

Pg 388 # 12a-d, 14, 16.  
(assume  $t=0$  is low tide)

Unit 6 lesson 8 Pg 0.

$$\left. \begin{array}{l} \text{12.a) depth at 21 m at high tide} \\ \text{11 m at low tide} \end{array} \right\} \text{Amplitude} = \frac{21-11}{2} = 5$$

Period is 12 hours  $\frac{360^\circ}{k} = 12 \text{ hours}$

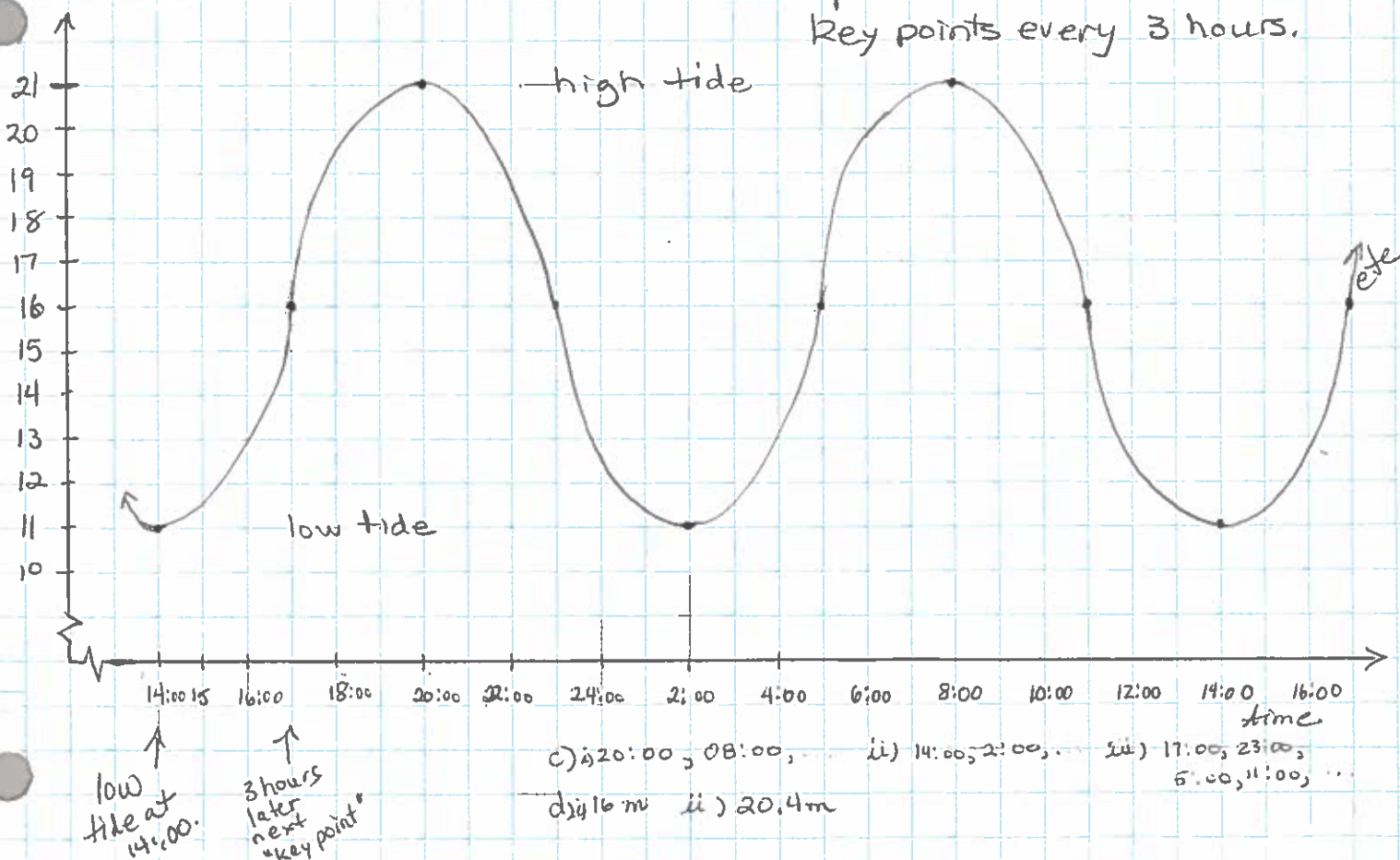
$$k = 30^\circ/\text{hour}$$

use  $k=30$  (this makes a lot more sense when we work with radians in grade 12).

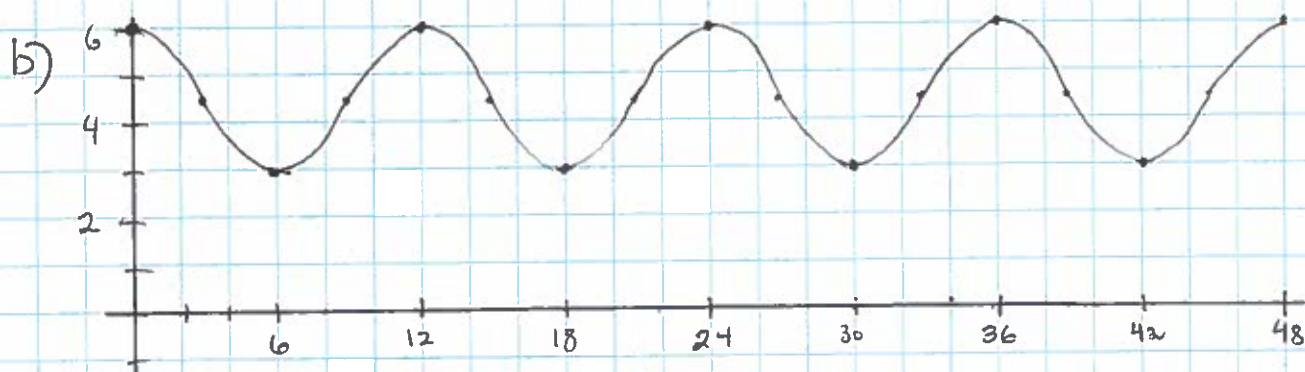
$$y = 5 \sin 30(t-5) + 16$$

← low tide at 14:00

$y = 5 \sin 30t + 16$  ← low tide at 0:00 hr.  
... use hours on the x-axis - 1 period is 12 hours so  
Key points every 3 hours.

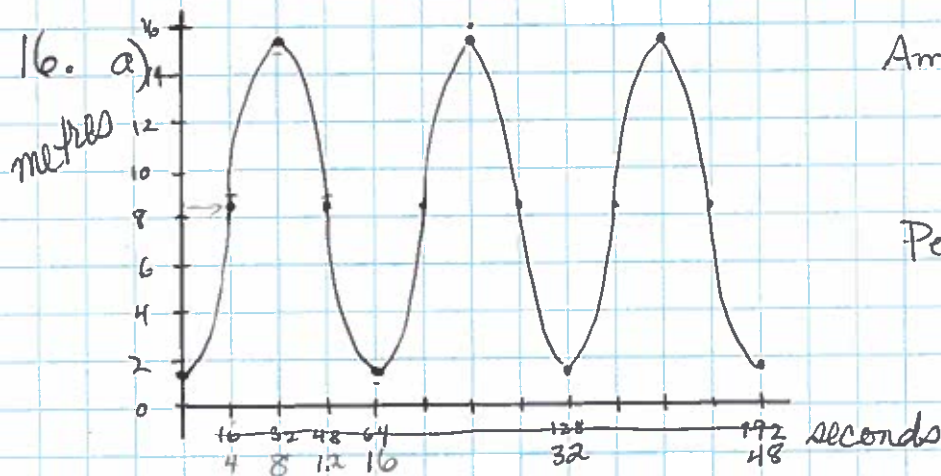


a)  $d(t) = \frac{3}{2} \cos 30t + 4.5$



c)  $d(8) = 3.75$   
 $d(15) = 4.5$   
 $d(20) = 3.75$   
 $d(30) = 3$

d) shift left 6 hours (or right 6 hours) for the graph to begin at low tide.



Amplitude = radius  
 $= \frac{15.5 - 1.5}{2}$   
 $= 7$

Period = 16 seconds  
 $\frac{360^\circ}{k} = 16 \text{ seconds}$

$k = 22.5 \text{ degrees/second}$   
 $\left(\frac{45^\circ}{2}\right)$

d) shift right 4 seconds  
 c) shift up 8.5  
 $C = 1.5 + 7$   
 $= 8.5$

b)  $y = 7 \sin \frac{45}{2}(x - 4) + 8.5$

c) as wheel revolves slower, period gets larger,  $k$  gets smaller.

## MCR 3UI

Applications of Sinusoidal & Other Trigonometric Functions

1. The human ear interprets the amplitude of a sound wave as loudness. Thus, a sound wave modelled by  $y = 2\sin kt$  is louder than a sound wave modelled by  $y = \sin kt$ . Because the human ear does not operate on a linear scale, the perceived loudness ratio is not actually 2:1. When an instrument is played, the sound wave spreads out in a spherical pattern. The amplitude decreases as the square of the distance. Suppose that a sound wave can be modelled as  $y = 64 \sin kt$  at a distance of 1 m from its source. At a distance of 2 m,  $a = \frac{64}{2^2} = 16$ . The modelling

equation becomes  $y = 16 \sin kt$ .

a) Write the modelling equation at a distance of 4 m from the source.

b) Write the modelling equation at a distance of 8 m from the source.

c) How far from the source does the modelling equation become  $y = \frac{1}{4} \sin kt$ ?

2. Is  $y = \tan x$  a function? Explain how you know. What is the domain and range of  $y = \tan x$  on the interval  $0^\circ \leq x \leq 360^\circ$ ?

3. At the end of a dock, high tide of 14 m is recorded at 9:00 a.m. Low tide of 6 m is recorded at 3:00 p.m. A sinusoidal function can model the water depth versus time.

a) Construct a model for the water depth using a cosine function, where time is measured in hours past high tide.

b) Construct a model for the water depth using a sine function, where time is measured in hours past high tide.

c) Construct a model for the water depth using a sine function, where time is measured in hours past low tide.

d) Construct a model for the water depth using a cosine function, where time is measured in hours past low tide.

4. The height,  $h$ , in metres, above the ground of a rider on a Ferris wheel after  $t$  seconds can be modelled by the sine function:  $h(t) = 10 \sin [3(t-30)] + 12$ .

a) Graph the function.

b) Determine the maximum and minimum heights of the rider above the ground.

c) Determine the height of the rider above the ground after 30 seconds.

d) Determine the time required for the Ferris wheel to complete one revolution.

5. The population,  $P$ , of a lakeside town with a large number of seasonal residents can be modelled using the function  $P(t) = 5000 \sin[30(t-7)] + 8000$ , where  $t$  is the number of months after New Year's Day.

a) Find the maximum and minimum values for the population over a whole year.

b) When is the population a maximum? When is it a minimum?

c) What is the population on September 30?

d) When is the population about 10 000?

6. The movement of a piston in an automobile engine can be modelled by the function  $y = 50\sin 10800t + 20$ , where  $y$  is the distance, in centimetres, from the crankshaft and  $t$  is the time, in seconds.

- What is the period of the motion?
- Determine the maximum, minimum and amplitude.
- When do the maximum and minimum values occur?
- What is the vertical position of the piston at  $t = \frac{1}{120}$  seconds?

7. Smog is a generic term used to describe pollutants in the air. A smog alert is usually issued when the air quality index is greater than 50. Air quality can vary throughout the day, increasing when more cars are on the road. Consider a model of the form  $I = 30 \sin[15(t-4)] + 25$ , where  $I$  is the value of the air quality index and  $t$  measure the time after midnight, in hours.

- What is the period of the modelled function? Explain why this makes sense.
- Determine the maximum, minimum and amplitude.
- When do the maximum and minimum occur?
- During what time interval would a smog alert be issued, according to this model?

8. A Ferris wheel has a diameter of 20 m and is 4 m above ground level at its lowest point. Assume that a rider enters a car from a platform that is located 30 degrees around the rim before the car reaches its lowest point.



- Model the rider's height above the ground versus angle using a transformed sine function.
- Model the rider's height above the ground versus angle using a transformed cosine function.
- Suppose that the platform is moved to 60 degrees around the rim from the lowest position of the car. How will the equations in parts a) and b) change? Write the new equations.

9. Suppose that the centre of the Ferris wheel in question 8 is moved upward 2 m, but the platform is left in place at a point 30 degrees before the car reaches its lowest point. How do the equations in parts a) and b) of question 8 change? Write the new equations.

Answers: (answers may vary due to the periodic nature of the sinusoidal functions)

1 a)  $y = 4\sin kt$  b)  $y = \sin kt$  c) 16 m 2. Yes.  $D = \{x \in R \mid x \neq 90^\circ, x \neq 270^\circ\}$ ,  $R = \{y \in R\}$

3 a)  $y = 4\cos 30x + 10$  b)  $y = 4\sin[30(x+3)] + 10$  c)  $y = 4\sin[30(x-3)] + 10$  d)  $y = 4\cos[30(x-6)] + 10$

4. b) Max = 22 m, min = 2 m c) 12 m d) 120 seconds

5 a) max 13 000, min 3000 b) max: end of Oct at  $t = 10$ , min: end of Apr at  $t = 4$  c) 12 330

d) Jan 6,  $t = 6$  days and <sup>Aug</sup> July 24,  $t = 7$  months 24 days. 6a)  $\frac{1}{30}$  s b) max 70 cm, min -30 cm,

amp 50 cm c) max at  $\frac{1}{120}$  s, min at  $\frac{1}{40}$  s d) 70 cm 7a) period 24 hours Sample answer: Smog

is often created from human activity that generally repeats from day to day, so a period of 24 hours is appropriate. b) max 55, min -5, amp 30 c) max at 10 a.m., min at 10 p.m.

d) sample answer: An interval around 10 a.m. (precisely: 7:46 a.m. until 12:14 p.m.)

8 a)  $y = 10\sin(x+240) + 14$  b)  $y = 10\cos(x+150) + 14$  c)

$y = 10\sin(x+210) + 14$  b)  $y = 10\cos(x+120) + 14$

9a)  $y = 10\sin(x+240) + 16$  b)  $y = 10\cos(x+150) + 16$

# 1. Applications Solutions. (Sinusoidal Functions).

$$\begin{aligned} \text{a) } \frac{64}{4^2} \\ = \frac{64}{16} \\ = 4 \end{aligned}$$

$$\therefore y = 4 \sin kt$$

$$\begin{aligned} \text{b) } \frac{64}{5^2} \\ = \frac{64}{25} \\ = 2.56 \end{aligned}$$

$$\therefore y = \sin kt$$

$$\text{c) } \frac{64}{d^2} \neq \frac{1}{4}$$

$$d^2 = 256$$

$$d = 16$$

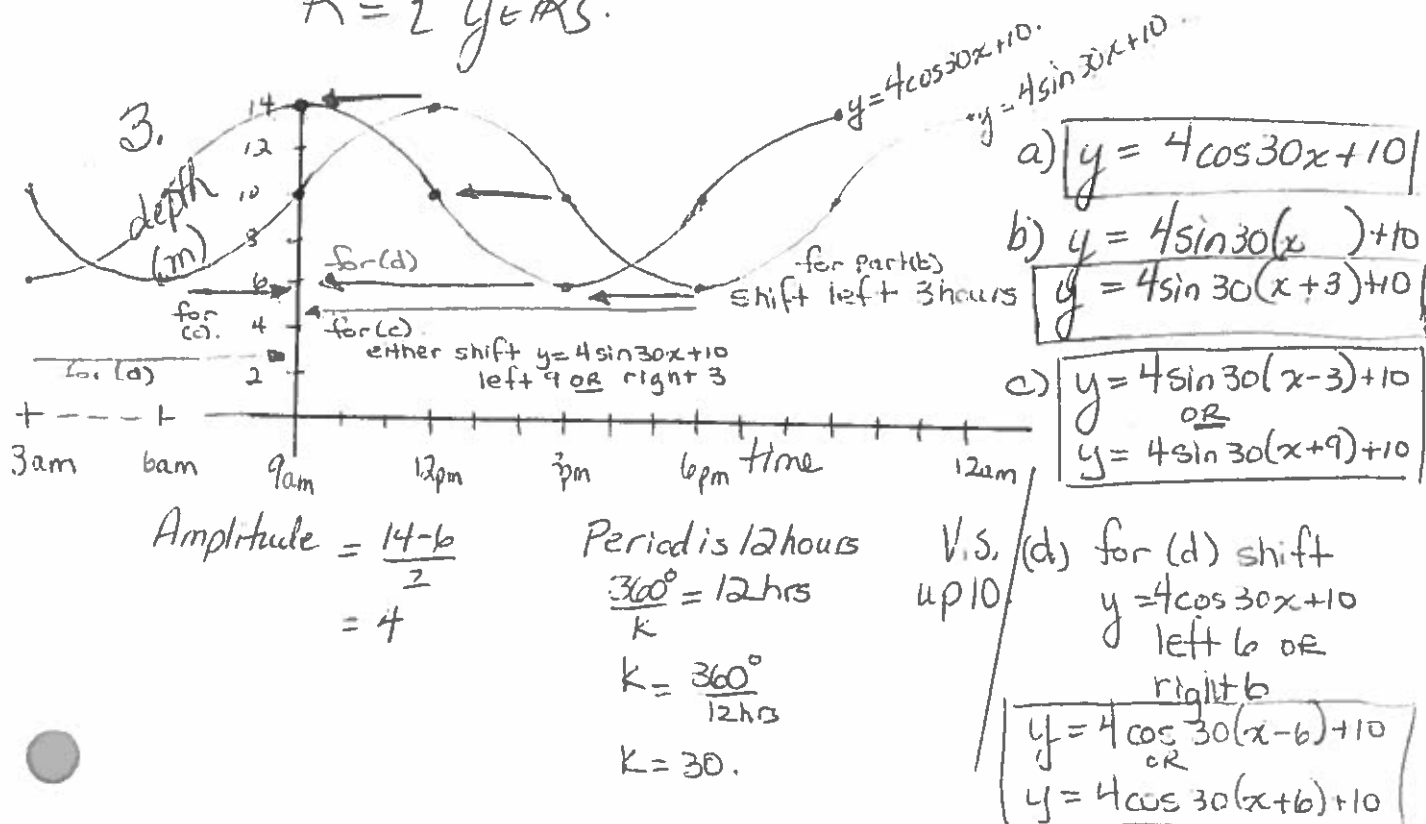
$\therefore$  the human ear is 16m from the source of the sound.

2. Yes,  $y = \tan x$  is a function.

There is no  $x$ -value with more than one  $y$ -value.

$$D = \{x \in \mathbb{R} \mid x \neq 90^\circ, x \neq 270^\circ\}$$

$$R = \{y \in \mathbb{R}\}.$$





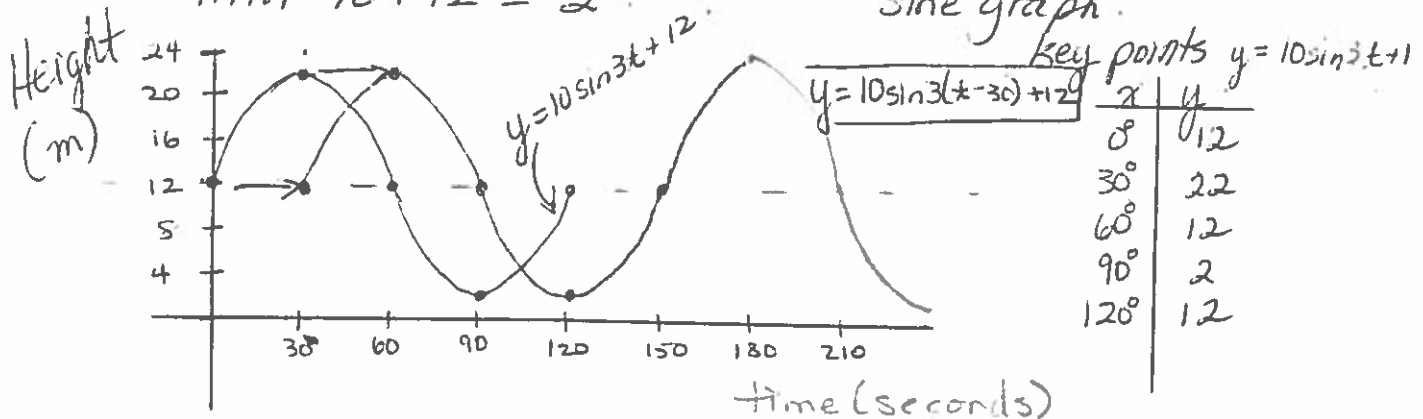
4. Period =  $\frac{360^\circ}{3^\circ/\text{second}}$   
 $= 120 \text{ seconds}$

Horizontal Translation  
 right 30.

Amplitude 10

Vertical translation  
 up 12.

Max  $10 + 12 = 22$   
 Min  $-10 + 12 = 2$



4b) Maximum 22 m  
 Minimum 2 m

c) from graph Height = 12 m at 30 seconds.

d) Period is 120 seconds  
 so it takes 120 seconds for the ferris wheel to complete one revolution.

5. a) max  $5000 + 8000 = 13000$  people.

b) min  $-5000 + 8000 = 3000$  people

b) Period is  $\frac{360^\circ}{30^\circ/\text{month}} = 12$  months.

~~Keep~~ Key points every 3 months.

$$y = 5000 \sin(30(t-7)) + 8000$$

It shifted right seven months.

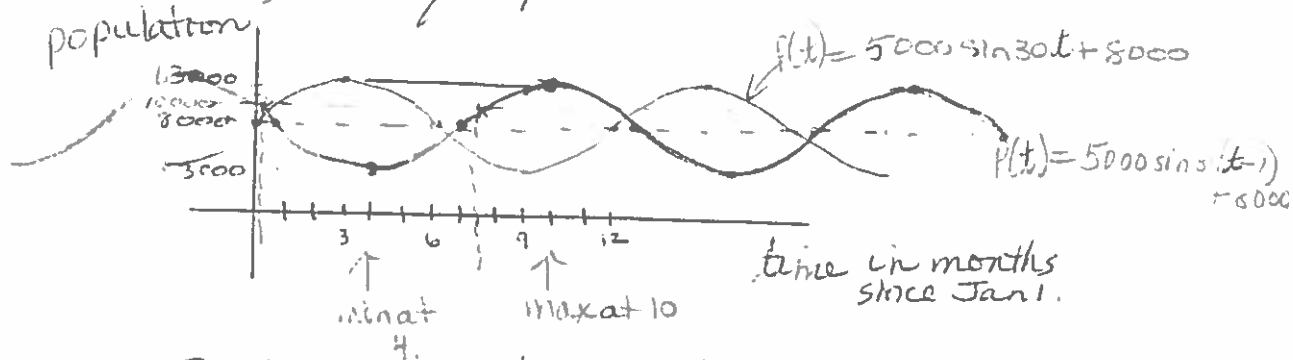
But shift max at 3 mas so

new max at  $7 + 3 = 10$  months.

$\Rightarrow$  end of October.

5b). cent'd.

min 3 months & 7 months = 4 months.  
 $\Rightarrow$  end of April.



c) September 30<sup>th</sup>  $\Rightarrow t=9$

$$\begin{aligned} P(q) &= 5000 \sin 30(q-7) + 8000 \\ &= 5000 \sin 60^\circ + 8000 \\ &= 5000 \left( \frac{\sqrt{3}}{2} \right) + 8000 \\ &= 2500\sqrt{3} + 8000 \\ &\approx 12330 \text{ people} \end{aligned}$$

d)  $P(t) = 10000$

\* note using an average of 30 days per month

$$\begin{aligned} 10000 &= 5000 \sin 30(t-7) + 8000 \\ 2000 &= 5000 \sin 30(t-7) \\ \frac{2000}{5000} &= \sin 30(t-7) \\ \frac{2}{5} &= \sin 30(t-7) \end{aligned}$$

$$\sin^{-1}\left(\frac{2}{5}\right) = 30(x-7)$$

$$\frac{\sin^{-1}(0.4)}{30} + 7 = t$$

$t = 7.7859$  end of July + 24 days  
 $\Rightarrow 7 \text{ months} + 24 \text{ days}$

August 24<sup>th</sup>

OR (look at graph)

↳ 24 days before the end of the 1<sup>st</sup> month

$$\Rightarrow 30 - 24 = 6$$

1. Jan 6<sup>th</sup>

If you use  
 $7359231$   
 $= 24.3024 \text{ days}$   
 $21 - 24.3024$   
 $= -3.3024$   
 $\rightarrow$  must  
 jump out you  
 are stuck in  
 a loop

$$y = 50 \sin 10800t + 20$$

b) a) period =  $\frac{360^\circ}{10800/\text{second}}$   
 $= \frac{1}{30}$  seconds.

b) max  $50 + 20 = 70\text{cm}$   
 min  $-50 + 20 = -30\text{cm}$   
 Amplitude =  $50\text{cm}$

c)  $y = 70$ , find  $t$

$$70 = 50 \sin 10800t + 20$$

$$50 = 50 \sin 10800t$$

$$1 = \sin 10800t$$

$$\sin^{-1} 1 = 10800t$$

$$90^\circ = 10800t$$

$$t = \frac{90^\circ}{10800^\circ/\text{second}}$$

$$t = \frac{1}{120} \text{ second}$$

$y = -30$ , find  $t$

$$-30 = 50 \sin 10800t + 20$$

$$-50 = 50 \sin 10800t$$

$$-1 = \sin 10800t$$

$$\sin^{-1}(-1) = 10800t$$

$$270^\circ = 10800t$$

$$\frac{270^\circ}{10800^\circ/\text{second}} = t$$

$$t = \frac{1}{40} \text{ s}$$

max occurs after  $\frac{1}{20}$  second, min occurs after  $\frac{1}{40}$  second.

d) Find  $y$  when  $t = \frac{1}{120}$  seconds.

(that is at the max (from part c) so we know answer should be  $70\text{cm}$ ).

Algebraically,

$$y = 50 \sin 10800\left(\frac{1}{120}\right) + 20$$

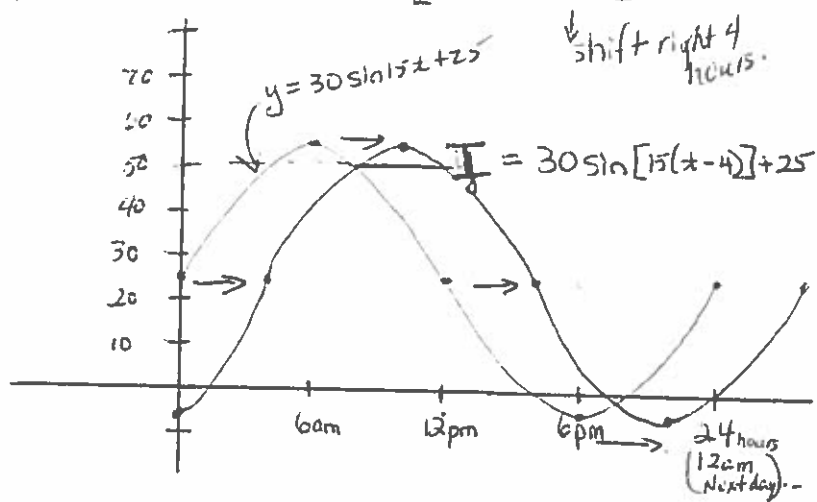
$$= 50 \sin 90^\circ + 20$$

$$= 50 + 20$$

$$= 70 \text{ cm.}$$



7.  $I = 30 \sin[15(t-4)] + 25$



a) Period =  $\frac{360}{15}$   
 $= 24$

Pattern repeats daily.

b) Max  $30 + 25 = 55$   
 Min  $-30 + 25 = -5$   
 Amplitude = 30

c)  $\frac{\text{Max}}{55} = 30 \sin 15(t-4) + 25$   
 $30 = 30 \sin 15(t-4)$   
 $1 = \sin 15(t-4)$   
 $\sin^{-1} 1 = 15(t-4)$   
 $\frac{90}{15} = t-4$

$6 + 4 = t$  Max at  
 $t = 10$  10am

c) Min  
 $\frac{-5 - 25}{30} = \sin 15(t-4)$

$\sin^{-1}(-1) = 15(t-4)$

$270^\circ = 15(t-4)$

$\frac{270}{15} + 4 = t$

$t = 22$  hours past midnight

$\Rightarrow 10 \text{ pm}$

So, Min at 10pm

d)  $50 = 30 \sin 15(t-4) + 25$   
 $\frac{25}{30} = \sin 15(t-4)$

$\sin^{-1}\left(\frac{5}{6}\right) = 15(t-4)$

$\frac{\sin^{-1}\left(\frac{5}{6}\right)}{15} + 4 = t$

$t = 7.76$

$\Rightarrow 7:46 \text{ am}$

$0.76 \times 60$   
 $\approx 46$

OR  
 $\frac{[180^\circ - \sin^{-1}\left(\frac{5}{6}\right)]}{15} + 4 = t$

$t \approx 12.24$

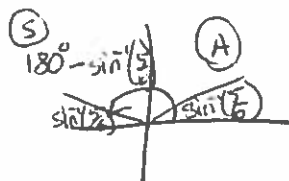
$\Rightarrow 12:14$

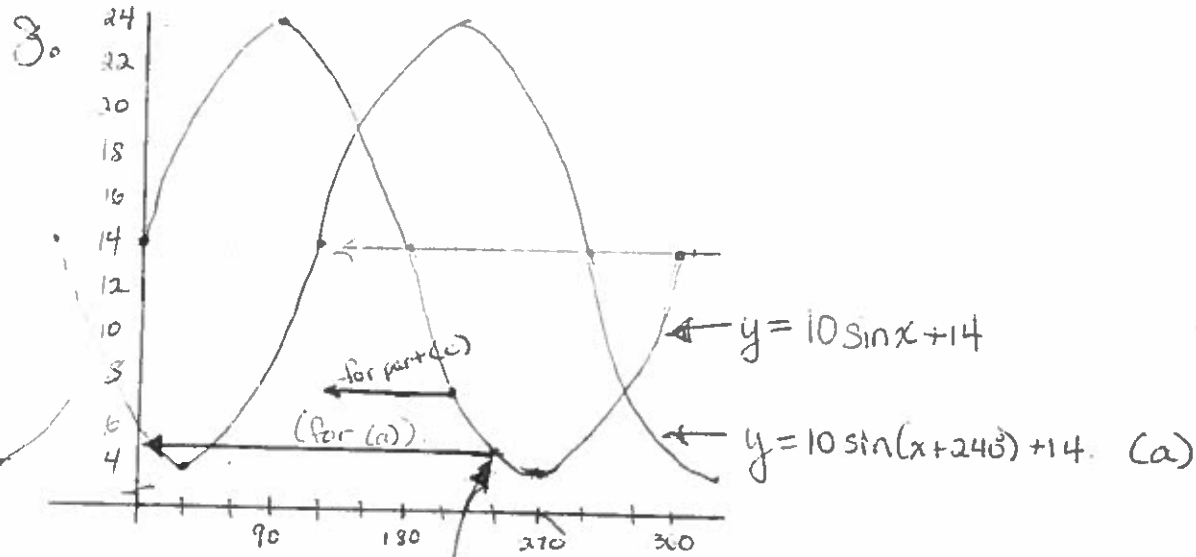
12 hrs 14 mins past midnight

$\rightarrow 12:14 \text{ pm}$

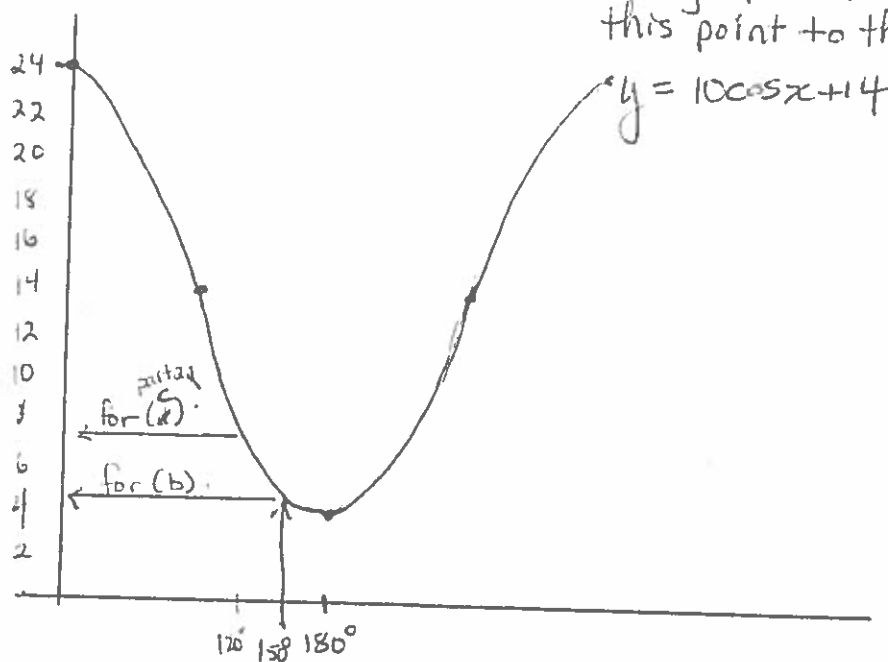
$0.24 \times 60$   
 $= 14$

$\therefore$  the index is over 50  
 from 7:46 am to 12:14 pm.





this is the point  $30^\circ$  before the lowest point  
 ... shift graph left  $240^\circ$  to move  
 this point to the 'start'.



$30^\circ$  before lowest point  
 shift graph so this point is at the start - the 'initial point'.  
 ... shift left  $150^\circ$ .

(b)  $y = 10 \cos(x + 150) + 14$

(c) see <sup>first</sup> graph above  $\rightarrow y = 10 \sin(x + 210) + 14$ .

(X) see second graph above  $\rightarrow y = 10 \cos(x + 120) + 14$

#9 just translate 8(a), 8(b) up another 2m (a)  $y = 10 \sin(x + 240) + 16$   
 (b)  $y = 10 \cos(x + 150) + 16$