U7D4-T Arithmetic Series

Sunday, December 10, 2017 4:10 PM

U7D4-T Arithmeti...

U7D4 MCR 3UI SEQUENCES AND SERIES

Arithmetic Series

An arithmetic series is the sum of the terms of an arithmetic sequence.

If the sequence is $t_1, t_2, t_3, t_4, \ldots, t_n$, then the series is $S_1, S_2, S_3, S_4, \ldots, S_n$ where :

$$S_1 = t_1 = a$$
$$S_2 = t_1 + t_2$$
$$S_3 = t_1 + t_2 + t_3$$
$$S_4 = t_1 + t_2 + t_3 + t_4$$
$$\ldots$$

In general,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

a is the first term

d is the common difference

n is the number of terms

also (the term number of the last term in the series)

(the term number of the last term in the series)

Or the formula can be written as :

$$\boxed{s_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

$$s_n = \frac{n}{2}\left[a + \underbrace{a + (n-1)d}_{t_n}\right]$$

$$s_n = \frac{n}{2}\left[t_1 + t_n\right]$$

$$s_n = n\left[\frac{t_1 + t_n}{2}\right]$$

$$\boxed{S_n = \frac{n}{2}(a + t_n)}$$

And so, we have two different versions of the same formula.

Examples:

1. Find the sum of the first 100 terms of
$8 + 11 + 14 + \ldots$

$a = 8 \qquad d = 3 \qquad n = 100$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(8) + \underbrace{99(3)}\right]$$

$$= 50(16 + 300 - 3)$$

$$= 50(313)$$

$$= 15650$$

$\therefore$ the sum is 15650.

2. Find the sum of

1.1 + 1.2 + 1.3 + 1.4 + . . . + 8.9

a $\quad d = 0.1 \quad t_n$

* we don't know what n is *

recall: $t_n = a + (n-1)d$

So,
$$1.1 + (n-1)(0.1) = 8.9$$
$$1.1 + 0.1n - 0.1 = 8.9$$
$$1 + \frac{1}{10}n = 8.9$$
$$\frac{1}{10}n = 7.9$$
$$n = 79$$

$$S_n = \frac{n}{2}(a + t_n)$$
$$S_{79} = \frac{79}{2}(1.1 + 8.9)$$
$$S_{79} = \frac{79}{2}(10)$$
$$S_{79} = 395$$

$\therefore$ the sum is 395.

3. If the sum of n terms of a sequence is given by $S_n = n^2 + n$, find t_{11}.

$S_{11} = 11^2 + 11$

$= 121 + 11$

$S_{11} = 132$

$S_1 = 1^2 + 1$

$S_1 = 2$

So, $t_1 = 2$

$a = 2$

$t_{11} = ?$

$n = 11$

$S_n = \frac{n}{2}(a + t_n)$

$132 = \frac{11}{2}(2 + t_{11})$

$132 \times \frac{2}{11} = 2 + t_{11}$

$12 \times 2 = 2 + t_{11}$

$24 - 2 = t_{11}$

$t_{11} = 22$

U7D4 HW: p. 469 #1-5(eoo)