U7D7_T Binomial Exp _ Pascal

Sunday, December 10, 2017 4:11 PM

U7D7_T Binomial ...

J7D7 MCR 3UI SEQUENCES AND SERIES

Pascal's Triangle and Binomial Theorem

Preamble A binomial is an algebraic expression containing two terms.

Ex. $3x+1$, $1-x^2$, $3x-7y$

Today we will learn how to expand binomials raised to any power without the use of tedious and lengthy calculations.

Ex. $(x-3)^2 = (x-3)(x-3) = x^2-6x+9$

$(a+b)^2 = a^2+2ab+b^2$

$(3x-2y)^{10} =$

Part A Expand and simplify the following. Place your final answer below.

Keep in mind that $(x+y)^n = (x+y)(x+y)^{n-1}$, in other words you may use your previous answer to proceed.

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

$(x+y)(x^2+2xy+y^2)$

$(x+y)^3 = (x+y)(x+y)^2 = x^3+2x^2y+xy^2$
$+x^2y+2xy^2+y^3$
$= x^3+3x^2y+3xy^2+y^3$

$(x+y)^4 = (x+y)(x^3+3x^2y+3xy^2+y^3)$
$= x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(x+y)^5 = x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

Part B List all of the patterns you see in the final answers in the expansions. (done the long way).

If n is the exponent on $(x+y)$,

1. x exponents pattern: $n, n-1, n-2, \ldots, 1, 0$
2. y exponents pattern: $0, 1, 2, \ldots, n-1, n$
3. the sum of the exponents in each term is n.
4. the first and last term have a coefficient 1
5. the second & second last term have a coefficient n
6. there are $n+1$ terms in the expansion.

Part C Write the coefficients of the expansions below centering each row in the space. Then add the coefficients in each row.

n		row sum
0	1	$1 = 2^0$
1	1 1	$2 = 2^1$
2	1 2 1	$4 = 2^2$
3	1 3 3 1	$8 = 2^3$
4	1 4 6 4 1	$16 = 2^4$
5	1 5 10 10 5 1	$32 = 2^5$
6	1 6 15 20 15 6 1	$64 = 2^6$
7	1 7 21 35 35 21 7 1	
8	1 8 28 56 70 56 28 8 1	

This is Pascal's Triangle.

Part D List some of the characteristics of the numbers in Pascal's triangle.

1. The sum in any row is 2^n

2. Each row begins and ends with a one

3. To find any entry add the two entries above.

4. It is symmetric

linear
quad.
cubic, etc.

Part E Using Pascal's triangle and the patterns you have discovered today, expand :

$$(x+y)^8 = x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$$

From Δ, 1 8 28 56 70 56 28 8 1

Part F How is the expansion of $(x-y)^2$ different from $(x+y)^2$?

$$(x-y)^2 = x^2 - 2xy + y^2$$

Part G Expand the following. Recall: $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$$(x-y)^3 = \boxed{x^3 - 3x^2y + 3xy^2 - y^3}$$

$$= [x+(-y)]^3 = x^3 + 3x^2(-y) + 3(x)\underbrace{(-y)^2}_{+} + (-y)^3$$

$$(x-y)^4 =$$

1 4 6 4 1

$$(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$$

1 5 10 10 5 1

$$(2x+3)^5 = (2x)^5 + 5(2x)^4(3) + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)(3)^4 + (3)^5$$

$$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243.$$

$$(x^2-1)^6 =$$

$$\left(x^2 - \sqrt{x}\right)^4 =$$

Part H We can generate the coefficients on your calculator if you have a special button.

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

$$(a+b)^6 = \binom{6}{0}a^6 b^0 + \binom{6}{1}a^5 b^1 + \binom{6}{2}a^4 b^2 + \ldots + \binom{6}{5}a^1 b^5 + \binom{6}{6}b^6$$

$$(m+n)^6 = m^6 + 6m^5 n + 15m^4 n^2 + 20m^3 n^3 + \ldots$$

$\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ ${}_6C_2$ $\binom{6}{3}$ $\binom{6}{4}$

Part H Using binomial Theorem expand the following:

$(\ldots)^5$

$\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ ${}_6C_2$ $\binom{6}{3}$ $\binom{6}{4}$

Part H Using binomial Theorem expand the following:

$$(2x+1)^5 =$$

$$(x^2-3)^6 =$$

$n=0$ $\binom{0}{0}$

$n=1$ $\binom{1}{0}$ $\binom{1}{1}$

$n=2$ $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

$\frac{8\times 7\times 6\times 5}{4\times 3\times 2}$

$\vdots$

$n=8$ $\binom{8}{0}$ $\binom{8}{1}$ $\binom{8}{2}$ $\binom{8}{3}$ $\binom{8}{4}$ $\binom{8}{5}$ $\binom{8}{6}$ $\binom{8}{7}$ $\binom{8}{8}$

1 8 28 56 70 56 28 8 1

${}_8C_0$ ${}_8C_1$ ${}_8C_2$ ${}_8C_3$ ${}_8C_4$ ${}_8C_5$ ${}_8C_6$ ${}_8C_7$ ${}_8C_8$