MCR3UI - Unit 5 Day 1

Exploring Properties of Exponential Functions

Practice Questions

1. An insect colony, with an initial population of 20, quadruples every day.

   (a) Copy and complete the table.

Day	Population	First Differences	Second Differences
0	20		
1	80		
2			
3			
4			
5			

   (b) Is the relationship between the insect population and the number of days exponential? Explain how you can tell.

   (c) Examine the finite differences. Describe how the first differences and second differences are related.

   (d) Will the pattern of first and second differences observed in part (c) continue with the third and fourth differences? Write down your conjecture.

   (e) Calculate the third and fourth differences. Was your conjecture to part (d) correct? Explain.

2. Suppose that there is a rumour going around your school that next year all weekends will be extended to three days. Initially, on day 0, five students know the rumour. Suppose that each person who knows the rumour tells two more students the day after they hear about it. Also assume that no-one hears the rumour more than once.

   (a) How many people will learn about the rumour:

      (i) on day 1?

      (ii) on day 2?

   (b) Assume the population at WO is 1400 students. How long will it take for this rumour to spread throughout the entire school? Assume the same pattern for the weekends.

   (c) Is this an example of exponential growth? Explain your reasoning.

3. Suppose you just won the choice of one of three prizes at a game show:

   – Everyday Deal: On day 1, the prize is worth \$1. Then, every day for two weeks, the value of the prize is one more dollar than it was the day before.
   – Square Deal: On day 1, the prize is worth 1^2, or \$1. On day 2, the prize is worth 2^2 or \$4, and so on, for two weeks.
   – Double Deal: On day 1, the prize is worth \$1. On day 2, the prize value doubles to $2 \times \$1$, or \$2. On day 3, the value doubles again to $2 \times \$2$, or \$4, and so on, for two weeks.

   Which prize should you take at the end of the two-week period? Why? Use an algebraic method to justify your choice.

4. A bacterial colony has an initial population of 200. The population triples every week.

   (a) Write an equation to relate population, p, to time, t, in weeks.
   (b) Sketch the graph of this relationship for the first month.
   (c) Determine the approximate population after 10 days. Which tool do you prefer to use for this: the equation or the graph? Explain why.
   (d) Determine the approximate population after 3 months. Which tool to you prefer to use for this: the equation or the graph? Explain why.

Extra Fun Practice!

5. Bacteria A has an initial population of 500 and doubles every day, while bacteria B has an initial population of 50 and triples daily.

   (a) After how long will the population of B overtake the population of A? What will their populations be at this point?
   (b) How much faster would B overtake A if A's doubling period were twice as long?

6. The doubing period of a type of yeast cell is 3 days. A jar starts off with one yeast cell. After 27 days, there are 512 cells. It takes 30 days to fill the jar. The number of cells in the jar when it is full is:

   A 512 **B** 4096 **C** 1024 **D** $512(2)^{30}$