Unit 7: Exponential Functions

Lesson 6: Exponential Growth & Decay

RECALL:

Last class we compared linear, quadrac and exponenal funcons.

Success Criteria:

Aer last class you should be able to...

- idenfy whether a funcon is linear, quadrac or exponenal from its table of values, graph and equaon.

TODAY:

Success Criteria:

At the end of today's lesson, can you...

- use the informaon given to come up with the equaon that models real world exponenal growth and decay problems?
- solve problems involving real world applicaon of exponenal decay and growth?

RECALL: Exponenal funcons are funcons in the form

$$y = a(b)^x$$

where $b > 0$ and $b \neq 1$

Exponenal funcons are oen used in the real-world to help describe the growth and decay of populaons, investments, etc.

NOTE: It cannot always be used long-term. This becomes parcularly evident with models of populaon growth as eventually the populaon would become unsustainable.

KEY CONCEPTS:

Exponenal growth...	is modeled by an increasing exponenal funcon (i.e. $b > 1$)
Exponenal decay...	is modeled by a decreasing exponenal funcon (i.e. $0 < b < 1$) Fractions as base.

How to use the equaon $y = a(b)^x$ to model exponenal growth & decay:

$y =$ the **total** (populaon/investment amount/etc.)

***NOTE: Oen we use the variable $'P'$ instead of y when represenng populaon

$a =$ **the inial/starng amount** ***(think ABCs...a starts it off)

$b =$ **the growth rate** ***(b > 1 for growth and 0 < b < 1 for decay)

- For exponenal growth: $b = 1 + r$, where r is the rate
- For exponenal decay: $b = 1 - r$

$x =$ **number of growth/decaying periods** (days/weeks/years/etc.)

***NOTE: Oen we use the variables $'n'$ instead of x when represenng populaon

For word problems...

STEP 1: Find the important informaon (usually numbers).

Some key words to keep an eye out for...

* *half-life:* this means that the *b* is $\frac{1}{2}$

* *doubles:* this means that the *b* is 2

* *triples:* this means that the *b* is 3

* *depreciates:* this means that the *b* must be <1

STEP 2: Take what you know and sub it into the equaon.

STEP 3: Solve for what's missing.

EX. 1. A bacteria colony has an inial populaon of 200. The populaon increases by 20% each day.

$\hookrightarrow 0.2$

a) How many bacteria will there be in one week?

$P =$ Final Population

$a = 200$

$b = 1.2$

$t = 7$

$y = a(b)^x$

$P = a(b)^n$

$= 200(1.2)^7$

$= 200(\boxed{3.5831808})$

$= 716.6$

Keep this # in calc.

$\therefore$ there would be 716 bacteria.

b) When will the populaon reach 2500?

$P = 2500$

$a = 200$

$b = 1.2$

$t = ??$

$y = a(b)^x$

$P = a(b)^t$

$$\frac{2500}{200} = \frac{\cancel{200}(1.2)^t}{\cancel{200}}$$

$12.5 = (1.2)^t$

Guess and check.

$1.2^{10} = 6.19$

$1.2^{14} = 12.8$

$1.2^{15} = 15.4$

$1.2^{13.5} = 11.7$

$1.2^{13.9} = 12.6$

$1.2^{20} = 38.3$

$\therefore$ 13.9 days until pop is 2500.

$0.9 \times 24 \doteq 22.$

13 days and 22 hours.

EX. 2. It is esmated that a new car $\boxed{\text{depreciates by 20\% per year.}}$ $1-0.2 = 0.8$

a) Determine the value of a car aer 4 years if it costs $27,000 new.

$P =$ Final Value.
$a =$ 27 000
$b =$ 0.8
$t =$ 4

$P = a(b)^t$
$= 27\,000(0.8)^4$
$= 27\,000(0.4096)$
$= 11\,059.20$

$\therefore$ the car would be worth $11 059.20

b) When will the car be worth $8,000?

$P =$ 8 000
$a =$ 27 000
$b =$ 0.8
$t =$??

$$\frac{8000}{27\,000} = \frac{\cancel{27\,000}(0.8)^t}{\cancel{27\,000}}$$

$0.296 = 0.8^t$

Guess & check:

$0.8^6 = 0.262$
$0.8^5 = 0.32768$
$0.8^{5.5} = 0.293$

$\therefore$ after 5 years and 6 months it will be worth $8000

EX. 3. Water lilies in a pond double each day. It takes 30 days for the lilies to completely cover the pond. On what day was the pond half full of water lilies?

$P = \boxed{}$

$a = 1$

$b = 2$

$t = 30$

$$P = a(b)^t$$
$$= (1)(2)^{30}$$
$$= (1)(1\ 073\ 741\ 824)$$
$$= 1\ 073\ 741\ 824$$

To determine how many days to get half the pond covered. it would have happened the 29th day.

$\frac{1}{2}$ (half life) t

EX. 4. The remaining mass of a drug in a person's bloodstream is modelled by $M = 500(0.5)^{\frac{t}{2}}$, where M is the remaining mass in milligrams, and t is the me, in hours, that the drug is in the bloodstream.

a) What is the half-life of the drug?

$\frac{1}{2}$ hour

b) What was the dosage of the drug?

500 mg

c) What will be the concentraon of the drug in the bloodstream

i. aer 2 hours?

$M = 500\left(\frac{1}{2}\right)^2$

$= 500(0.25)$

$= 125$

$\therefore$ 125mg are still in blood.

ii. aer 6 hours?

$M = 500\left(\frac{1}{2}\right)^6$

$= 500(0.015625)$

$= 7.8125$

$\therefore$ 7.8mg in blood after 6 hours.

Practice: p. 429 #2, 4, 7, 10

p. 437 #1, 3, 8, 9