

# Day 1 Unit 2 Geometry: Optimization Problems

For a 4-sided rectangle:

1. a) A square has the minimum perimeter,  $6m \times 3$

$$\begin{aligned} A &= s^2 & A &= 36 & P &= 4s \\ s^2 &= 36 & & & &= 4(6) \\ s &= 6m & & & &= 24m \end{aligned}$$

b) A 6m by 6m square gives minimum fencing

c) A square.

- d) 1. Use  $A = s^2$  formula for area of a square  
 2. Sub. for A, calculate s.  $s = \sqrt{A}$   
 3. Use  $P = 4s$  formula for perimeter  
 4. Sub for s. Calculate P.

$$\begin{aligned} A &= 220.25 m^2 & 3. P &= 4s \\ 1. s^2 &= 220.25 & 4. P &= 4(14.8) \\ 2. s &= \sqrt{220.25} & &= 59.2 \\ & s = 14.8 & & \end{aligned}$$

$\therefore$  minimum perimeter of the garden is 59.2.

$$\begin{aligned} A &= 27 m^2 \\ 1. A &= s^2 & 3. P &= 4s \\ 2. s^2 &= 27 & 4. P &= 4(5.2) \\ & s = \sqrt{27} & &= 20.8 \\ & s = 5.2 & & \end{aligned}$$

$\therefore$  minimum perimeter of the garden is 20.8

2. For a 3-sided rectangle when the length is twice the width it gives a minimum perimeter.

a)  $A = lw$   $A = 72$   
 $A = (2w)w$

$$\begin{aligned} (2w)(w) &= 72 & l &= 2w \\ 2w^2 &= 72 & &= 2(6) \\ w^2 &= 36 & &= 12 \\ w &= 6 & & \end{aligned}$$

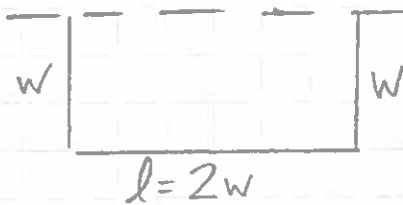
w	A = 72 playground
school	
$P = l + 2w$ $= 12 + 2(6)$ $= 24$	

$\therefore$  minimum perimeter is 24m

2 b) Dimensions 6m by 12m give the minimum amount of fencing.

c) A rectangle with a length that is twice the width gives a minimum amount of fencing. ( $2 \text{ widths} + 1 \text{ length}$ )

d)  $l = 2w$



$P = l + 2w$   
 $= 2w + 2w$   
 $= 4w$

\* e)  $A = lw$   
 $A = (2w)w$   
 $A = 2w^2$  is the formula for area of a rectangle minimum perimeter.

① \*  $A = 2w^2$   $A = 50 \text{ m}^2$   $P = l + 2w$   
 $2w^2 = 50$   $l = 2w$   $= 2w + 2w$   
 $w^2 = 25$   $= 2(5)$   $= 4w$   
 $w = 5$   $= 10$   $= 4(5)$   
 $= 20$

∴ minimum perimeter for fencing is 20 m

②  $A = 2w^2$   $A = 112.5 \text{ m}^2$   $P = 4w$   
 $2w^2 = 112.5$   $= 4(7.5)$   
 $w^2 = 56.25$   $= 30$   
 $w = \sqrt{56.25}$   
 $w = 7.5$

∴ minimum perimeter for fencing is 30 m

3. Surface Area of a rectangular based prism.

a) The optimum shape of a rectangular based prism to minimize the surface area (and maximize it) is a cube (height = side length of base).

b)  $V = s^3$   $V = 343 \text{ cm}^3$  ∴ the cube has dimens.  
 $s^3 = 343$  7cm by 7cm by 7cm.  
 $s = \sqrt[3]{343}$

3.c) The cube with height = side length of base is a maximum Volume

d) Formula for minimum surface area and maximum volume:

$$\text{SA of each face} = s^2$$

$$\text{SA of 6 faces} = 6s^2$$

∴  $SA = 6s^2$  is the formula for the surface of a cube, which gives a minimum surface area and maximum volume.

①  $V = 729$     $V = s^3$

$$s^3 = 729$$

$$s = \sqrt[3]{729}$$

$$s = 9$$

$$\text{SA} = 6s^2$$

$$= 6(9)^2$$

$$= 486$$

∴ minimum surface area of the cube is 486

②  $V = 1500$     $V = s^3$

$$s^3 = 1500$$

$$s = \sqrt[3]{1500}$$

$$s \approx 11.4$$

$$\text{SA} = 6s^2$$

$$= 6(11.4)^2$$

$$\approx 779.8$$

∴ minimum surface area of the cube is