multiply \(x\) by \(-0.5\) and subtract 3, and multiply \(y\) by 3 and add 1) \(\therefore\) the new point is \((-5, -14)\)

7. a) Given: \(f(x) = \sqrt{x+1}\)

i) Write the image equation for the transformation: \(y = f(x-2)+4\)
ii) State the domain and range of each function.
\[ f(x) = \sqrt{x+1} \quad D = \{ x \in R \mid x \geq -1 \} \quad R = \{ y \in R \mid y \geq 0 \} \]
\[ y = \sqrt{x-1} + 4 \quad D = \{ x \in R \mid x \geq 1 \} \quad R = \{ y \in R \mid y \geq 4 \} \]

iii) Graph both functions on the same grid.

b) Given: \( f(x) = \sqrt{x} - 1 \)
   i) Write the image equation for the transformation: \( y = f(x - 3) + 2 \)

   \[ y = \sqrt{(x-3) - 1} + 2 \]
   \[ y = \sqrt{x - 3 + 1} \]

   ii) State the domain and range of each function.

   \[ f(x) = \sqrt{x - 1} \quad D = \{ x \in R \mid x \geq 0 \} \quad R = \{ y \in R \mid y \geq -1 \} \]
   \[ y = \sqrt{x - 3 + 1} \quad D = \{ x \in R \mid x \geq 3 \} \quad R = \{ y \in R \mid y \geq 1 \} \]
   iii) Graph both functions on the same grid.

   c) Given: \( f(x) = \frac{1}{x+1} \)
   i) Write the image equation for the transformation: \( y = f(x+1) \)

   \[ y = \frac{1}{(x+1) + 1} \]
   \[ y = \frac{1}{x + 2} \]

   ii) State the domain and range of each function.

   \[ f(x) = \frac{1}{x+1} \quad D = \{ x \in R \mid x \neq -1 \} \quad R = \{ y \in R \mid y \neq 0 \} \]
\[ y = \frac{1}{x+2} \quad D = \{x \in R \mid x \neq -2\} \quad R = \{y \in R \mid y \neq 0\} \]

Graph both functions on the same grid.

\[ y = \frac{1}{x+2} \quad f(x) = \frac{1}{x+1} \]

\[ \frac{1}{x} + 2 \]

i) Write the image equation for the transformation: \( y = f(x-3) + 4 \)
\[ y = \frac{1}{(x-3)} + 2 + 4 \]
\[ y = \frac{1}{x-3} + 6 \]

ii) State the domain and range of each function.
\[ f(x) = \frac{1}{x} + 2 \quad D = \{x \in R \mid x \neq 0\} \quad R = \{y \in R \mid y \neq 2\} \]
\[ y = \frac{1}{x-3} + 6 \quad D = \{x \in R \mid x \neq 3\} \quad R = \{y \in R \mid y \neq 6\} \]

Graph both functions on the same grid.