MCR 3UI Name: ______________

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UNIT 3 TEST: Transformations of Functions

[5] 1. For each statement below, circle T if the statement is true, or F if the statement is false.

T F a) When the function $f(x) = x^2$ is transformed to $y = -f(x)$, there is one invariant point.

$2(x-3)$

T F b) For the function $y = -f(2x - 6)$, there is a horizontal translation right 6 units.

T F c) An asymptote is a line that a curve approaches, but never touches.

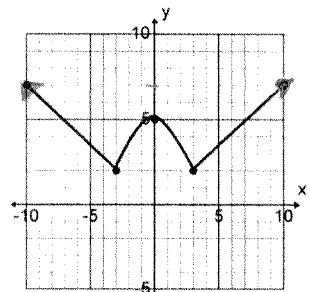
T F d) A vertical line is a relation, but not a function.

T F e) Horizontal and vertical translations are always completed first when applying transformations to any function.

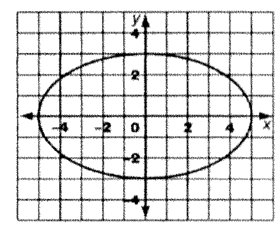
[9] 2. For each relation below, determine whether it is a function and state its domain and range.

a) {(-5, 2), (-3, 3), (-1, 4), (1, 5)}

b)



c)



#1 → Function? (circle) Yes / No

#2 → Domain: $\{-5, -3, -1, 1\}$

→ Range: $\{2, 3, 4, 5\}$

Function? (circle) Yes / No

Domain: $\{x \in \mathbb{R} \mid -10 \le x \le 10\}$

Range: $\{y \in \mathbb{R} \mid 2 \le y \le 7\}$

or $D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \geq 3\}$.

Function? (circle) Yes / No

Domain: $\{x \in \mathbb{R} \mid -5 \le x \le 5\}$

Range: $\{y \in \mathbb{R} \mid -3 \le y \le 3\}$

[5] 3. If $f(x) = \sqrt{-x+7}$, find:

a) $f(3)$

$f(3) = \sqrt{-3+7}$
$= \sqrt{4}$
$= 2$

b) $f(1-a)$

$f(1-a) = \sqrt{-(1-a)+7}$
$= \sqrt{-1+a+7}$
$= \sqrt{a+6}$

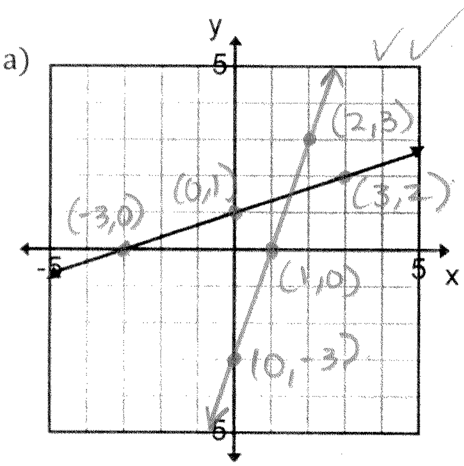
c) x when $f(x) = 4$

$\sqrt{-x+7} = 4$
$-x+7 = 16$
$-x = 9$
$x = -9$

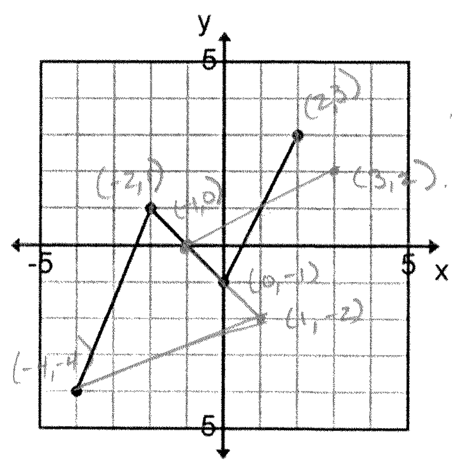
[4] 4. Sketch the inverses of the following functions on the same grids they are drawn.

#7

a)



b)



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[6] 5. For each function below, find its inverse, $f^{-1}(x)$. Identify if the inverse is a function or not. Show your work.

#8

a) $f(x) = \frac{4x-3}{9}$

for f^{-1};

$x = \frac{4y-3}{9}$

$9x = 4y - 3$

$9x + 3 = 4y$

$y = \frac{9x+3}{4}$

$\therefore f^{-1}(x) = \frac{9x+3}{4}$

Function (Y/N): Yes.

b) $f(x) = (x+2)^2 - 5$

for f^{-1};

$x = (y+2)^2 - 5$

$x + 5 = (y+2)^2$

$\pm\sqrt{x+5} = y + 2$

$y = \pm\sqrt{x+5} - 2$

$\therefore f^{-1}(x) = \pm\sqrt{x+5} - 2$

Function (Y/N): No.

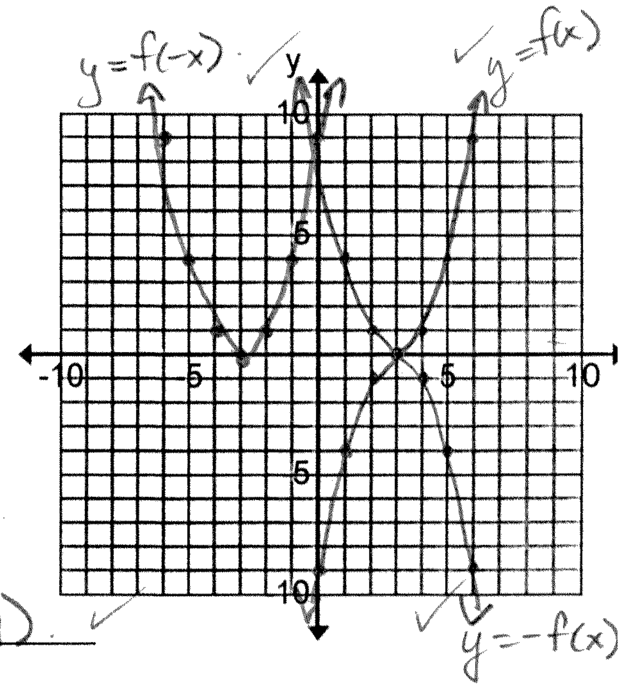
[11] 6. Given ~~the graph of~~ $f(x) = (x-3)^2$,

a) Write equations for:

$-f(x) =$ $-(x-3)^2$

$f(-x) =$ $(-x-3)^2$ OR $(x+3)^2$

b) Sketch the graphs of $y = f(x)$, $y = -f(x)$ and $y = f(-x)$ on the same set of axes. Label each function.



c) Determine any points that are invariant for each reflection.

$-f(x)$: $(3,0)$ $f(-x)$: $(0,9)$

d) State the domain and range for the reflected functions.

$-f(x)$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \leq 0\}$

$f(-x)$:

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

[4] 7. Given a point (-4, 6) that lies on the graph of $y = f(x)$, determine its new co-ordinates as you apply each of the following transformations.

$y = f(x)$	$y = f(2x)$	$y = f(-2x)$	$y = 3f(-2x)$	$y = 3f(-2(x+1))$
(-4, 6)	$x \times \frac{1}{2}$ (-2, 6)	$x \times (-1)$ (2, 6)	$y \times 3$ (2, 18)	$x-1$ (1, 18)

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8. The graph of $g(x) = x^3$ is reflected in the x-axis, compressed vertically by a factor of ¼ then translated left 4 units and up 7 units.

[2] What is the equation of the new image (the transformed function)?

#5 $y = -\frac{1}{4}(x+4)^3 + 7$ ½ mark each

9. Given $h(x) = \frac{1}{x-2} + 3$,

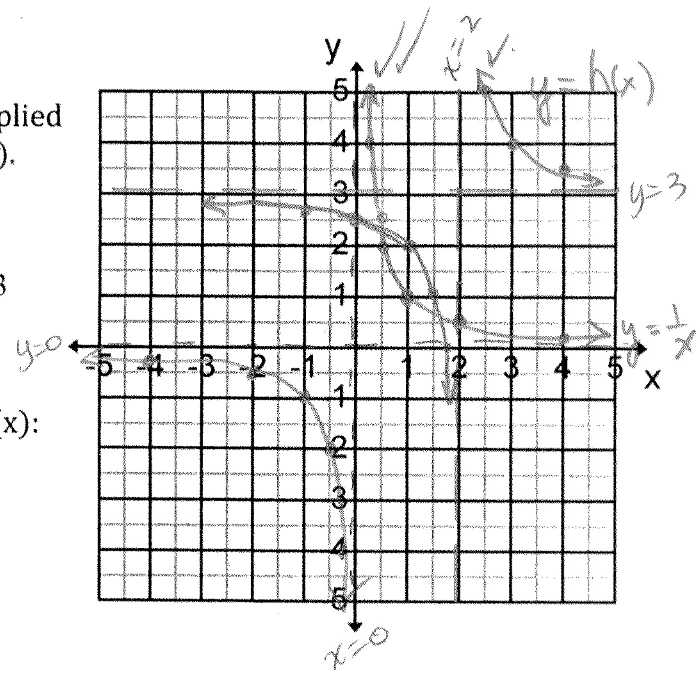
[6]

a) List the transformations that have been applied to the reciprocal function to obtain $y = h(x)$. $(y = \frac{1}{x})$

right 2 up 3.

#6a

b) Graph the base function and $h(x) = \frac{1}{x-2} + 3$ on the grid provided.



c) State the equations of the asymptotes for h(x):

Horizontal Asymptote: x=2

Vertical Asymptote: y=3

10. Consider $f(x) = \sqrt{x}$.

[9]

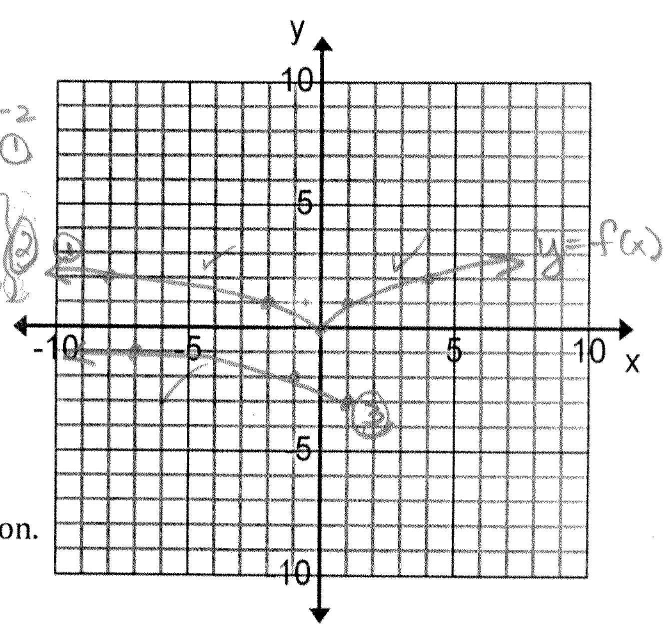
a) List the transformations in the order you would apply them to the function $f(x) = \sqrt{x}$ to graph $y = f(-\frac{1}{2}(x-1)) - 3$.

#4

1. reflection in y-axis
2. horizontal stretch factor 2 } $x \times -2$
3. shift right 1 $x+1$
4. shift down 3. $y-3$

b) Graph the original image $f(x) = \sqrt{x}$ and the transformed image. Show all work/graphs for full marks. Label the original function and the final graph.

#6b



c) Write the equation for the transformed function.

#3 $y = \sqrt{-\frac{1}{2}(x-1)} - 3$

d) State the domain and range of $y = f(-\frac{1}{2}(x-1)) - 3$

D: { $x \in \mathbb{R} \mid x \le 1$ } R: { $y \in \mathbb{R} \mid y \ge -3$ }

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11. Given $f(x)=x$, $g(x)=x^2$, $h(x)=\sqrt{x}$

[6] 45

Write the simplified equation of the following transformations.

a.) $y=\frac{2}{3}f(x-1)$

$y=\frac{2}{3}(x-1)$ ✓

$=\frac{2}{3}x-\frac{2}{3}$

b) $y=g\left(-\frac{1}{4}x\right)-5$

$y=\left(-\frac{1}{4}x\right)^2-5$ ✓

$y=\frac{1}{16}x^2-5$ ✓

c) $y=6h(x)+2$

$y=6\sqrt{x}+2$ ✓

[2] [3] 12. A catering company charges \$250, plus a variable rate of \$20/person for any event they are hired to cater.

a) Write a function to represent the total cost of hiring the catering company, *c(p)* dollars, in terms of the number of people, *p*, that will be attending a catered event.

$C(p)=250+20p$ ✓

b) Determine the domain of the function.

$D=\{p\in W \mid p\geq 0\}$. ✓

c) If we were to find the inverse of the function, what would it represent in the context of this relationship?

x1

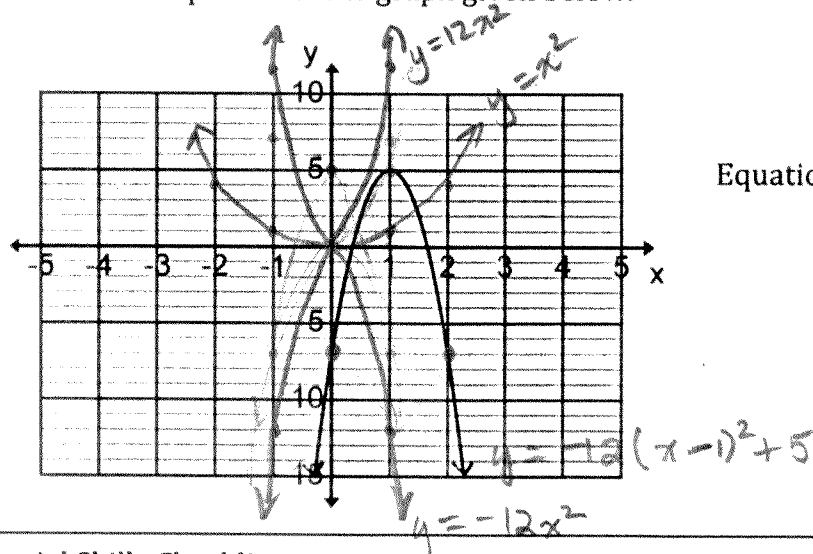
$20p=C-250$

$p=\frac{C-250}{20}$

The number of people is a function of cost of the banquet.

BONUS:

Determine the equation of the graph given below.



Equation:

$y=-12(x-1)^2+5$

$-12(x^2-2x+1)+5$

$=-12x^2+24x-12+5$

$=-12x^2+24x-7$

Essential Skills Checklist

- ☐ Determine whether a relation is a function or not
- ☐ Determine domain and range of functions
- ☐ Interpret and Apply Function Notation
- ☐ Identify and interpret transformations of functions – graphically
- ☐ Identify and interpret transformations of functions – algebraically
- ☐ Graph transformations of basic functions
- ☐ Find the inverse of a function – graphically
- ☐ Find the inverse of a function – algebraically