

Unit 2 Lesson 1.

MAP 4CI

Trigonometry Intro

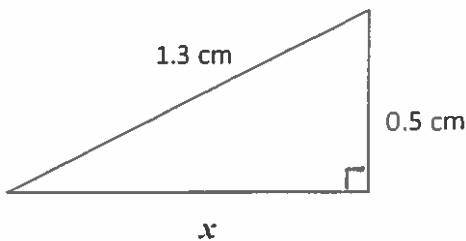
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**Set your calculator to DEGREE mode

1. **Pythagorean Theorem.** Draw a right triangle. Label the sides a, b and c (c must be the longest side). Side c is called the hypotenuse.

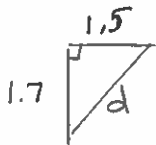
Now draw a square on each side of the triangle. State the relationship between the squares on the sides of the right triangle. $a^2 + b^2 = c^2$

Ex. 1 Determine the length of the indicated side.



$$\begin{aligned} x^2 + 0.5^2 &= 1.3^2 \\ x^2 &= 1.3^2 - 0.5^2 \\ x^2 &= 1.69 - 0.25 \\ x^2 &= 1.44 \\ x &= \sqrt{1.44} \\ \boxed{x = 1.2 \text{ cm}} \end{aligned}$$

Ex. 2 Brad walks 1.7 km North and then 1.5 km East along the sides of a park. Dan starts at the same point and takes a shortcut along the diagonal. How much shorter is Dan's walk?



Brad's walk is $1.7 \text{ km} + 1.5 \text{ km} = 3.2 \text{ km}$

Dan's walk is 2.3 km

So, Brad walked 0.9 km farther than Dan.

$$\begin{aligned} 1.7^2 + 1.5^2 &= d^2 \\ d^2 &= 2.89 + 2.25 \\ d^2 &= 5.14 \\ d &= 2.3 \end{aligned}$$

2. Solving Equations.

Ex. 1 Solve for x to the nearest tenth.

a) $\frac{12}{x} = \frac{20}{3}$

$$20x = 36$$

$$\frac{20x}{20} = \frac{36}{20}$$

$$x = 1.8$$

b) $\frac{6.7}{2.8} = \frac{x}{4.2}$

$$2.8x = 6.7 \times 4.2$$

$$2.8x = 28.14$$

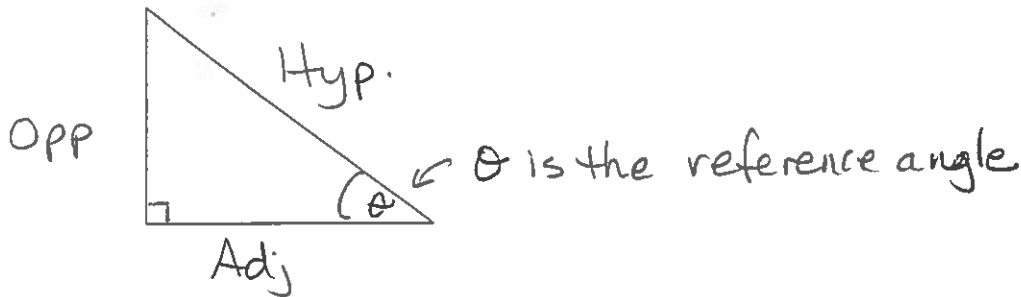
$$\frac{2.8x}{2.8} = \frac{28.14}{2.8}$$

$$x = 10.1$$

10.05

2.267

3. Primary Trig Ratios. Given a right triangle with angle θ (theta), label the sides "hypotenuse", side "opposite" to angle θ , and side "adjacent" to angle θ .



To remember the 3 primary trig. ratios of the sides of a right triangle relative to angle θ use SOH CAH TOA

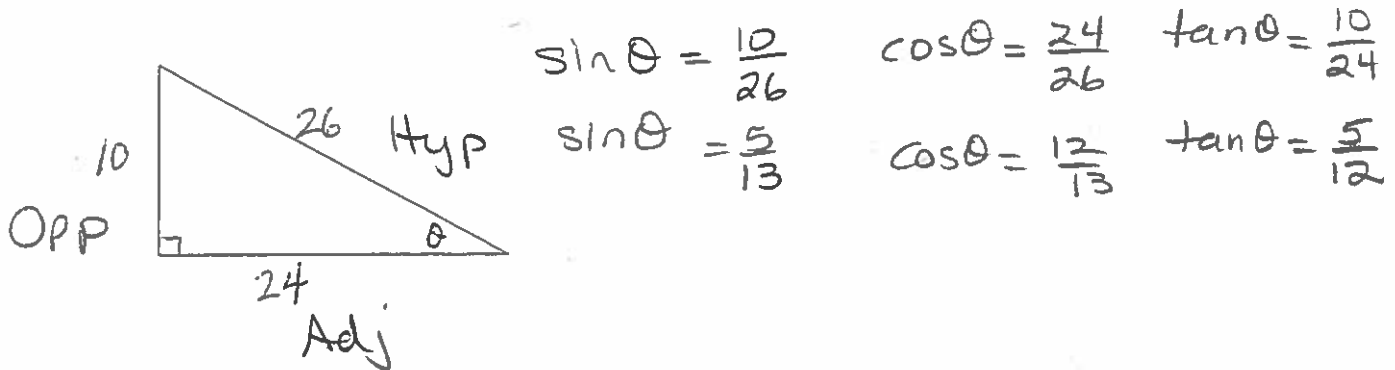
The 3 primary trig ratios are:

$$\text{sine } \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \sin \theta = \frac{O}{H}$$

$$\text{cosine } \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \cos \theta = \frac{A}{H}$$

$$\text{tangent } \theta = \frac{\text{Opposite}}{\text{Adjacent}} \quad \tan \theta = \frac{O}{A}$$

Ex. 1 Write the 3 primary trig ratios relative to θ .



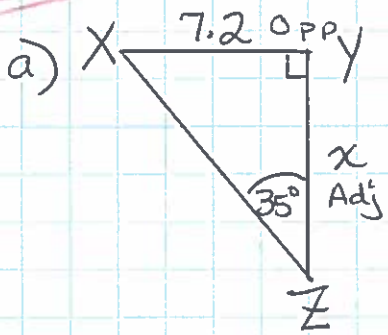
Ex. 2 Evaluate to four decimal places.

a) $\sin 54^\circ = 0.8090$ b) $\cos 14^\circ = 0.9703$ c) $\tan 61^\circ = 1.8040$

Practice: Pg 72 # 4 - 7, 9, 10 ab Check Answers: Pg. 540

Determining Lengths of Sides in Right Triangles

Ex. 1 Determine the length of x to the nearest tenth.



Have: $O = 7.2$

Need: side A

Use $\tan 35^\circ = \frac{O}{A}$

$$\tan 35^\circ = \frac{7.2}{x}$$

$$\frac{\tan 35^\circ}{1} \times \frac{7.2}{x}$$

Cross multiply.

$$\tan 35^\circ x = 7.2$$

$$1 \frac{(\tan 35^\circ)x}{\tan 35^\circ} = \frac{7.2}{\tan 35^\circ}$$

$$x = 10.28$$

$$x \approx 10.3$$

With \tan (if only with \tan) if the unknown is in denominator, you may switch the reference angle so the unknown is in the numerator

$$\angle X = 180^\circ - 90^\circ - 35^\circ$$

$$\angle X = 55^\circ$$

then $A = 7.2$
 $x = \text{Opp.}$

$$\tan 55^\circ = \frac{x}{7.2}$$

$$x = 7.2 \tan 55^\circ$$

$$x \approx 10.3$$

Recall:

Angle of Elevation/Inclination is measured UP from the HORIZONTAL

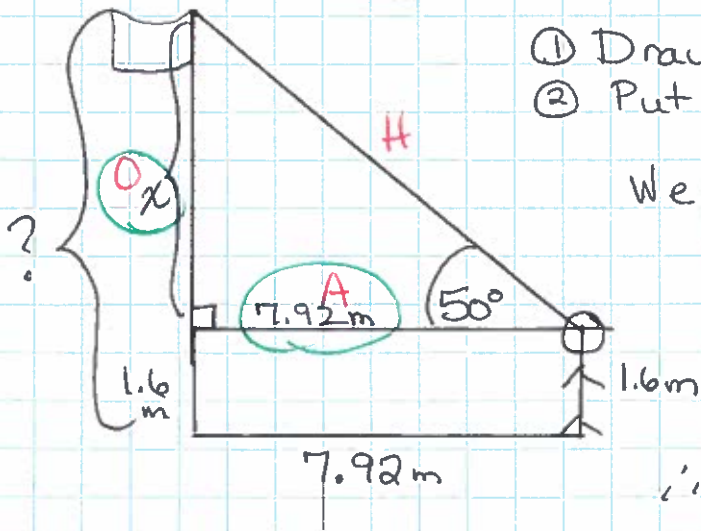


Angle of Depression/Declination is always measured DOWN from the HORIZONTAL



Ex. 2 Tanya is standing 7.92 m from the flagpole.

She is holding a clinometer at eye level 1.6 m above the ground. How tall is the flagpole if she measures a 50° angle of elevation?



① Draw and label diagram.

② Put A, H, O on diagram using 50° as reference angle.

We have A, looking for O so use \tan

$$\tan 50^\circ = \frac{x}{7.92}$$

$$x = 7.92 \tan 50^\circ$$

$$x \approx 9.4$$

\therefore the flagpole is $1.6 + 9.4 = 11.0$ m tall.

unit 2 lesson 3

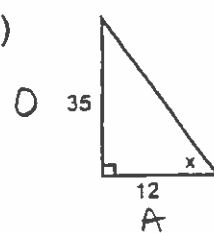
Determining Measures of Angles in Right Triangles

Trig ratios can also be used to find the measures of angles of a right triangle that are not known.

Examples: For the following triangles, identify the trig ratio to use, write the equation and solve it to one decimal place using the inverse trig buttons

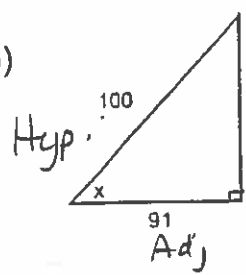
\sin^{-1} \cos^{-1} \tan^{-1} on your calculator.

a)



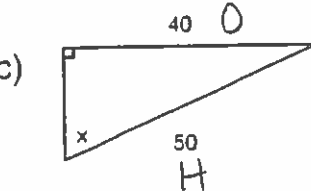
Have: $A=12$, $O=35$
 Need: x
 Use: $\tan x = \frac{O}{A}$
 $\tan x = \frac{35}{12}$
 $x = \tan^{-1}(35/12)$
 $x \approx 71.1^\circ$

b)



Have: $A=91$, $H=100$
 Need: x
 Use: $\cos x = \frac{A}{H}$
 $\cos x = \frac{91}{100}$
 $x = \cos^{-1}(0.91)$
 $x \approx 24.5^\circ$

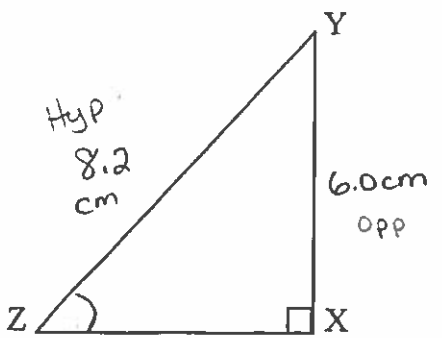
c)



Have: $O=40$, $H=50$
 Need: x
 Use: $\sin x = \frac{O}{H}$
 $\sin x = \frac{40}{50}$
 $x = \sin^{-1}(0.8)$
 $x \approx 53.1^\circ$

Ex. 2 Solve $\triangle XYZ$ given that $\angle X = 90^\circ$, $x = 8.2 \text{ cm}$, $z = 6.0 \text{ cm}$.

To solve means to determine the values of all missing sides and angles.



$\sin z = \frac{6}{8.2}$
 $z = \sin^{-1}(0.7317)$
 $z \approx 47^\circ$
 $y = 180^\circ - 90^\circ - 47^\circ$
 $y \approx 43^\circ$

$y^2 = x^2 - z^2$
 $y^2 = 8.2^2 - 6.0^2$
 $y^2 = 67.24 - 36$
 $y^2 = 31.24$
 $y = \sqrt{31.24}$
 $y \approx 5.6$

Unit 2 Lesson 4: Investigating Obtuse Angles

Introduction to the Activity:

In this activity, you will use your calculator and the following chart to investigate the trigonometric ratios of obtuse angles. Then, you will analyze the results to determine any patterns.

Performing the Activity

- 1) Refer to the chart that follows. For each of the listed angles, use your calculator to determine the value of each primary trigonometric ratio in the chart.
- 2) After you have completed the chart, answer the questions that follow.

Round values to 3 decimal places. There will be some rounding error.

Primary Angle, B	$\sin B$	$\cos B$	$\tan B$
5°	$\frac{\text{opp}}{\text{hyp}} \approx 0.087$	$\frac{\text{adj}}{\text{hyp}} \approx 0.996$	$\frac{\text{opp}}{\text{adj}} \approx 0.087$
10°	0.174	0.985	0.177
25°	0.423	0.906	0.466
30°	0.500	0.866	0.577
89°	1.000	0.017	57.290
Supp \angle 91°	1.000	-0.017	-57.290
150°	0.500	-0.866	-0.577
155°	0.423	-0.906	-0.466
170°	0.174	-0.985	-0.177
175°	0.087	-0.996	-0.087

Investigating Obtuse Angles (Continued)

After you have completed the chart, answer the following questions.

- 1) What do you notice about the signs (positive? negative?) of the values of $\sin B$? Be as specific as possible. Why does this happen?

$\sin B$ is always positive. (O, H are both positive)

- 2) What do you notice about the signs (positive? negative?) of the values of $\cos B$? Be as specific as possible. Why does this happen?

A, H both $> 0 \rightarrow \cos B$ is positive when B is acute (between 0° and 90°)
 A or $H < 0 \rightarrow \cos B$ is negative when B is obtuse (between 90° and 180°)

- 3) What do you notice about the signs (positive? negative?) of the values of $\tan B$? Be as specific as possible. Why does this happen?

O and H both $> 0 \rightarrow \tan B$ is positive when B is acute.
 O or $H < 0 \rightarrow \tan B$ is negative when B is obtuse.

- 4) Write down pairs of $\angle B$ that have approximately the same value for $\sin B$. Verify that the values are actually the same using your calculator. For example, check that $\sin 5^\circ$ and $\sin 175^\circ$ give the same value. How are the angles related to each other?

$\sin 10^\circ = \sin 170^\circ$ $\sin 89^\circ = \sin 91^\circ$
 $\sin 25^\circ = \sin 155^\circ$ Sine of Supplementary angles (angles that add to 180°)
 $\sin 30^\circ = \sin 150^\circ$ are equal.

Using the same pairs of angles, what do you notice about the values of $\cos B$? (Verify on your calculator if needed.)

Cosine of supplementary angles are opposites.

Using the same pairs of angles, what do you notice about the values of $\tan B$? (Verify on your calculator if needed.)

Tangent of supplementary angles are opposites.

- 5) Use \sin^{-1} on your calculator to solve for angle B in $\sin B = 0.5$. What value does your calculator give? 30°

What other value for B is possible? $180^\circ - 30^\circ = 150^\circ$

How can you quickly determine the value of the second angle? $180^\circ - \text{first angle}$.

Complete the following using a calculator and what you have learned:

$$\sin B = 0.7660$$

$$B = \underline{50^\circ} \quad \text{or} \quad B = \underline{180^\circ - 50^\circ = 130^\circ}$$

$$\sin B = 0.9205$$

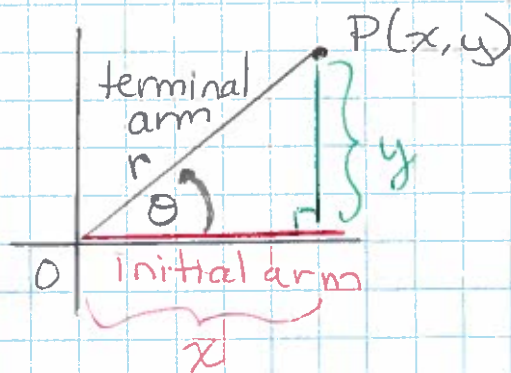
$$B = \underline{67^\circ} \quad \text{or} \quad B = \underline{180^\circ - 67^\circ = 113^\circ}$$

Unit 2 Lesson 4 Obtuse Angles In Standard Position.

Angles in standard position.

- * You will be given an ordered pair
- * plot that point on the Cartesian Plane
- * join that point to the origin
(this line segment is called the terminal arm)
- * draw the 'initial arm' on the positive x-axis beginning at the origin
- * θ is measured from the initial arm counter-clockwise to the terminal arm.

To find the primary trig ratios, drop a vertical line segment from the plotted point to the x-axis. This will form a right triangle.



Unit 2 Lesson 4 part 2

2.2

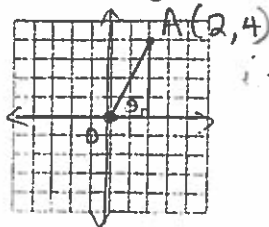
Trigonometric Ratios With Obtuse Angles

Section
2.2

Practise

1. The terminal arm of an angle, θ , in standard position passes through A(2, 4).

a) Sketch a diagram for this angle in standard position.



- b) Determine the length of OA.



$$h^2 = 2^2 + 4^2$$

$$h = \sqrt{20}$$

$$\overline{OA} = \sqrt{20}$$

- c) Determine the primary trigonometric ratios to three decimal places.

$$\sin \theta = \frac{4}{\sqrt{20}}$$

$$\approx 0.894$$

$$\cos \theta = \frac{2}{\sqrt{20}}$$

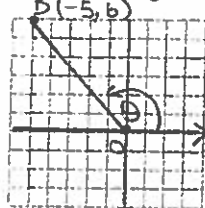
$$\approx 0.447$$

$$\tan \theta = \frac{4}{2}$$

$$= 2$$

2. The terminal arm of an angle, θ , in standard position passes through B(-5, 6).

a) Sketch a diagram for this angle in standard position.



- b) Determine the length of OB.



$$|\overline{OB}|^2 = 5^2 + 6^2$$

$$= 25 + 36$$

$$|\overline{OB}| = \sqrt{61}$$

- c) Determine the primary trigonometric ratios to three decimal places.

$$\sin \theta = \frac{6}{\sqrt{61}}$$

$$\approx 0.768$$

$$\cos \theta = \frac{-5}{\sqrt{61}}$$

$$\approx -0.640$$

$$\tan \theta = \frac{-6}{5}$$

$$= -1.2$$

3. Complete the table. For each angle, indicate whether each trigonometric ratio is positive or negative. Round your answers to three decimal places.

Angle	Sine	Cosine	Tangent
60°	0.866	0.5	1.732
120°	0.866	-0.5	-1.732
98°	0.990	-0.139	-7.115
145°	0.574	-0.819	-0.700
162°	0.309	-0.951	-0.325

1. The sine of an obtuse angle, θ , in standard position is $\frac{3}{5}$.

a) Identify the coordinates of a point that lies on the terminal arm of $\angle\theta$.

$$\sin \theta = \frac{3}{5} \quad \leftarrow \begin{array}{l} y \\ r \end{array}$$

$$x^2 + y^2 = r^2$$

$$x^2 + 3^2 = 5^2$$

$$x^2 = 25 - 9$$

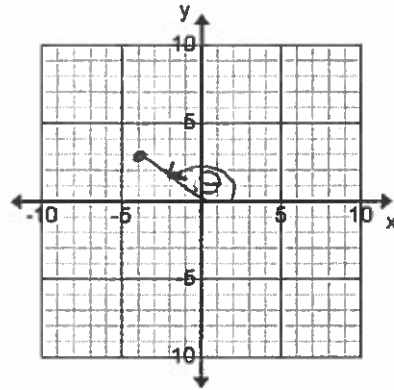
$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = 4$$

Point is at $(-4, 3)$

b) Sketch a diagram of $\angle\theta$.



c) Determine $\cos \theta$ and $\tan \theta$.

$$\cos \theta = \frac{-4}{5}$$

$$\tan \theta = \frac{-3}{4}$$

$$\cos \theta = -0.8$$

$$= -0.75$$

d) Determine the measure of $\angle\theta$, using a calculator.

$$\theta = \cos^{-1}(-0.8)$$

$$\theta \approx 143^\circ$$

2. The tangent of an obtuse angle, θ , in standard position is -1 .

a) Identify the coordinates of a point that lies on the terminal arm of $\angle\theta$.

$$\tan \theta < 0 \text{ so } x < 0, y > 0$$

$$\tan \theta = \frac{-1}{1}$$

$$\text{so } \frac{y}{x} = \frac{1}{-1}$$

$$x = -1, y = 1$$

$$r = \sqrt{(-1)^2 + 1^2}$$

$$r = \sqrt{2}$$

c) Determine $\sin \theta$ and $\cos \theta$. Round your answers to three decimal places.

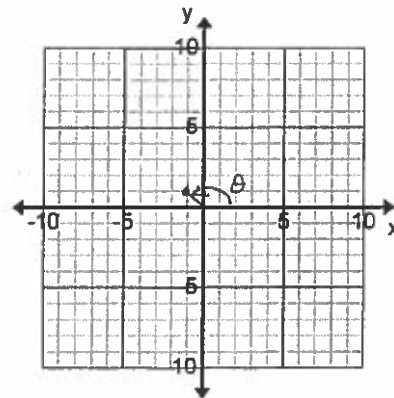
$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{-1}{\sqrt{2}}$$

$$\approx 0.707$$

$$\approx -0.707$$

b) Sketch a diagram of $\angle\theta$.



d) Determine the measure of $\angle\theta$, using a calculator.

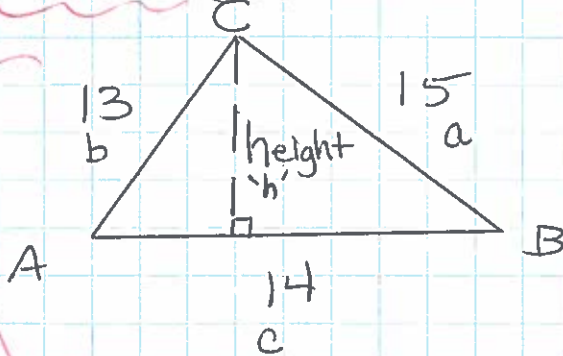
$$\theta = \cos^{-1}(-0.707)$$

$$\theta \approx 135^\circ$$

Sine Law for Oblique Triangles

↳ non right-angled triangles.

General Case



Create two right triangles
So we can use
trig. ratios.

• where
sine
law
comes
from

$$\sin A = \frac{O}{H}$$

$$\frac{\sin A}{1} = \frac{h}{13}$$

$$h = 13 \sin A$$

$$\text{So, } 13 \sin A = 15 \sin B$$

$$\frac{\sin A}{15} = \frac{\sin B}{13}$$

↑ ↑
a b

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin B = \frac{O}{H}$$

$$\frac{\sin B}{1} = \frac{h}{15}$$

$$h = 15 \sin B$$

↳ ÷ 13, ÷ 15
(undo the cross
multiplying).

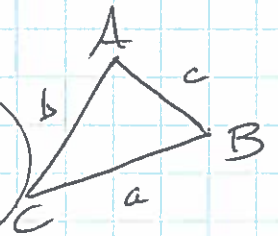
This can be extended to include C.

Sine Law

(02)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

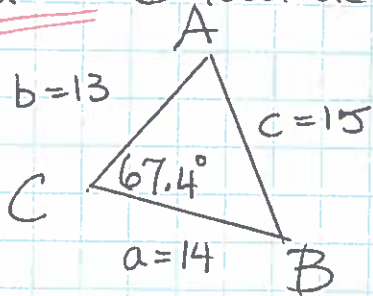
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



To use the sine law you need one complete
angle/side pair. (You need A, a @ B, b @ C, c)

Unit 2 lesson 6 Sine Law (cont'd) pg. 2:

Ex. 1 Calculate the value of angle A and angle B.



Round to one decimal place

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{14} = \frac{\sin B}{13} = \frac{\sin 67.4^\circ}{15}$$

$$\frac{\sin A}{14} \rightarrow \frac{\sin 67.4^\circ}{15}$$

$$\sin A = \frac{\sin 67.4^\circ \times 14}{15}$$

$$\sin A = 0.8616628$$

$$A = \sin^{-1}(0.8616628)$$

$$A = 59.50$$

$$\boxed{A \approx 59.5^\circ}$$

$$B = 180^\circ - 59.5^\circ - 67.4^\circ$$

$$\boxed{B = 53.1^\circ}$$

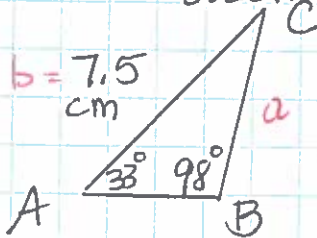
→ check →

$$\frac{\sin B}{13} = \frac{\sin 67.4^\circ}{15}$$

$$B = \sin^{-1}\left(\frac{\sin 67.4^\circ \times 13}{15}\right)$$

$$B = 53.1^\circ$$

Ex. 2. Determine the length of a to one decimal place.



$$\frac{a}{\sin 33^\circ} = \frac{7.5}{\sin 98^\circ}$$

$$a = \frac{7.5 \times \sin 33^\circ}{\sin 98^\circ}$$

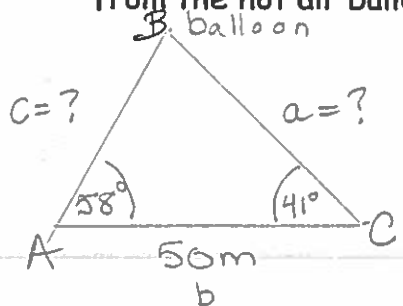
$$a = 7.5 \times \sin 33^\circ \div \sin 98^\circ$$

$$a \approx 4.1249$$

$$\boxed{a \approx 4.1 \text{ cm}}$$

APPLICATIONS OF SINE LAW

1. Two people stand approximately 50 m apart on level ground. One person measures the angle of elevation of a hot air balloon to be 58° . The other person measures the angle of elevation to be 41° . How far is each person from the hot air balloon?



$$\frac{a}{\sin 58^\circ} = \frac{50}{\sin 81^\circ} = \frac{c}{\sin 41^\circ}$$

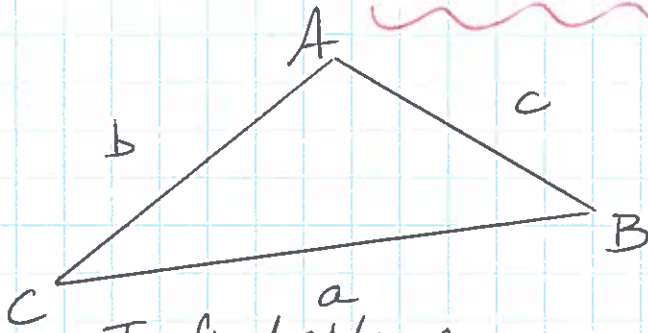
$$a = \frac{50 \sin 58^\circ}{\sin 81^\circ} = 42.9 \text{ m}$$

$$c = \frac{50 \sin 41^\circ}{\sin 81^\circ} = 33.2 \text{ m}$$

$$\angle B = 180^\circ - 58^\circ - 41^\circ$$

\therefore the first person is 33.2 m from the balloon, the second person is 42.9 m from the balloon.

Unit 2 lesson 8 Cosine Law (for oblique triangles).



To find side a:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

adjust formula accordingly if you are looking for another side

For E.g., q in ΔPQR

$$q^2 = p^2 + r^2 - 2pr \cos Q.$$

To find angle A

$$\cos A = \frac{(b^2 + c^2 - a^2)}{(2bc)}$$

OR

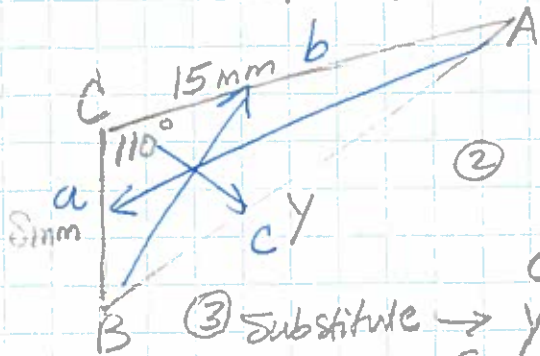
$$A = \cos^{-1} \left[\frac{(b^2 + c^2 - a^2)}{(2bc)} \right]$$

Use Cosine law if you have all 3 sides and you are looking for an angle.

- if you have 2 sides and the contained angle and you are looking for third side.

Cosine Law Examples Unit 2 lesson 8. pg. ②

Ex. 1 Determine length of indicated side to the nearest tenth.



① Label diagram

② Formula:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

③ Substitute $\rightarrow y^2 = 8^2 + 15^2 - 2(8)(15)\cos 110$
 $= 64 + 225 - (-82.0848344)$

④ Calculate

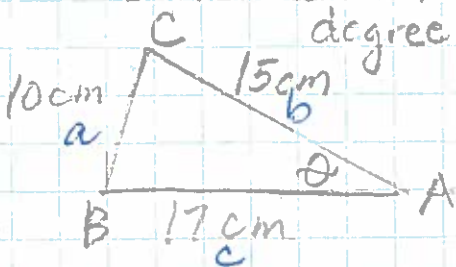
$$y^2 = 289 + 82.0848...$$

$$y = \sqrt{371.0848...}$$

$$y = \underline{19.3 \text{ mm}}$$

↑
Keep all decimal places

Ex. 2 Determine measure of indicated angle to nearest degree.



① Label diagram

② Formula:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\angle A = \cos^{-1} \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$$

③ Substitute $\angle A = \cos^{-1} \left(\frac{15^2 + 17^2 - 10^2}{2(15)(17)} \right)$
 $= \cos^{-1} \left(\frac{225 + 289 - 100}{510} \right)$

$$= \cos^{-1} \left(\frac{414}{510} \right)$$

$$= \cos^{-1} (0.81176...)$$

$$= 35.7^\circ$$

$$= 36^\circ$$

Practice: Pg. 110 # 1a, 3, 4a, 5, 6, 7a.
 ✓ Answers Pg. 541-542 2, 4

Unit 2 Lesson 9: Cosine Law Applications

Review Cosine Law:

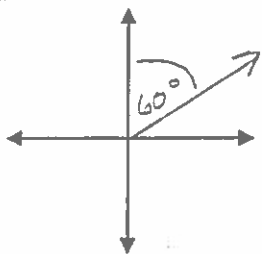
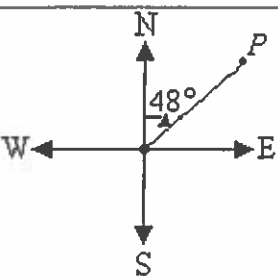
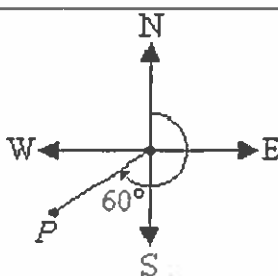
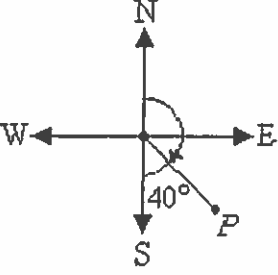
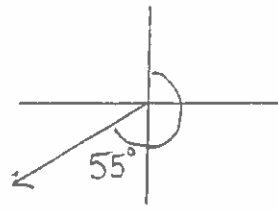
The Cosine Law can be used to solve for an unknown side, if you are given two sides and a contained angle:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

It can also be re-arranged to solve for an unknown angle:

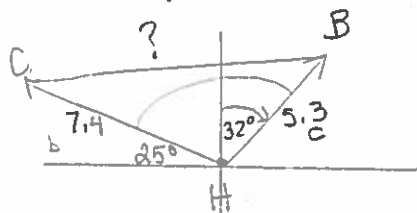
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Bearings: Direction can be written in several ways

Direction	bearing	Diagram			
N60°E	060° measured clockwise from North.				
Diagram	Bearing	Direction	Diagram	Bearing	Direction
	48°	N48°E		180° + 60° 240°	S60°W
	180° - 40° 140°	S40°E	Provide a sketch here. 	235°	S55°W

Applications of Cosine Law lesson 9 page 2

1. A harbour master uses radar to monitor two ships, B and C, as they approach the harbour, H. One ship is 5.3 miles from the harbour on a bearing of 032° . The other ship is 7.4 miles away from the harbour on a bearing of 295° . How far apart are the two ships?



$$h^2 = b^2 + c^2 - 2bc \cos H$$

$$h^2 = 7.4^2 + 5.3^2 - 2(7.4)(5.3) \cos 97^\circ$$

$$h^2 = 92.4094313$$

$$h = 9.6$$

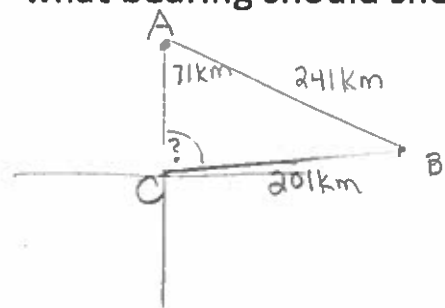
$$\angle BHC = 32^\circ + (90^\circ - 25^\circ)$$

$$= 32^\circ + 65^\circ$$

$$= 97^\circ$$

\therefore the two ships are about 9.6 miles apart.

2. An aircraft navigator knows that town A is 71 km due north of the airport, town B is 201 km from the airport, and towns A and B are 241 km apart. On what bearing should she plan the course from the airport to town B?



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{201^2 + 71^2 - 241^2}{2(201)(71)}$$

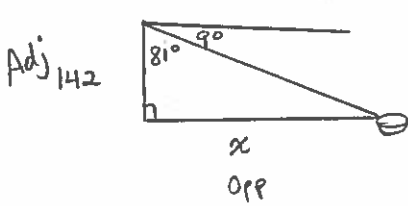
$$C = \cos^{-1}(-12639 \div 28542)$$

$$C = \cos^{-1}(-0.44282)$$

$$C \approx 116^\circ$$

\therefore the plane should plan the course on a bearing of 116° .

Ex. 1 From the top of a vertical cliff a person measures the angle of depression of a boat as 9° . The height of the cliff is 142 m. How far is the boat from the base of the cliff? Round your answer to the nearest m.



$$\tan 81^\circ = \frac{x}{142}$$

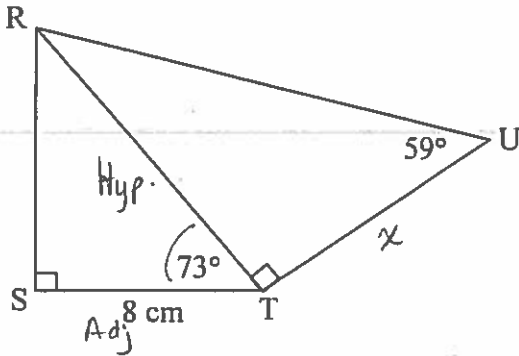
$$x = 142 \tan 81^\circ$$

$$x = 896.5527$$

$$x \approx 897 \text{ m}$$

\therefore the boat is about 897 m from the base of the cliff.

Ex. 2 Find the length of TU to the nearest tenth.

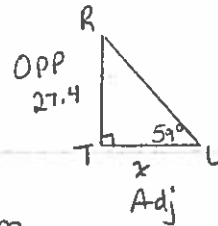


In ΔRTU ,

$$\cos 73^\circ = \frac{8}{H}$$

$$H = \frac{8}{\cos 73^\circ}$$

$$H \approx 27.362 \text{ cm}$$



In ΔRTU

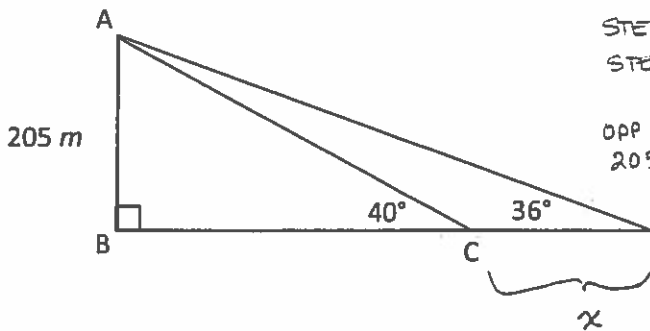
$$\tan 59^\circ = \frac{27.362}{x}$$

$$x = \frac{27.362}{\tan 59^\circ}$$

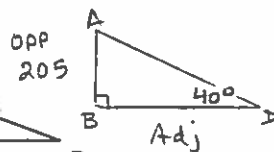
$$x \approx 16.4$$

\therefore TU is 16.4 cm

Ex. 3 A smokestack, AB, is 205m high. From two points C and D on the same side of the smokestack's base B, the angles of elevation to the top of the smokestack are 40° and 36° respectively. Find the distance between C and D to the nearest metre.



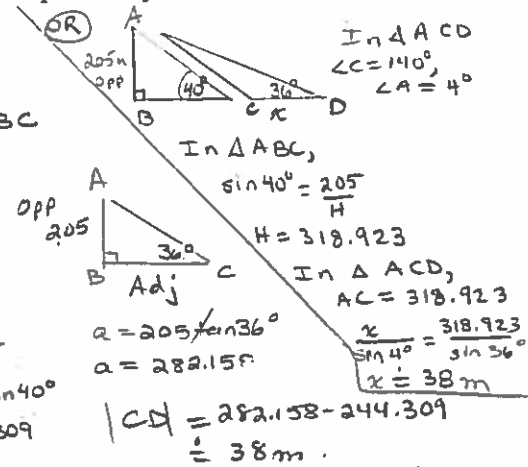
- STEP ① Find BD
- STEP ② Find BC
- STEP ③ $CD = BD - BC$



$$\tan 40^\circ = \frac{205}{a}$$

$$a = \frac{205}{\tan 40^\circ}$$

$$a = 244.309$$



In ΔABC ,

$$\sin 40^\circ = \frac{205}{H}$$

$$H = 318.923$$

In ΔACD ,

$$AC = 318.923$$

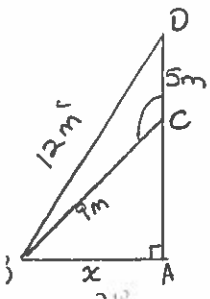
$$\frac{x}{\sin 40^\circ} = \frac{318.923}{\sin 36^\circ}$$

$$x \approx 38 \text{ m}$$

$$|CD| = 282.158 - 244.309$$

$$\approx 38 \text{ m}$$

Ex. 4 Two guy-wires are anchored at the same point. The first guy-wire is 12 m in length and is attached to the top of a tower. The second guy-wire is 9 m in length and is attached to a point 5 m below the top of the tower. How far are the wires anchored from the base of the tower? Round your answer to the nearest tenth of a metre.



In ΔBCD

$$C = \cos^{-1} \left(\frac{5^2 + 9^2 - 12^2}{2(5)(9)} \right)$$

$$C = \cos^{-1} \left(-\frac{38}{90} \right)$$

$$C = 114.97^\circ$$

In ΔABC ,

$$\angle C = 180^\circ - 114.97^\circ$$

$$\angle C = 65.03^\circ$$

$$\sin 65.03^\circ = \frac{x}{9}$$

$$x = 9 \sin 65.03^\circ$$

$$x \approx 8.15$$

$$x \approx 8.2 \text{ metres}$$

\therefore the guy wires are attached 8.2 m from the base of the tower.

Unit 2 Lesson 11 part 1: Review

FOR RIGHT ANGLE TRIANGLES

PYTHAGOREAN Formula

$$a^2 + b^2 = c^2 \text{ where } c \text{ must be the hypotenuse}$$

SOHCAHTOA

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}, \cos(\theta) = \frac{\text{adj}}{\text{hyp}}, \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

FOR OBLIQUE TRIANGLES (non-right angled, including obtuse triangles)

Sine Law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ or } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use when you have

- 2 sides and one opposite angle
- 2 angles and one opposite side

** need a complete pair.*

Cosine Law

$$a^2 = b^2 + c^2 - 2bc \cos A \text{ or } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Use when you have

- 2 sides and a contained angle (find the opposite side)
- All three sides (find an angle)

Obtuse Angles

Obtuse angle - $90^\circ \leq \theta \leq 180^\circ$

Supplementary Angles $A+B=180^\circ$

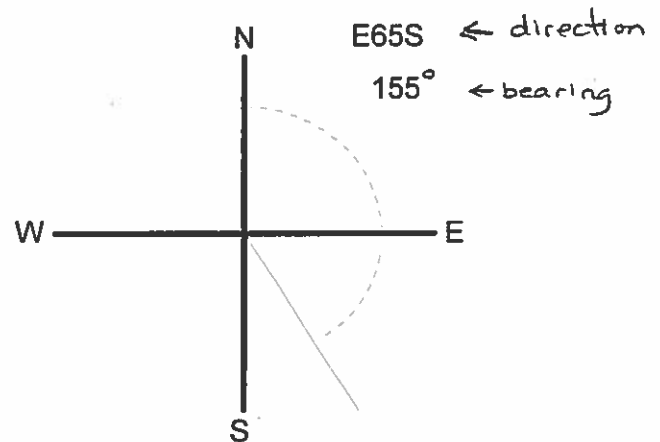
$$\sin A = \sin B, \quad \cos A = -\cos B, \quad \tan A = -\tan B$$

The primary trigonometric ratios of an angle, θ , in standard position are defined in terms of the coordinates of a point, (x, y) , on the terminal arm, as follows:

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \quad \text{where } r = \sqrt{x^2 + y^2}$$

Bearing and Directions

Bearings - 050° , Directions - $N50^\circ E$



Types of Problems

Directions,

Solve a Triangle

Area

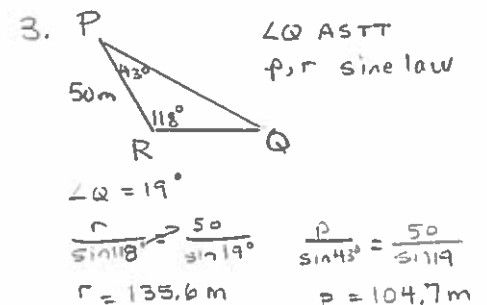
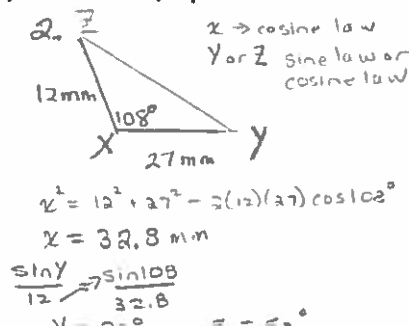
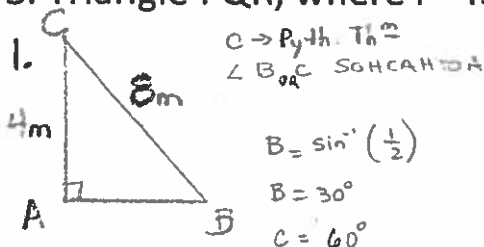
Practice Drawing Triangles.

Draw the following triangles, state unknowns and approach to solving:

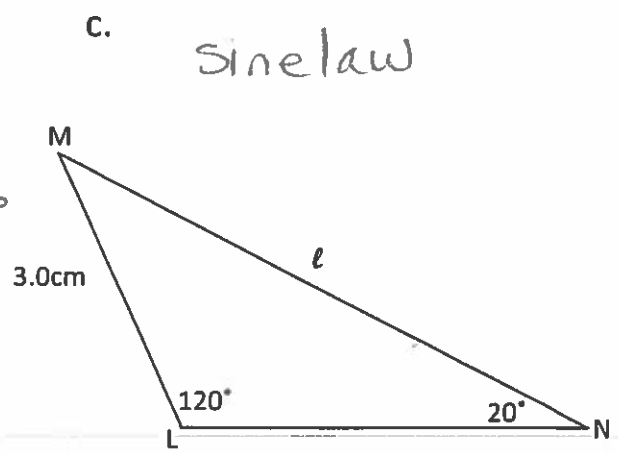
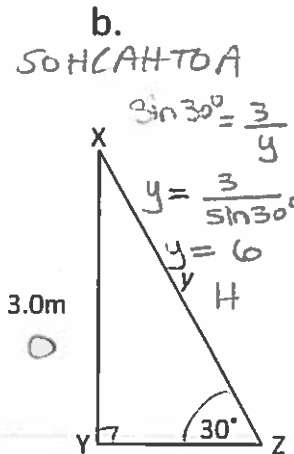
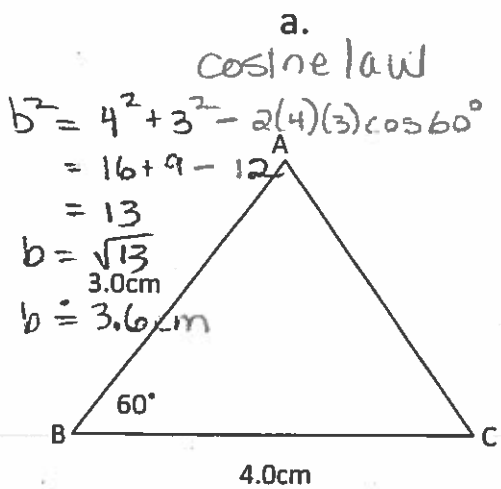
1. Triangle ABC, where $a=8m$, $b=4m$, $A=90^\circ$

2. Triangle XYZ, where $X=108^\circ$, $z=27mm$, $y=12mm$.

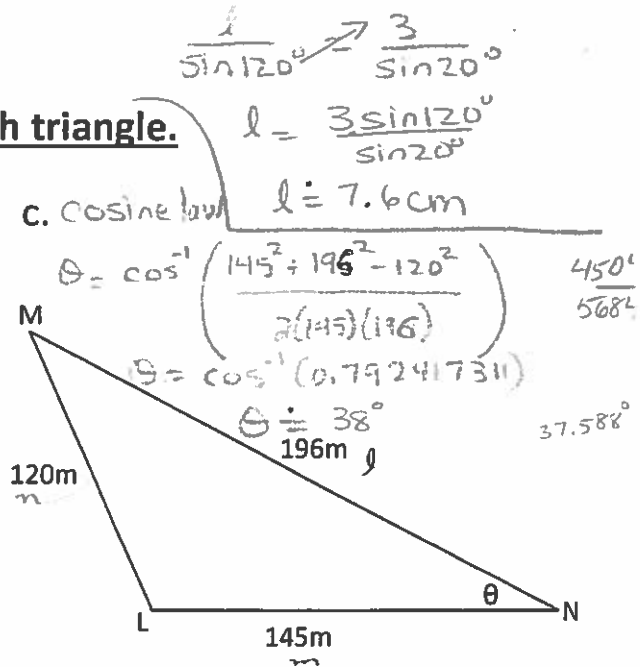
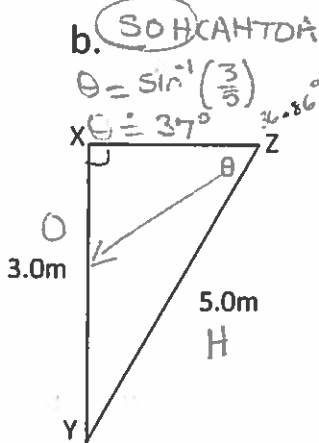
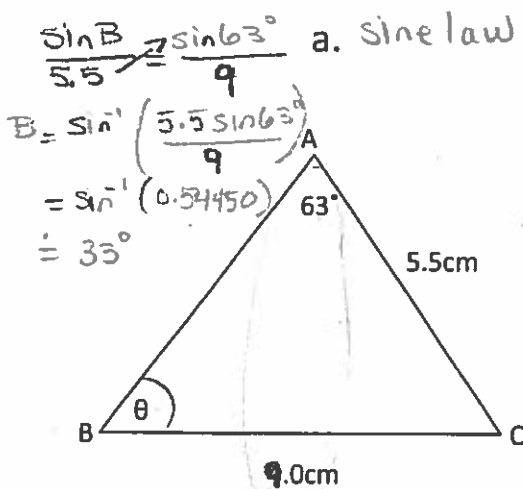
3. Triangle PQR, where $P=43^\circ$, $R=118^\circ$, $q=50m$.



Example #1 : Calculate the length of the unknown side in each triangle.



Example #2 : Calculate the indicated angle in each triangle.



Example #3

A boat is proceeding on a bearing of 045° at 12 km/hr . At $3:00 \text{ PM}$ the captain sees a navigation buoy at 020° . He sights the same buoy at 230° at $4:15$. How many km's is the boat from the buoy at $4:15 \text{ PM}$?

$$1.25 \times 12 = 15$$

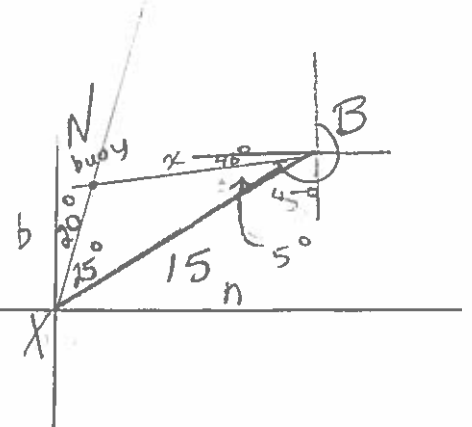
- Draw the figure
- Determine what Trig Rules to use sine law
- Solve for unknown.

$$\angle N = 180^\circ - 25^\circ - 5^\circ$$

$$\angle N = 150^\circ, n = 15 \quad X = 25^\circ$$

$$\frac{x}{\sin 25^\circ} = \frac{15}{\sin 150^\circ}$$

$$x = \frac{15 \sin 25^\circ}{\sin 150^\circ}$$



Deciding how to solve a triangle?

Formula	Picture	When to use	
Pythagorean $a^2 = b^2 + c^2$		Right angle triangle - given 2 sides	- asked to find third side
Trig Ratios SOHCAHTOA		Right angle triangle - given two sides - given one side and an angle	- asked to find angle - asked to find side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$		No right angle - given two angles and one opposite side - given two sides and one opposite angle	- asked to find other opposite side - asked to find other opposite angle
Cosine Law $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$		No right angle - given two sides & a contained angle - given three sides	- calculate the third side - can calculate angle