UNIT 3 TEST: Transformations of Functions

1. For each statement below, circle T if the statement is true, or F if the statement is false.
   a) When the function $f(x) = x^2$ is transformed to $y = -f(x)$, there is one invariant point. **T**
   b) For the function $y = -f(2x - 6)$, there is a horizontal translation right 6 units. **F**
   c) An asymptote is a line that a curve approaches, but never touches. **T**
   d) A vertical line is a relation, but not a function. **T**
   e) Horizontal and vertical translations are always completed first when applying transformations to any function. **F**

2. For each relation below, determine whether it is a function and state its domain and range.
   a) \{(5, 2), (3, 3), (4, 4), (1, 5)\} **Function? Circle** Yes / No
      Domain: \{-5, -3, -1\} Range: \{2, 3, 4, 5\}
   b) **Function? Circle** No
      Domain: \{-10 \leq x \leq 10\} Range: \{y \leq 7\}
   c) **Function? Circle** Yes / No
      Domain: \{-5 \leq x \leq 5\} Range: \{-3 \leq y \leq 3\}

3. If $f(x) = \sqrt{x} + 7$, find:
   a) $f(3)$
      $f(3) = \sqrt{3} + 7$
      $= \sqrt{4}$
      $= 2$
   b) $f(1 - a)$
      $f(1 - a) = \sqrt{1 - a} + 7$
      $= \sqrt{-a + 14}$
      $= \sqrt{a + 16}$
   c) $x$ when $f(x) = 4$
      $\sqrt{x} + 7 = 4$
      $x = 9$
      $x = -9$

4. Sketch the inverses of the following functions on the same grids they are drawn.
   a) 
   b)
For each function below, find its inverse, \( f^{-1}(x) \). Identify if the inverse is a function or not. Show your work.

a) \( f(x) = \frac{4x - 3}{9} \)

For \( f^{-1} \):

\[
\begin{align*}
\chi &= \frac{4\chi - 3}{9} \\
9\chi &= 4\chi - 3 \\
\chi &= \frac{3}{5}
\end{align*}
\]

\[
\begin{align*}
y &= \frac{4\chi + 3}{4} \\
y &= \frac{4\cdot \frac{3}{5} + 3}{4} \\
\therefore f^{-1}(x) &= \frac{4x + 3}{4}
\end{align*}
\]

Function (Y/N): \(\text{Yes} \)

b) \( f(x) = (x + 2)^2 - 5 \)

For \( f^{-1} \):

\[
\begin{align*}
\chi &= (y + 2)^2 - 5 \\
\chi + 5 &= (y + 2)^2 \\
\pm \sqrt{\chi + 5} &= y + 2 \\
y &= \pm \sqrt{\chi + 5} - 2
\end{align*}
\]

\[
\begin{align*}
\therefore f^{-1}(x) &= \pm \sqrt{x + 5} - 2
\end{align*}
\]

Function (Y/N): \(\text{No} \)

6. Given the graph of \( f(x) = (x - 3)^2 \),

a) Write equations for:

\[
\begin{align*}
f(x) &= -(x - 3)^2 \\
f(-x) &= -(x + 3)^2
\end{align*}
\]

b) Sketch the graphs of \( y = f(x) \), \( y = -f(x) \) and \( y = f(-x) \) on the same set of axes. Label each function.

c) Determine any points that are invariant for each reflection.

\(- f(x): (3, 0) \ ✓ \)

\(- f(-x): (0, 9) \ ✓ \)

d) State the domain and range for the reflected functions.

\(- f(x) \)

D: \( \{x \in \mathbb{R} \} \ ✓ \)

R: \( \{y \in \mathbb{R}, y \leq 0 \} \ ✓ \)

\(- f(-x) \)

D: \( \{x \in \mathbb{R} \} \ ✓ \)

R: \( \{y \in \mathbb{R}, y > 0 \} \ ✓ \)

7. Given a point \((-4, 6)\) that lies on the graph of \( y = f(x) \), determine its new co-ordinates as you apply each of the following transformations.

<table>
<thead>
<tr>
<th>( y = f(x) )</th>
<th>( y = f(2x) )</th>
<th>( y = f(-2x) )</th>
<th>( y = 3f(-2x) )</th>
<th>( y = 3f(-2(x + 1)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-4, 6))</td>
<td>((-2, 6))</td>
<td>((2, 6))</td>
<td>((3, 18))</td>
<td>((1, 18))</td>
</tr>
</tbody>
</table>
8. The graph of \( g(x) = x^3 \) is reflected in the x-axis, compressed vertically by a factor of \( \frac{1}{4} \) then translated left 4 units and up 7 units. What is the equation of the new image (the transformed function)?

\[
y = -\frac{1}{4}(x+4)^3 + 7
\]

9. Given \( h(x) = \frac{1}{x-2} + 3 \),

a) List the transformations that have been applied to the reciprocal function to obtain \( y = h(x) \).

b) Graph the base function and \( h(x) = \frac{1}{x-2} + 3 \) on the grid provided.

c) State the equations of the asymptotes for \( h(x) \):
   - Horizontal Asymptote: \( x = 2 \)
   - Vertical Asymptote: \( y = 3 \)

10. Consider \( f(x) = \sqrt{x} \).

a) List the transformations in the order you would apply them to the function \( f(x) = \sqrt{x} \) to graph \( y = f(-\frac{1}{2}(x-1)) - 3 \).

   1. reflection in y-axis
   2. horizontal stretch factor 2
   3. shift right 1
   4. shift down 3

b) Graph the original image \( f(x) = \sqrt{x} \) and the transformed image. Show all work/graphs for full marks. Label the original function and the final graph.

c) Write the equation for the transformed function.

\[
y = \sqrt{-\frac{1}{2}(x-1)} - 3
\]

d) State the domain and range of \( y = f(-\frac{1}{2}(x-1)) - 3 \)

\[ D: \{ x \in \mathbb{R} \mid x \leq 1 \} \quad R: \{ y \in \mathbb{R} \mid y \geq -3 \} \]
11. Given $f(x) = x$, $g(x) = x^2$, $h(x) = \sqrt{x}$

Write the simplified equation of the following transformations.

a.) $y = \frac{2}{3}f(x - 1)$

\[ y = \frac{2}{3}(x - 1) \]

\[ y = \frac{2}{3}x - \frac{2}{3} \]

b.) $y = g\left(-\frac{1}{4}x\right) - 5$

\[ y = \left(-\frac{1}{4}x\right)^2 - 5 \]

\[ y = \frac{1}{16}x^2 - 5 \]

c.) $y = 6h(x) + 2$

\[ y = 6\sqrt{x} + 2 \]

12. A catering company charges $250, plus a variable rate of $20/person for any event they are hired to cater.

a) Write a function to represent the total cost of hiring the catering company, $c(p)$ dollars, in terms of the number of people, $p$, that will be attending a catered event.

\[ c(p) = 250 + 20p \]

b) Determine the domain of the function.

\[ D = \{ p \in \mathbb{R} | p > 0 \} \]

c) If we were to find the inverse of the function, what would it represent in the context of this relationship?

\[ p = \frac{c - 250}{20} \]

The number of people is a function of the cost of the banquet.

BONUS:
Determine the equation of the graph given below.

Equation:

\[ y = -12(x - 1)^2 + 5 \]