

## Exponential Growth

- Solve each exponential equation. Express answers to the nearest hundredth of a unit.
  - $A = 250(1.05)^{10}$
  - $P = 7500(1.067)^{15}$
  - $500 = N_0(1.25)^{1.25}$
  - $84,000 = a(1.005)^{28}$
- The growth in population of a small town since 2000 is given by the function  $P(n) = 1250(1.03)^n$ 
  - What is the initial population?
  - What is the growth rate?
  - Determine the population in 2011?
  - In which year does the population reach 2000 people?
- In 1990, a sum of \$1,000 is invested at a rate of 6% per year for 15 years.
  - What is the growth rate?
  - What is the initial amount?
  - How many growth periods are there?
  - Write an equation that models the growth of the investment, and use it to determine the value of the investment after 15 years.
- A species of bacteria has a population of 500 at noon. It doubles every 10 hours. The function that models the growth of the population  $P$  at any hour  $t$  is  $P(t) = 500(2^{\frac{t}{10}})$ 
  - Why is the exponent  $\frac{t}{10}$ ?
  - Why is the base 2? What is the rate of growth?
  - Why is 'a' value 500?
  - Determine the population at midnight.
  - Determine the population at noon the next day.
  - Determine the time at which the population first exceeds 2000?
- A town with a population of 12,000 has been growing at an average rate of 2.5%.
  - Write an equation that models the population of the town.
  - Determine the population of the town in 10 years.
  - Determine the number of years until the population doubles.
  - Determine what the population was 8 years ago.
- A population of yeast cells can double in as little as 1 hour. Assume an initial population of 80 cells.
  - What is the growth rate, in percent per hour, of this colony of yeast cells?
  - Write an equation that can be used to determine the population of cells in  $t$  hours.
  - Use your equation to determine the population after 6 hours.
  - Use your equation to determine the population after 90 minutes.
  - Approximately how many hours would it take for the population to reach 1 million cells?
  - What are the domain and range for this situation?
- A town has a population of 8400 in 1990. Fifteen years later, its population grew to 12500. Determine the average annual growth rate of this town's population.
- A collector's hockey card is purchased in 1990 for \$5. The value increases by 6% every year.
  - Write an equation that models the value of the card, given the number of years since 1990.
  - Determine the value of the card in the 20<sup>th</sup> year after it was purchased.

## Exponential Decay

- Solve each exponential equation. Express answers to the nearest hundredth of a unit.
  - $A = 2505(0.85)^{10}$
  - $P = 7500(0.67)^4$
  - $500 = N_0(0.75)^{11}$
- The population of a small town since 2000 is given by the function  $P(n) = 1250(0.94)^n$ 
  - What is the initial population?
  - What is the decay rate?
  - Determine the population in 2011?
  - In which year does the population reach 385 people?
- Which of these functions describe exponential decay? Explain.
  - $g(x) = -4(3)^x$
  - $f(x) = 0.8(0.75)^{2x}$
  - $P = 7500(0.067)^4$
  - $500 = N_0(1.05)^{11}$
- A computer loses its value each month after it is purchased. Its purchase price was \$1,900 and it loses 4.5% of its value after each month.
  - Write an equation that models the value of the computer as a function of time, in months.
  - Determine the value of the computer after 3 months.
  - Determine the value of the computer after 2 years.
  - Determine the time at which the value of the computer drops under \$400.
- A student records the internal temperature of a hot sandwich that has been left to cool on a kitchen counter. The room temperature is  $19^\circ\text{C}$ . An equation that models this situation is  $T(t) = 63(0.5)^{\frac{t}{10}} + 19$ , where  $T$  is the temperature in degrees Celsius and  $t$  is the time in minutes.
  - What is the temperature of the sandwich when she began to record its temperature?
  - Determine the temperature, to the nearest degree, of the sandwich after 20 min.
  - How much time did it take for the sandwich to reach an internal temperature of  $30^\circ\text{C}$ ?
- In each case, write an equation that models the situation described. Explain what each part of each equation represents.
  - The percent of colour left if blue jeans lose 1% of their colour every time they are washed?
  - The population if a town had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for  $t$  years.
  - The population of a colony if a single bacterium takes 1 day to divide into two; the population is  $P$  after  $t$  days.
- A town has a population of 8400 in 1990. Eighteen years later, its population was 4850. Determine the average annual decay rate of this town's population.

**APPLICATION PROBLEMS - ANSWERS**

**EXPONENTIAL GROWTH SOLUTIONS**

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|--|--|---|--|
| 1. a) 407.22<br>b) 19,839.36<br><br>c) 378.30<br>d) 73,051.57                    | 2. a) 1250 people<br>b) 3%<br><br>c) 1730 people<br>d) 16 years                              | 3. a) 6%<br>b) \$1000<br><br>c) 15<br>d) $A = 1000(1.06)^{15}$<br>A = \$2396.56 | 4. a) takes 10 hours to double<br>b) 2 b/c it is doubling<br>growth rate is 100%<br>c) initial pop is 500<br>d) 1149 bacteria<br>e) 2639 bacteria<br>f) 20 hours |
| 5. a) $P(t)=12000(1.025)^t$<br>b) 15,361 people<br>c) 28 years<br>d) 9849 people | 6. a) 100%<br>b) $P(t) = 80(2)^t$<br>c) 5210 cells<br>d) $P(1.5) = 226$ cells<br>e) 14 hours | 7. 2.70%  | 8. a) $V(t)=5(1.06)^t$<br>b) \$16  |

**EXPONENTIAL DECAY SOLUTIONS**

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|--|---|---|---|
| 1.a. 493.17<br><br>b. 1511.33<br><br>c. 11838.48 | 2.a. 1250<br><br>b. 6%<br><br>c. 633<br><br>d. 2019                       | 3.b&c Bases between zero and one  | 4.a. $V(t) = 1900(0.955)^t$<br><br>b. $V(3)=1655$<br><br>c. $V(24)=629$<br><br>d. 34 months |
| 5.a. 82°C<br><br>b. 34.75°C<br><br>c. 25 minutes | 6.a. $C = 100(0.99)^w$<br><br>b. $P=2500(1.005)^t$<br><br>c. $P = 1(2)^t$ | 100 is the % of colour at the beginning, 99 refers to the fact that 1% is lost during every wash, & w refers to the # washes<br><br>2500 is the initial population, 1.005 refers to the fact that the population grows 0.5% every year, t refers to the number of years after 1990<br><br>2 refers to the fact that the population doubles in one day<br><br>t refers to the number of days |   |
| 7. 3%  |   |   |   |