Exponential Growth

1. Solve each exponential equation. Express answers to the nearest hundredth of a unit.
   a. \[ A = 250(1.05)^{10} \]
   b. \[ P = 7500(1.067)^{15} \]
   c. \[ 500 = N_0(1.25)^{1.25} \]
   d. \[ 84,000 = a (1.005)^{28} \]

2. The growth in population of a small town since 2000 is given by the function \[ P(n)= 1250(1.03)^n \]
   a. What is the initial population?
   b. What is the growth rate?
   c. Determine the population in 2011?
   d. In which year does the population reach 2000 people?

3. In 1990, a sum of $1,000 is invested at a rate of 6% per year for 15 years.
   a. What is the annual growth rate?
   b. What is the initial amount?
   c. How many growth periods are there?
   d. Write an equation that models the growth of the investment, and use it to determine the value of the investment after 15 years.

4. A species of bacteria has a population of 500 at noon. It doubles every 10 hours. The function that models the growth of the population \( P \) at any hour \( t \) is \( P(t) = 500(2^{\frac{t}{10}}) \)
   a. Why is the exponent \( \frac{t}{10} \)?
   b. Why is the base 2? What is the rate of growth?
   c. Why is ‘a’ value 500?
   d. Determine the population at midnight.
   e. Determine the population at noon the next day.
   f. Determine the time at which the population first exceeds 2000?

5. A town with a population of 12,000 has been growing at an average rate of 2.5%.
   a. Write an equation that models the population of the town.
   b. Determine the population of the town in 10 years.
   c. Determine the number of years until the population doubles.
   d. Determine what the population was 8 years ago.

6. A population of yeast cells can double in as little as 1 hour. Assume an initial population of 80 cells.
   a. What is the growth rate, in percent per hour, of this colony of yeast cells?
   b. Write an equation that can be used to determine the population of cells in \( t \) hours.
   c. Use your equation to determine the population after 6 hours.
   d. Use your equation to determine the population after 90 minutes.
   e. Approximately how many hours would it take for the population to reach 1 million cells?
   f. What are the domain and range for this situation?

7. A town has a population of 8400 in 1990. Fifteen years later, its population grew to 12500. Determine the average annual growth rate of this town’s population.

8. A collector’s hockey card is purchased in 1990 for $5. The value increases by 6% every year.
   a. Write an equation that models the value of the card, given the number of years since 1990.
   b. Determine the value of the card in the 20th year after it was purchased.
Exponential Decay

1. Solve each exponential equation. Express answers to the nearest hundredth of a unit.
   a. \( A = 2505(0.85)^{10} \)
   b. \( P = 7500(0.67)^4 \)
   c. \( 500 = N_0(0.75)^{11} \)

2. The population of a small town since 2000 is given by the function \( P(n) = 1250(0.94)^n \)
   a. What is the initial population?
   b. What is the decay rate?
   c. Determine the population in 2011?
   d. In which year does the population reach 385 people?

3. Which of these functions describe exponential decay? Explain.
   a. \( g(x) = -4(3)^x \)
   b. \( f(x) = 0.8(0.75)^{2x} \)
   c. \( P = 7500(0.067)^4 \)
   d. \( 500 = N_0(1.05)^{11} \)

4. A computer loses its value each month after it is purchased. Its purchase price was $1,900 and it loses 4.5% of its value after each month.
   a. Write an equation that models the value of the computer as a function of time, in months.
   b. Determine the value of the computer after 3 months.
   c. Determine the value of the computer after 2 years.
   d. Determine the time at which the value of the computer drops under $400.

5. A student records the internal temperature of a hot sandwich that has been left to cool on a kitchen counter. The room temperature is 19°C. An equation that models this situation is \( T(t) = 63(0.5)^{10} + 19 \), where \( T \) is the temperature in degrees Celsius and \( t \) is the time in minutes.
   a. What is the temperature of the sandwich when she began to record its temperature?
   b. Determine the temperature, to the nearest degree, of the sandwich after 20 min.
   c. How much time did it take for the sandwich to reach an internal temperature of 30°C?

6. In each case, write an equation that models the situation described. Explain what each part of each equation represents.
   a. The percent of colour left if blue jeans lose 1% of their colour every time they are washed?
   b. The population of a town if it had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for \( t \) years.
   c. The population of a colony if a single bacterium takes 1 day to divide into two; the population is \( P \) after \( t \) days.

7. A town has a population of 8400 in 1990. Eighteen years later, its population was 4850. Determine the average annual decay rate of this town’s population.
APPLICATION PROBLEMS - ANSWERS

EXPONENTIAL GROWTH SOLUTIONS
1. a) 407.22  
   b) 19,839.36  
   c) 378.30  
   d) 73,051.57  
2. a) 1250 people  
   b) 3%  
   c) 1730 people  
   d) 16 years  
3. a) 6%  
   b) $1000  
   c) 15  
   d) $2396.56  
4. a) takes 10 hours to double  
   b) 2 b/c it is doubling  
   c) initial pop is 500  
   d) 1149 bacteria  
   e) 2639 bacteria  
   f) 20 hours
5. a) $P(t)=12000(1.025)^t$  
   b) 15,361 people  
   c) 28 years  
   d) 9849 people  
6. a) 100%  
   b) $P(t)=80(2)^t$  
   c) 5210 cells  
   d) $P(1.5)=226$ cells  
   e) 14 hours
7. a) $V(t)=5(1.06)^t$  
   b) $V(3)=1655$  
   c) $V(24)=629$  
   d) 34 months

EXPONENTIAL DECAY SOLUTIONS
1. a. 493.17  
   b. 1511.33  
   c. 11838.48  
   d. 2019  
2. a. 1250  
   b. 6%  
   c. 633  
3. b&c Bases between zero and one
4. a. $V(t)=1900(0.955)^t$  
   b. $V(3)=1655$  
   c. $V(24)=629$  
   d. 34 months
5. a. $82^\circ$C  
   b. $34.75^\circ$C  
   c. 25 minutes  
6. a. $C=100(0.99)^w$  
   b. $P=2500(1.005)^t$  
   c. $P=1(2)^t$  
100 is the % of colour at the beginning, 99 refers to the fact that 1% is lost during every wash, & w refers to the # washes  
2500 is the initial population, 1.005 refers to the fact that the population grows 0.5% every year, t refers to the number of years after 1990  
2 refers to the fact that the population doubles in one day  
t refers to the number of days
7. 3%