MCR 3UI

Name: ______________________

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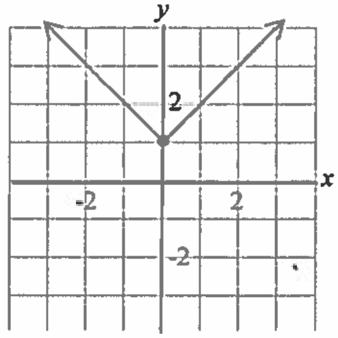
UNIT 3 TEST

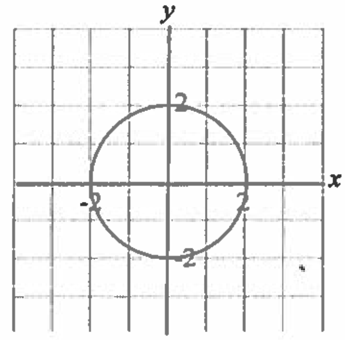
[9] 1. For each relation below, state whether it is a function and its domain and range.

a) $\{(1, 9), (2, 7), (3, 5), (4, 3)\}$

b)



c)



a) Function (Y/N): Yes
Domain: $\{1, 2, 3, 4\}$
Range: $\{3, 5, 7, 9\}$

b) Function (Y/N): Yes
Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y \geq 1\}$

c) Function (Y/N): No
Domain: $\{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$
Range: $\{y \in \mathbb{R} \mid -2 \leq y \leq 2\}$

[5] 2. If $f(x) = 3 - 2x$, find:

a) $f(1)$

$f(1) = 3 - 2(1)$
$= 1$

b) $f(3a - 1)$

$f(3a-1) = 3 - 2(3a-1)$
$= 3 - 6a + 2$
$= 5 - 6a$

c) x when $f(x) = -7$

$-7 = 3 - 2x$
$-10 = -2x$
$x = 5$

[4] 3. For each function below, find its inverse $f^{-1}(x)$. Show your work.

a) $f(x) = \dfrac{2x-5}{7}$

$y = \dfrac{2x-5}{7}$

for f^{-1},

$\dfrac{2y-5}{7} = x$

$2y - 5 = 7x$

$2y = 7x + 5$

$y = \frac{7}{2}x + \frac{5}{2}$

$\therefore f^{-1} = \frac{7}{2}x + \frac{5}{2}$

b) $f(x) = 1 + \sqrt{x+2}$

note: $y = 1 + \sqrt{x+2}$, $x \geq -2$

for f^{-1},

$1 + \sqrt{y+2} = x$

$\sqrt{y+2} = x - 1$

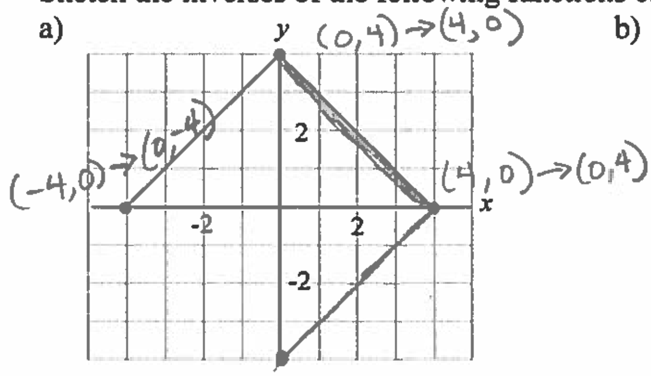
$y + 2 = (x-1)^2$

$y = (x-1)^2 - 2$ for $x \geq -2$

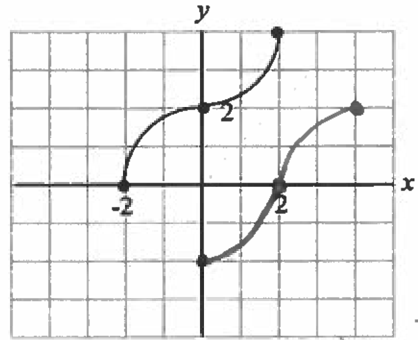
$\therefore f^{-1} = (x-1)^2 - 2$, $x \geq -2$.

[4] 4. Sketch the inverses of the following functions on the same grids they are drawn.

a)



b)



← be sure curve is correct direction for reflection in line $y=x$

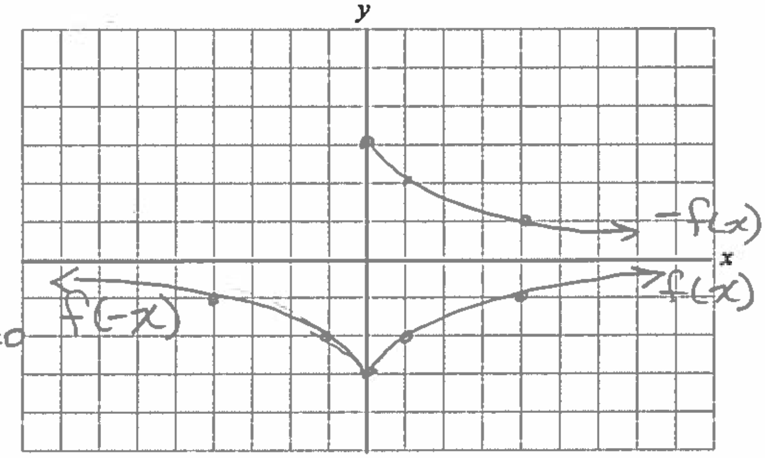
[10] 5. Given $f(x)=\sqrt{x}-3$,

a) Write equations for:

$-f(x) =$ $-(\sqrt{x}-3) = -\sqrt{x}+3$

$f(-x) =$ $\sqrt{-x}-3$

b) Sketch all three graphs on the same set of axes. Label each curve.



c) Determine any points that are invariant for each reflection.

$\sqrt{x}-3=0$
$\sqrt{x}=3$
$x=9$

invariant points when $y=0$

$-f(x)$: $(9,0)$

invariant points when $x=0$

$f(-x)$: $(0,-3)$

$\sqrt{0}-3$
$=-3$

d) State the domain and range for all three functions.

$f(x)$:
D: $\{x\in\mathbb{R} \mid x\geq 0\}$
R: $\{y\in\mathbb{R} \mid y\geq -3\}$

$-f(x)$:
D: $\{x\in\mathbb{R} \mid x\geq 0\}$
R: $\{y\in\mathbb{R} \mid y\leq 3\}$

$f(-x)$:
D: $\{x\in\mathbb{R} \mid x\leq 0\}$
R: $\{y\in\mathbb{R} \mid y\geq -3\}$

[9] 6. For each function below, list the transformations, in the order you would apply them from the graph of $y=f(x)$.

a) $y=f(x-2)-3$

shift right 2
shift down 3

b) $y=-f(2x)$

reflect in x-axis
horizontal compression factor $\frac{1}{2}$ (or factor 2)

c) $y=\frac{1}{3}f(x+9)$

Vertical Compression factor $\frac{1}{3}$
shift left 9.

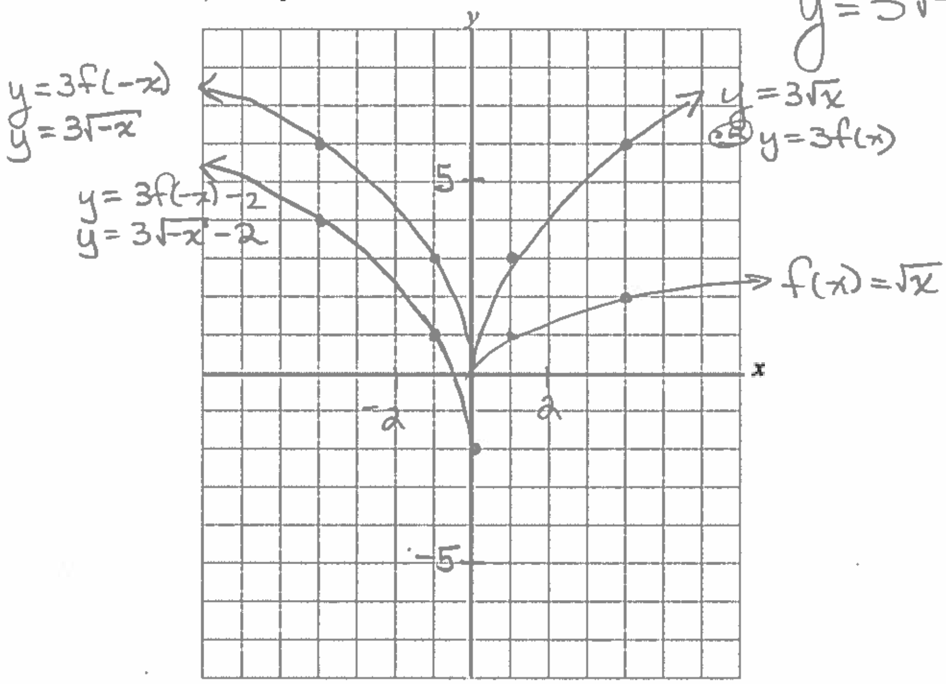
d) $y=f(-\frac{1}{5}x)+7$

reflect in y-axis
horizontal stretch factor 5
shift up 7

[5] 7. The graph of $f(x) = \sqrt{x}$ is stretched vertically by factor 3, reflected in the y-axis, and then translated 2 units down.

$y = 3f(-x) - 2$ ← description of transformations in function notation... you were not asked for this in this question.

a) Sketch the graph of the base curve and each individual transformation. Label your final curve.



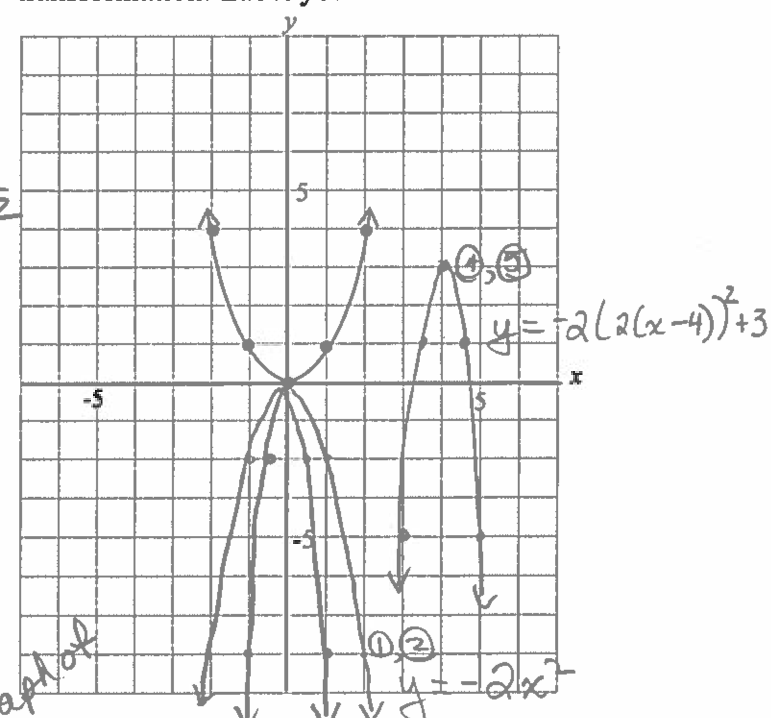
b) Write the equation of the new function.

$y = 3\sqrt{-x} - 2$

[10] 8. a) List the transformations, in the order you would apply them for the graph of $y = f(x)$ to the graph of $y = -2f(2(x-4)) + 3$.

① reflect in x-axis
② vertical stretch factor 2
③ horizonal compression factor $\frac{1}{2}$
④ shift right 4
⑤ shift up 3

b) Start with the graph of the base curve, $f(x) = x^2$ provided and sketch each individual transformation. Label your final curve.



note: $y = -2(2x)^2$
$y = -2(2x)(2x)$
$y = -8x^2$
That is why the graph of $y = -2(2x)^2$ looks like $y = -8x^2$!! ⌣ → $y = -2(2x)^2$

[5] 9. A manufacturing company produces garage doors. The number of garage doors, g, produced per week is related to the number of hours of labour, h, required per week to produce them by the function $g(h) = 1.8\sqrt{h}$.

a) How many doors can be produced per week using 500 hours of labour?

$h = 500, \quad g(500) = ?$

$g(500) = 1.8\sqrt{500}$

$=$

b) Determine the inverse of the function.

$1.8 = \frac{18}{10} = \frac{9}{5}$

$g \div \frac{9}{5} = g \times \frac{5}{9}$

$g = 1.8\sqrt{h}$

$\frac{g}{1.8} = \sqrt{h}$

$(\sqrt{h})^2 = \left(\frac{5g}{9}\right)^2$

$h = \frac{25g^2}{81}$

c) Explain its meaning (i.e. what it can be used to calculate).

If the company needs to produce a certain number of garage doors, this formula will determine the number of hours of labour that are required.

d) How many hours of labour are needed each week to keep production at or above 25 doors a week?

$h = \frac{25(25)^2}{81}$

$= \frac{15625}{81}$

$= 192.9$

∴ it will take 193 hours of labour each week.

BONUS

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[2] 10. Consider the exponential function $f(x) = 2^x$.

a) What point is invariant when it is reflected in the y-axis?

axis of symmetry is $x=0$

- function will be invariant for points on the axis of symmetry $f(0) = 2^0 = 1$ ∴ $(0,1)$ is the invarient point.

b) What is the equation of the horizontal asymptote of the transformed function $y = f(x) - 5$?

$y = -5$

[1] 11. Write the equation of a function that is its own inverse.

$y = -x$, $y = x$, $y = \frac{1}{x}$