Pg. 413 # 10, 11, 13, 14, 32, 33, 34
Pg 316 # 1–11.

#10 a) $\sin\theta = \frac{y}{r}$ $\quad r = \sqrt{16+25}$ $\quad \cos\theta = \frac{4}{\sqrt{41}}$ $\quad \tan\theta = \frac{5}{4}$

$= \frac{5}{\sqrt{41}}$ $\quad = \sqrt{41}$

b) $P(-2,7)$ $\quad \sin\theta = \frac{7}{\sqrt{53}}$ $\quad \cos\theta = \frac{-2}{\sqrt{53}}$ $\quad \tan\theta = -\frac{7}{2}$

$r = \sqrt{4+49}$
$= \sqrt{53}$

c) $P(-3,-6)$ $\quad \sin\theta = \frac{-6}{\sqrt{45}}$ $\quad \cos\theta = \frac{-3}{\sqrt{45}}$ $\quad \tan\theta = \frac{-6}{-3}$

$r = \sqrt{9+36}$ $\quad = \frac{-6}{3\sqrt{5}}$ $\quad = \frac{-3}{3\sqrt{5}}$ $\quad = 2$

$= \sqrt{45}$
$= 3\sqrt{5}$ $\quad = -\frac{2}{\sqrt{5}}$ $\quad = \frac{-1}{\sqrt{5}}$

d) $P(7,-4)$

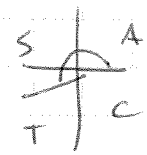
$r = \sqrt{49+16}$ $\quad \sin\theta = \frac{-4}{\sqrt{65}}$ $\quad \cos\theta = \frac{7}{\sqrt{65}}$ $\quad \tan\theta = -\frac{4}{7}$

$= \sqrt{65}$

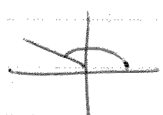
#11 a) $\cos 30^\circ = \frac{\sqrt{3}}{2}$

b) $\tan 225^\circ$
$= \tan 45^\circ$
$= 1$

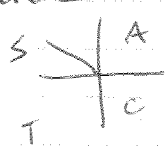
c) $\sin 210^\circ$
$= -\sin 30^\circ$
$= -\frac{1}{2}$



d) $\cos 150^\circ$
$= -\cos 30^\circ$
$= -\frac{\sqrt{3}}{2}$



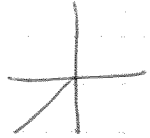
#13 a) $\sin\theta = \frac{2}{5}$ Quad II $\quad \cos\theta = \frac{-\sqrt{21}}{5}$ $\quad \tan\theta = \frac{-2}{\sqrt{21}}$



$x = \sqrt{5^2 - 2^2}$
$= \sqrt{25-4}$
$= \sqrt{21}$ ← use $-\sqrt{21}$ in quad. II.

$y = 2$
$r = 5$

b) $\cos\theta = -\frac{4}{7}$, Quad III $\quad \sin\theta = \frac{-\sqrt{33}}{7}$ $\quad \tan\theta = \frac{\sqrt{33}}{4}$

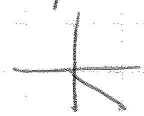


$x = -4$, $r = 7$,
$y = \sqrt{49-16}$
$= \sqrt{33}$
use $y = -\sqrt{33}$ in quad. III.

Pg. 413 #13, 14, 32, 33, 34.

13c) $\tan\theta = -\frac{5}{6}$ quad IV $\quad \sin\theta = \frac{-5}{\sqrt{61}} \quad \cos\theta = \frac{6}{\sqrt{61}}$

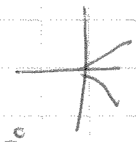
$r = \sqrt{25+36}$
$= \sqrt{61}$
$x = 6$
$y = -5$



14. a) $\sin A = \frac{1}{2}$

$A = 30°, \; 180° - 30°$
$= 150°$

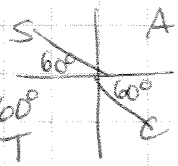
b) $\cos A = \frac{1}{\sqrt{2}}$



$A = 45°, \; 360° - 45°$
$= 315°$

c) $\tan A = -\sqrt{3}$

$A = 180° - 60°$ or $360° - 60°$
$= 120° \qquad = 300°$



d) $\cos A = \frac{\sqrt{3}}{2}$

$A = 30°, \; 360° - 30°$
$= 330°.$

32. a) $\frac{1-\sin^2 x}{\cos x} = \cos x$

LS
$\frac{1-\sin^2 x}{\cos x}$
$= \frac{\cos^2 x}{\cos x}$ (PI)
$= \cos x$
$= RS$

$\therefore \frac{1-\sin^2 x}{\cos x} = \cos x$

b) $\frac{\tan x}{\sin x} = \frac{1}{\cos x}$

LS
$\tan x \cdot \frac{1}{\sin x}$
$= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$ (QI).
$= \frac{1}{\cos x}$
$= RS$

$\therefore \frac{\tan x}{\sin x} = \frac{1}{\cos x}$

c) $\frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$

LS
$\sin x \cos x \cdot \frac{1}{\tan x}$
$= \sin x \cos x \cdot \frac{\cos x}{\sin x}$ (QI)
$= \cos^2 x$
$= 1 - \sin^2 x$ (PI).
$= RS$

$\therefore \frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$

d) $\cos^2 x + \frac{\sin x \cos x}{\tan x} = 2\cos^2 x$

LS
$\cos^2 x + \sin x \cos x \left(\frac{1}{\tan x}\right)$
$= \cos^2 x + \sin x \cos x \left(\frac{\cos x}{\sin x}\right)$ QI
$= \cos^2 x + \cos^2 x$
$= 2\cos^2 x$
$= RS$

$\therefore \cos^2 x + \frac{\sin x \cos x}{\tan x} = 2\cos^2 x$

Pg 413 #32, 33, 34

32e) $1+\tan^2 x = \dfrac{1}{\cos^2 x}$

LS

$1 + \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$ (PI)

$= RS$ $\quad \therefore 1+\tan^2 x = \dfrac{1}{\cos^2 x}$

f) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

LS

$\cos^2 x - (1-\cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$

$= RS$

$\therefore \cos^2 x - \sin^2 x = 2\cos^2 x - 1.$

g) $\dfrac{1}{\sin x} - \sin x = \dfrac{\cos x}{\tan x}$

LS

$\dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x}$

$= \dfrac{1-\sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x}{\sin x}$ (PI)

$= \cos x \left(\dfrac{\cos x}{\sin x}\right)$

$= \cos x \div \dfrac{\sin x}{\cos x}$

$= \cos x \div \tan x$ (QI)

$= RS.$ $\quad \therefore \dfrac{1}{\sin x} - \sin x = \dfrac{\cos x}{\tan x}$

h) $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$

LS

$\left(1 - \dfrac{\sin^2 x}{\cos^2 x}\right) \div \left(1 + \dfrac{\sin^2 x}{\cos^2 x}\right)$

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x} \div \dfrac{(\cos^2 x + \sin^2 x)}{\cos^2 x}$

$= \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \dfrac{\cos^2 x}{\cos^2 x + \sin^2 x}$

$= \dfrac{\cos^2 x - \sin^2 x}{1} \times \dfrac{1}{1}$ (PI)

$= \cos^2 x - \sin^2 x$

$= RS$

$\therefore \dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$

33a) $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = \dfrac{2}{\sin^2 x}$

LS

$\dfrac{(1-\cos x)+(1+\cos x)}{(1+\cos x)(1-\cos x)}$

$= \dfrac{2}{1-\cos^2 x}$

$= \dfrac{2}{\sin^2 x}$ (PI) $\quad \therefore \dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = \dfrac{2}{\sin^2 x}$

$= RS.$

Pg 413 # 33, 34

33b) $(\sin x-\cos x)^2 = 1-2\sin x\cos x$

oops! that's 34b.

LS

$(\sin x-\cos x)^2$

$= \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= \sin^2 x + \cos^2 x - 2\sin x\cos x$ (PI)

$= 1 - 2\sin x\cos x$

$=$ RS.

$\therefore (\sin x-\cos x)^2 = 1-2\sin x\cos x$

* Don't forget to HAVE A BLAST!

34 a) $(\sin x-\cos x)(\sin x+\cos x) = 2\sin^2 x-1$

LS $= \sin^2 x - \cos^2 x$ (difference of squares).

$= \sin^2 x - (1-\sin^2 x)$ (PI)

$= \sin^2 x - 1 + \sin^2 x$

$= 2\sin^2 x - 1$

$=$ RS

$\therefore (\sin x-\cos x)(\sin x+\cos x) = 2\sin^2 x-1$.

b) $(\sin x-\cos x)^2 = 1-2\sin x\cos x$

LS

$(\sin x-\cos x)^2$

$= \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= \sin^2 x + \cos^2 x - 2\sin x\cos x$ (PI)

$= 1 - 2\sin x\cos x$

$=$ RS

$\therefore (\sin x-\cos x)^2 = 1-2\sin x\cos x$

33b) $\dfrac{1+\cos x}{\sin x} - \dfrac{\sin x}{1-\cos x} = 0$

LS

$\dfrac{(1+\cos x)(1-\cos x) - \sin x(\sin x)}{\sin x\,(1-\cos x)}$

$= \dfrac{1-\cos^2 x - \sin^2 x}{\sin x(1-\cos x)}$

$= \dfrac{1-(\cos^2 x + \sin^2 x)}{\sin x(1-\cos x)}$

$= \dfrac{1-1}{\sin x(1-\cos x)}$ (PI)

$= 0.$ $=$ RS.

$\therefore \dfrac{1+\cos x}{\sin x} - \dfrac{\sin x}{1-\cos x} = 0$

34 c) $1 + \tan^2 x = \frac{1}{\cos^2 x}$

LS

$1 + \tan^2 x$

$= 1 + \frac{\sin^2 x}{\cos^2 x}$ (QI)

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$ (PI) $\therefore\ 1 + \tan^2 x = \frac{1}{\cos^2 x}$.

$= RS$

d) $\cos^2 x - \cos^4 x = \cos^2 x \sin^2 x$

LS

$\cos^2 x - \cos^4 x$

$= \cos^2 x (1 - \cos^2 x)$

$= \cos^2 x \sin^2 x$ (PI).

$= RS$ $\therefore\ \cos^2 x - \cos^4 x = \cos^2 x \sin^2 x$

e) $(1 - \cos^2 x)(1 + \tan^2 x) = \tan^2 x$

LS

$(1 - \cos^2 x)(1 + \tan^2 x)$

$= 1 + \tan^2 x - \cos^2 x - \cos^2 x \tan^2 x$

$= 1 + \tan^2 x - \cos^2 x - \cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$ (QI)

$= 1 + \tan^2 x - \cos^2 x - \sin^2 x$

$= 1 + \tan^2 x - (\cos^2 x + \sin^2 x)$

$= 1 + \tan^2 x - 1$ (PI)

$= \tan^2 x$

$= RS$ $\therefore\ (1 - \cos^2 x)(1 + \tan^2 x) = \tan^2 x$

f) $\frac{\sin x}{1 - \cos x} - \frac{1 + \cos x}{\sin x} = 0$

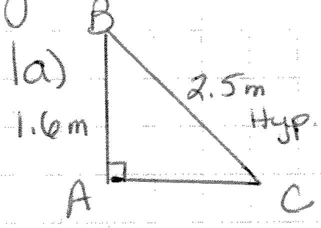
LS $\frac{\sin^2 x - (1 + \cos x)(1 - \cos x)}{\sin x (1 - \cos x)}$

$= \frac{\sin^2 x - (1 - \cos^2 x)}{\sin x (1 - \cos x)}$ $\therefore\ \frac{\sin x}{1 - \cos x} - \frac{1 + \cos x}{\sin x} = 0.$

$= \frac{\sin^2 x - \sin^2 x}{\sin x (1 - \cos x)} = 0 = RS.$

Pg. 316 #1–11

1a)



$\sin C = \frac{1.6}{2.5}$

$C = \sin^{-1}\left(\frac{1.6}{2.5}\right)$

$C = \sin^{-1}(0.64)$

$C = 39.8^\circ$

$\angle B = 90^\circ - 39.8^\circ$
$= 50.2^\circ$ (ASTT).

$\cos 39.8^\circ = \frac{b}{2.5}$

$b = 2.5\cos 39.8^\circ$

$b \doteq 1.9\,m$

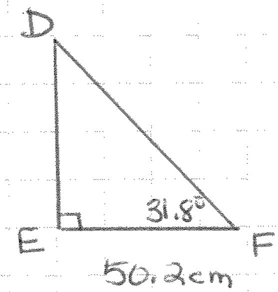
OR

$b = \sqrt{2.5^2 - 1.6^2}$

$b = \sqrt{3.69}$

$b \doteq 1.9\,m$

b)



$\cos 31.8^\circ = \frac{50.2}{e}$

$e = \frac{50.2}{\cos 31.8^\circ}$

$e = 59.1\,cm.$

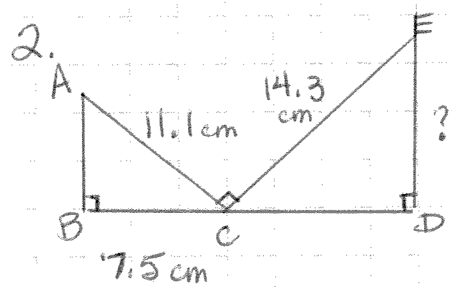
$\angle D = 58.2^\circ$ (ASTT)

$\tan 31.8^\circ = \frac{f}{50.2}$

$f = 50.2\tan 31.8^\circ$

$f = 31.1\,cm.$

2.



$\cos\angle BCA = \frac{7.5}{11.1}$

$\angle BCA = \cos^{-1}\left(\frac{7.5}{11.1}\right)$

$\angle BCA \doteq 47.5^\circ$

$\angle ECD = 180^\circ - 90^\circ - 47.5^\circ$ (supplementary).
$= 42.5^\circ$

$\sin 42.5^\circ = \frac{?}{14.3}$

$? = 14.3\sin 42.5^\circ$

$? = 9.7\,cm.$

3. $(-12, 9)$

$r = \sqrt{12^2 + 9^2}$
$= \sqrt{225}$
$r = 15$
$x = -12$
$y = 9$

$\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{9}{15}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{-12}{15}$

$\cos\theta = \frac{-4}{5}$

Pg. 316 #4-11

4. a) $\sin 82.3^\circ \doteq 0.9910$ b) $\cos 19.9^\circ \doteq 0.9403$ c) $\sin 149.5^\circ = 0.5075$ d) $\cos 159.2^\circ = -0.9348$

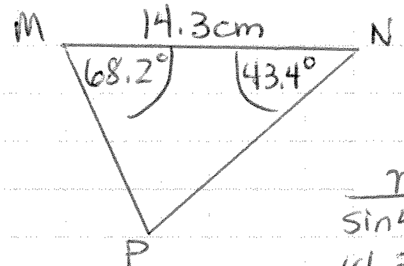
5. a) $\sin A = 0.6678$
$A = 41.9^\circ$ OR 138.1°

b) $\cos A = -0.5519$
$A = 123.5^\circ$

6. Acute angles are all in quadrant I. In quadrant I, cosine, sine and tangent are all positive. It is not possible to have cosine of an acute angle equal to a negative value.
No. when $\cos\theta = -0.4328$
$\theta = 115.6^\circ$ or 244.4°

7. An obtuse angle has a terminal arm in quadrant II. In quadrant II sine is positive.
So it is possible to have $\sin\theta = 0.3781$
$(\theta = 157.8^\circ)$

8 a)



$\angle P = 180^\circ - 68.2^\circ - 43.4^\circ$ (ASTT)
$= 68.4^\circ$

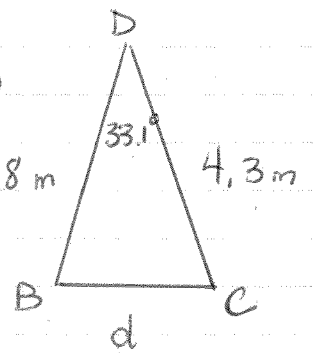
$$\frac{n}{\sin 43.4^\circ} = \frac{14.3}{\sin 68.4^\circ} = \frac{m}{\sin 68.2^\circ}$$

$$n = \frac{14.3 \sin 43.4^\circ}{\sin 68.4^\circ} \qquad m = \frac{14.3 \sin 68.2^\circ}{\sin 68.4^\circ}$$

$n = 10.6$ cm. $= 14.3$ cm

(*)
$B = 63^\circ$
$C = 83.9^\circ$
d ✓

b)



$d = \sqrt{4.8^2 + 4.3^2 - 2(4.8)(4.3)\cos 33.1^\circ}$
$= 2.6$

$$\frac{2.6}{\sin 33.1^\circ} = \frac{4.3}{\sin B}$$

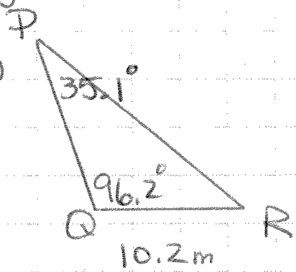
$$\sin B = \frac{4.3 \sin 33.1^\circ}{2.6}$$

$B \doteq 64.6^\circ$

$C \doteq 180^\circ - 64.6^\circ - 33.1^\circ$
$= 82.3^\circ$

* no ambiguous case.

Pg. 316 # 8c-11

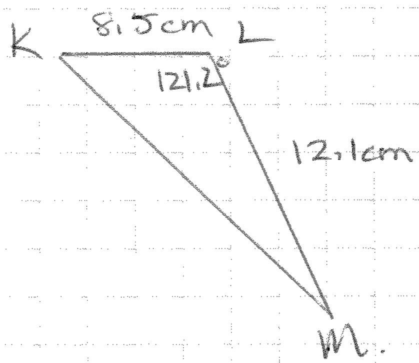
8c) 

$R = 48.7°$ (ASTT).

$$\frac{r}{\sin 48.7°} = \frac{10.2}{\sin 35.1°} = \frac{q}{\sin 96.2°}$$

$$r = \frac{10.2 \sin 48.7°}{\sin 35.1°} \qquad q = \frac{10.2 \sin 96.2°}{\sin 35.1°}$$

$$r \doteq 13.3 \text{ m} \qquad q = 17.6 \text{ m}$$

d) 

$$l = \sqrt{8.5^2 + 12.1^2 - 2(8.5)(12.1)(\cos 121.2°)}$$

$$l \doteq 18.0 \text{ cm.}$$

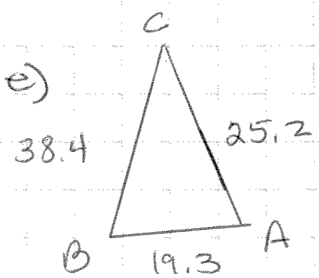
no worries about ambiguous case ... can only be one solution.

$$\frac{\sin M}{8.5} = \frac{\sin 121.2°}{18.0}$$

$$\sin M = \frac{8.5 \sin 121.2°}{18.0}$$

$$M \doteq 23.8°$$

$K \doteq 35.0°$ (ASTT).

e) 

$$C = \cos^{-1}\left(\frac{25.2^2 + 38.4^2 - 19.3^2}{2(25.2)(38.4)}\right)$$

$$\doteq 26.2°$$

$$A = \cos^{-1}\left(\frac{25.2^2 + 19.3^2 - 38.4^2}{2(25.2)(19.3)}\right)$$

$$= 118.7°$$

$B = 180° - 118.7° - 26.2°$ (ASTT).

$B = 35.1°$

NOTE: If you use sine law to find angle A you encounter the ambiguous case see next page for how to deal with this...

8e) First use cosine law as on previous page to find one angle ... BEST choice is to find the largest angle first (the angle across from the longest side). This is the only angle that <u>could</u> be obtuse.

<u>If</u> you did not do this but instead found B or C first, then when you use sine law for A, you get

$$\frac{\sin A}{38.4} = \frac{\sin 26.2^\circ}{19.3}$$

$$A = \sin^{-1}\left(\frac{38.4 \sin 26.2^\circ}{19.3}\right)$$

$$A \doteq 61.5^\circ$$ (notice not as accurate as using cosine law as we used an approximated value for C).

If $C = 26.2^\circ$, $A = 61.5^\circ$, then

$B = 180^\circ - 26.2^\circ - 61.5^\circ$
$= 92.3^\circ$

... not possible since a is the longest side ∴ A must be the largest angle ∴ b is the "middle length" side so B must be the "middle sized" angle.

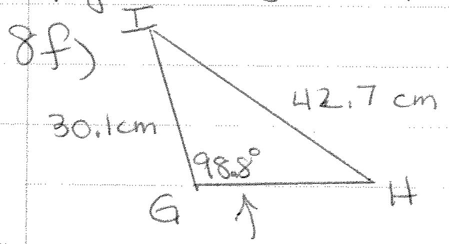
∴ ambiguous case and the correct angle A is <u>not</u> 61.5° but instead

$180^\circ - 61.5^\circ$
$= 118.5^\circ$ (notice not as accurate as cosine law answer).

So $A \doteq 118.5^\circ$, $B \doteq 180^\circ - 26.2^\circ - 118.5^\circ \doteq 35.3^\circ$, $C \doteq 26.2^\circ$

Pg. 317 8f – 11

8f)



>90 so no worries of ambiguous case.

$$\frac{\sin H}{30.1} = \frac{\sin 98.8^\circ}{42.7}$$

$$H = \sin^{-1}\left(\frac{30.1 \sin 98.8^\circ}{42.7}\right)$$

$$\doteq 44.2^\circ$$

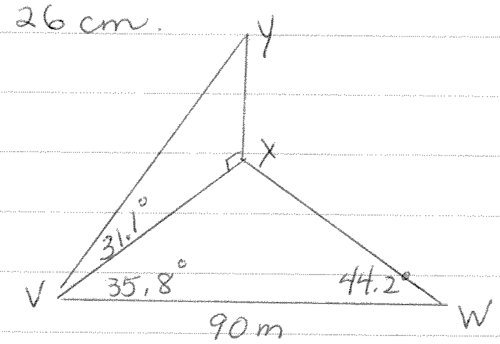
$$I = 180^\circ - 44.2^\circ - 98.8^\circ$$

$$= 37.0^\circ$$

$$i = \sqrt{30.1^2 + 42.7^2 - 2(30.1)(42.7)\cos 37^\circ}$$

$$\doteq 26 \text{ cm.}$$

9.



$\angle VXW = 100^\circ$ (ASTT)

In ΔXVW,

$$\frac{|XV|}{\sin 44.2^\circ} = \frac{90}{\sin 100^\circ}$$

$$|XV| = \frac{90 \sin 44.2^\circ}{\sin 100^\circ}$$

$$\doteq 63.7 m$$

In ΔXVY,

$$\tan 31.1^\circ = \frac{|XY|}{63.7}$$

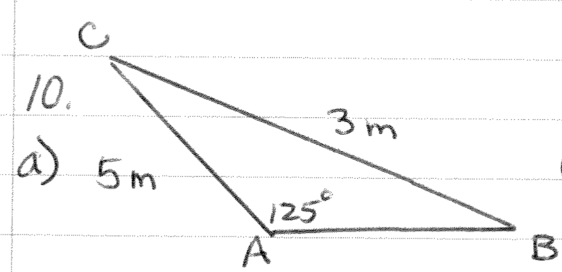
$$|XY| \doteq 38.4 \text{ m}$$

$$+ \ 1.7 \text{ m}$$

$$40.1 \text{ m.}$$

10.

a)

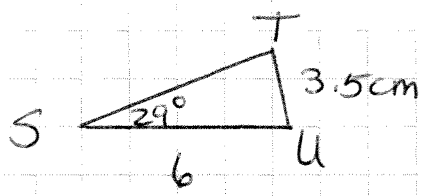


* not possible – no triangles. 125° would be the largest angle in the triangle (cannot have more than 1 obtuse angle).

The longest side is across from the largest angle ∴ 3 m is shorter than 5 m so this Δ is not possible.

Pg. 317 # 10b – 11.

10 b)



$\frac{\sin T}{6} = \frac{\sin 29^\circ}{3.5}$

$\sin T = \frac{6\sin 29^\circ}{3.5}$

$\sin T \doteq \left.\frac{2.9}{3.5}\right\} < 1$ so okay.

$T = \sin^{-1}\left(\frac{6\sin 29^\circ}{3.5}\right)$

* 2 Δ's possible.

$T \doteq 56.2^\circ$
$S = 29^\circ$
$U_1 \doteq 94.8^\circ$

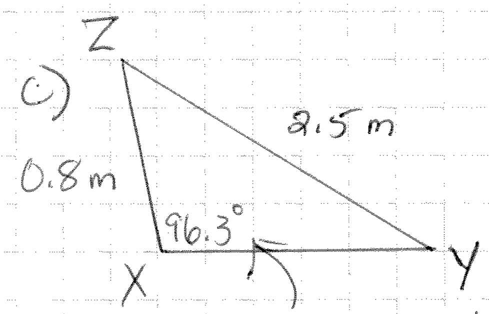
or

$T = 123.8^\circ$,
$S = 29^\circ$
$U_2 = 27.2^\circ$

$u_1 = \sqrt{6^2 + 3.5^2 - 2(6)(3.5)\cos 94.8^\circ}$
$\doteq 7.2\text{ m}$

$u_2 = \sqrt{6^2 + 3.5^2 - 2(6)(3.5)\cos 27.2^\circ}$
$\doteq 3.3\text{ m}$

c)



given obtuse angle so ambiguous not possible. (cannot have two obtuse ∠'s).

$\frac{\sin Y}{0.8} = \frac{\sin 96.3^\circ}{2.5}$

$Y = \sin^{-1}\left(\frac{0.8\sin 96.3^\circ}{2.5}\right)$

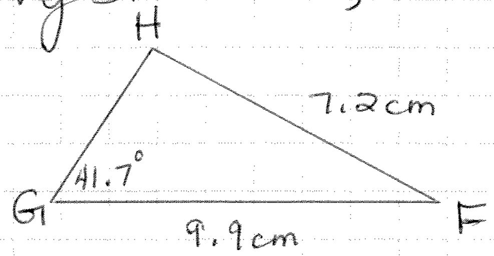
$= 18.5^\circ$

$Z = 65.2^\circ$ (ASTT)

$z = \sqrt{2.5^2 + 0.8^2 - 2(2.5)(0.8)\cos 65.2^\circ}$
$\doteq 2.3\text{ m}$

Pg 317 #10d, 11.

10d)



$$\frac{\sin H}{9.9} = \frac{\sin 41.7^\circ}{7.2}$$

$$\sin H = \frac{9.9 \sin 41.7^\circ}{7.2}$$

(this is <1 so may be 2 Δ's... must check!)

$H_1 \doteq 66.2^\circ$

$G = 41.7^\circ$

$F = 72.1^\circ$

$f_1 = \sqrt{9.9^2 + 7.2^2 - 2(9.9)(7.2)\cos 72.1^\circ}$

$\doteq 10.3$ cm

OR $H_2 \doteq 180^\circ - 66.2^\circ$

$H_2 \doteq 113.8^\circ$

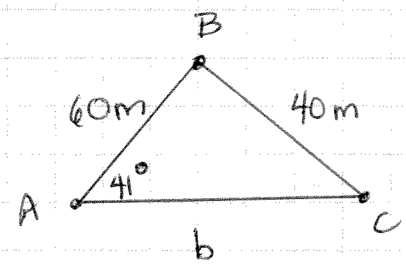
$G = 41.7^\circ$

$F = 24.5^\circ$

$f_2 = \sqrt{9.9^2 + 7.2^2 - 2(9.9)(7.2)\cos 24.5^\circ}$

$\doteq 4.5$ cm

11.



find all possible values of b.

$$\frac{b}{\sin B} = \frac{40}{\sin 41^\circ} = \frac{60}{\sin C}$$

$$C = \sin^{-1}\left(\frac{60 \sin 41^\circ}{40}\right)$$

$C_1 \doteq 79.8^\circ$ $\quad$ $C_2 = 180^\circ - 79.8$

$A = 41^\circ$ $\quad$ $= 100.2^\circ$

$B_1 = 59.2^\circ$ (ASTT) $\quad$ $A = 41^\circ$

$B_2 = 38.8^\circ$

$b_1 = \sqrt{60^2 + 40^2 - 2(60)(40)\cos 59.2^\circ}$

$\doteq 52.4$ m

$b_2 = \sqrt{60^2 + 40^2 - 2(60)(40)\cos 38.8^\circ}$

$\doteq 38.2$ m

∴ |AC| is either 52 metres or 38 metres.

means the length of the line segment joining A to C.