NAME: _____

MPM 2DI

EXAM REVIEW

03 – Wednesday, January 29, 2020
8:30 am – 10:00 am

*** A PENCIL, SCIENTIFIC CALCULATOR AND RULER ARE REQUIRED ***

Please Note: Your final mark in this course will be calculated as the better of:

MARK 1

Quizzes	15 %
Tests	55 %
Final Exam	30 %

OR

MARK 2

Final Exam	100 %

WATERLOO-OXFORD DISTRICT SECONDARY SCHOOL
FINAL EXAMINATION

Department:	Mathematics	Date:	Wednesday, January 29, 2020
Course:	MPM 2DI	Time:	8:30 AM
Section:	01	Length:	90 minutes
Teachers:	Mr. G. Albrecht		
		Page:	1 of 9

Student Name: _____ Mark: _____ /

INSTRUCTIONS

1. A scientific calculator is permitted; calculators built into phones or any other electronic devices are not. Check to make sure your calculator is set to Degrees.

2. Write all answers in pencil.

3. All formulas introduced this year are indicated on the front page. Formulas from previous years are expected to have been committed to memory.

4. You have 90 minutes to complete the exam. No extra time will be given. If you are done early, check each of your solutions carefully and double check your calculations.

5. For mathematical answers, write full solutions showing all important logical steps. Full marks may not be granted if only the final answer is shown.

FORMULAS

$y = a(x-s)(x-t)$
$y = ax^2 + bx + c$
$y = a(x-h)^2 + k$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x^2 + y^2 = r^2$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$
SOH-CAH-TOA
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
$c^2 = a^2 + b^2 - 2ab\cos C$

MPM 2DI COURSE OVERVIEW

Unit 1: Solving Linear Systems

- Find the point of intersection by graphing
- Find the point of intersection by substitution
- Find the point of intersection by elimination
- Verify if a point is on a line using a left side/right side check
- Understand when a system has one solution, no solution or many solutions

Unit 2: Analytic Geometry

- Distance between two points (length of a line segment): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Coordinates of the midpoint of a line segment: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- Equation of a circle with centre (0, 0) and radius *r*: $x^2 + y^2 = r^2$
- Slope of a line segment: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Perpendicular lines meet at a 90° (right) angle and have negative reciprocal slopes
- Use length of a line segment to determine if a triangle is isosceles, equilateral or scalene
- Use slope of a segment to determine if a triangle is a right-angled triangle
- Use both length of a line segment and slopes of lines to determine if a quadrilateral is a square, rectangle, parallelogram or rhombus
- Find the equation of a perpendicular bisector, an altitude and a median of a triangle.

Unit 3: Polynomials and Factoring

- Add and subtract polynomials
- Multiply polynomials
- Factor polynomials (common, simple trinomial, complex trinomial, difference of squares)

Units 4 & 5: Graphing Quadratic Relations

- Understand the terms: vertex, axis of symmetry, direction of opening, maximum value, minimum value, optimum value, roots, zeros
- Factor a quadratic in standard form: $y = ax^2 + bx + c$ to put it in factored form: $y = a(x - s)(x - t)$
- Complete the square on a quadratic in standard form to put it in vertex form: $y = a(x - h)^2 + k$
- Graph a parabola, given its equation in standard, factored or vertex form.
- Identify the effect of simple transformations on the graph of $y = x^2$

$$y = a(x - h)^2 + k$$

If $a < 0$, reflection in x-axis

If $a > 1$ or $a < -1$, vertical stretch by a factor of *a*

If $-1 < a < 1$, vertical compression by a factor of *1/a*

If $h > 0$, translation right *h* units.

If $h < 0$, translation left *h* units.

If $k > 0$, translation up k units.

If $k < 0$, translation down *k* units.

Units 4 & 5: Graphing Quadratic Relations Continued…

Exponent Laws

$a^{-n} = \left(\frac{1}{a}\right)^n$ $\quad a^0 = 1$ $\quad x^m \cdot x^n = x^{m+n}$

$x^m \div x^n = x^{m-n}$ $\quad \left(x^m\right)^n = x^{mn}$ $\quad (xy)^n = x^n y^n$ $\quad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Unit 6: Solving Quadratic Equations, Quadratic Word Problems

- Solving by factoring
- Solving by completing the square
- Solving using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Determine what you are asked to find
  - → If it requires finding a maximum or minimum, then complete the square.
  - → If it requires solving a quadratic equation, the factor or use the quadratic formula.
- Look at your answer and ask yourself: "Is this answer possible?" You may find that your answer is not possible because it does not fit with the facts presented in the problem.

Unit 7: Congruent and Similar Triangles and Trigonometry

- Conditions for congruency: SSS$\cong$, SAS$\cong$, ASA$\cong$
- Conditions for similarity: SSS~, SAS~, AA~
- Congruent triangles: corresponding sides equal, corresponding angles equal
- Similar triangles: corresponding sides proportional, corresponding angles equal
- Primary trigonometric ratios: SOH-CAH-TOA
- Know how to use primary trig ratios to solve right-angled triangles
- Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin \mathrm{B}}{b} = \frac{\sin C}{c}$
- Cosine Law: $a^2 = b^2 + c^2 - 2bc\cos A$ or $b^2 = a^2 + c^2 - 2ac\cos B$ or $c^2 = a^2 + b^2 - 2ab\cos C$
- Use Sine Law or Cosine Law as appropriate to solve non-right angled triangles.

Exam Details

- Double check your exam date, time and room number with your teacher.
- Come prepared with a scientific calculator, pencils, eraser and a ruler.
- Study!! Create a study schedule NOW. Arrange for extra help if necessary.
- Do ALL assigned review questions and check your answers. Do extra work on topics you struggle with.
- Old tests and quizzes are a great study aid. Redo and check your solutions with the corrections you have copied down.

MPM 2DI COURSE REVIEW

Unit 1: Number Systems and Solving Linear Systems

Please Note
Exponent Laws were actually covered in Unit 4.

1. Simplify:

a) $2[3(-4)-7+8]$ b) $\frac{-8}{9}-\left(\frac{-5}{8}\right)$ c) $5-6[3(-4)-(-1)]$ d) $\frac{7}{8}\div\left(-\frac{3}{4}\right)$

e) $3\left(\frac{-1}{4}\right)+2\left(\frac{2}{-3}\right)-5\left(\frac{-1}{4}\right)\left(\frac{2}{-3}\right)$ f) 2^4 g) $\left(\frac{2}{5}\right)^2$

h) $\left(\frac{3}{4}\right)^{-2}$ i) $(1.5)^0$ j) 2^3+2^{-3} k) $(3^2-2^2)^{-2}$

Additional Practice: page 16 #5, 6, page 18 #4

2. Solve the following system by graphing:
$y=-x+5$
$3x-y-3=0$

3. Solve each system by substitution:

a) $x+2y=4$ b) $y=2x+3$
$2x-3y=7$ $4x+y=8$

4. Solve each system by elimination:

a) $3x+8y=-3$ b) $11m+3n=25$ c) $3x-4y=10$ d) $4x-5y=-22$
$5x+8y=-5$ $-11m+7n=-15$ $2x-3y=7$ $5x+6y=-3$

5. The perimeter of a rectangle is 50 *cm*. The length is 7 *cm* more than the width. Find the dimensions of the rectangle.

6. Amy invested $20,000, part in bonds that paid 3% per annum and the remainder in term deposits that paid 5% per annum. The total interest after one year was $760. How much did she invest at each rate?

7. Brand A fertilizer is 32% phosphorus, while Brand B is 18% phosphorus. How much of each must be used to produce 56 *tonnes* of a 24% mixture?

8. The distance from Cambridge to Ottawa is 480 *km*. Gerry is able to drive at 100 *km/hour* on Highway 401 but can only average *60 km/hr* on Highway 7. If the trip takes 6 *hours*, how far did he travel on each highway?

9. Without graphing, determine if the linear system has zero, one, or many solutions:

a) $y=x+5$ b) $4x-y=-3$ c) $y=-x+4$
$y=x-1$ $2x-y=-1$ $3y=-3x+12$

Additional Practice: page 137 #1, 2a, 3a, 5, 6

Unit 2: Analytic Geometry

10. Find the length of the line segment joining each pair of points. Express each answer to the nearest tenth of a unit.
    a) (2, 5) and (5, 7) b) (-1, -3) and (-2, -4)

11. Find the coordinates of the midpoint of each line segment, given the endpoints.
    a) (-2, 2) and (4, 8) b) (3, 3) and (-9, 3)

12. The midpoint of *AB* is *M*(-2, 1). If one endpoint is *A*(-7, 3) what are the coordinates of *B*?

13. Write an equation for the circle with centre (0,0) and radius 15.

14. Determine the radius of the circle with centre (0, 0) and equation $x^2 + y^2 = 100$.

15. A rectangle has vertices *Q*(5, 6), *R*(5, -3), *S* (1, -3) and *T*(1,6). Find the:
    a) Perimeter b) Area

16. Triangle *ABC* has vertices *A*(3, 4), *B*(-5, 2) and *C*(1, -4). Determine:
    a) An equation for *CD*, the median from *C* to *AB*.
    b) An equation for *GH*, the right bisector of *AB*.
    c) An equation for *CE*, the altitude from *C* to *AB*.

17. Find the distance from the point *P*(3, 5) to the line $x = -2$.

18. Find the distance from the point *Q*(-2, -2) to the line $y = 3$.

19. A parallelogram has vertices *A*(-2, -2), *B*(3, 3), *C*(7, 4) and *D*(2, -1).
    Verify that the diagonals bisect each other.

Additional Practice: page 218 #1, 2, 3, 5

Unit 3: Polynomials and Factoring

20. Simplify:
    a) $(4x+3)+(5x+4)$ b) $(2x^2-3x+4)-(x^2+4x-1)$

21. Expand and simplify:
    a) $3(x+5)+4(x-3)$ b) $5(t+7)-9(t-2)$
    c) $3x(2x-4)-x(x+5)$ d) $4(x^2-2x+5)-(2x^2+3x-2)$
    e) $2(3x-2)(3x+2)$ f) $4(2x+1)(x-5)$
    g) $3(y+2)(y-3)+2(y-4)(y+1)$ h) $2(2x-3)(x+4)-(4x+1)(x+2)$
    i) $(x+2)^2$ j) $(3x-1)^2$

22. Factor, if possible:

a) $7x+42$ b) $4x^2-28x$ c) $6xy-7st$ d) $14r^2t-7rt+21rt^2$

23. Factor, if possible:

a) $x^2+7x+12$ b) y^2+y-6 c) k^2+k+1 d) $x^2-2x-15$

e) $y^2+10y+21$ f) $x^2-11x+18$

24. Factor fully:

a) $2x^2-6x-8$ b) $3x^2+12x+9$ c) $2xt-6x-3t+9$ d) $xy+x-2y-2$

25. Factor, if possible:

a) $3x^2+5x-2$ b) $2y^2+11y+12$ c) $4x^2+8x+5$ d) $4x^2-10x+3$

26. Factor, if possible:

a) x^2-16 b) $y^2+10y+25$ c) x^2+9 d) x^2-6x+9

27. Factor fully:

a) $4x^2+6x-10$ b) $18x^2-30x+12$ c) $7x^2-7$ d) $5x^2-10x+20$

Additional Practice: page 297 #3, page 307 #3, 4, 5

Units 4 & 5: Graphing Quadratic Relations

28. For each quadratic relation, find the zeros and the coordinates of the vertex and sketch the graph:

a) $y=(x-2)(x+4)$ b) $y=-2(x+1)(x-3)$ c) $y=x^2+6x$

29. The zeros of a parabola are -2 and 7 and it crosses the y-axis at -28.

a) What is the equation of the quadratic relation in factored form?
b) What are the coordinates of the vertex?

30. State the vertex, axis of symmetry and direction of opening of each parabola:

a) $y=(x-2)^2+1$ b) $y=-\frac{1}{2}(x+4)^2$

31. Express the following quadratic relations in vertex form by completing the square.

a) $y=x^2+6x-3$ b) $y=\frac{1}{2}x^2+5x-7$

32. Find, in vertex form, the equation of the quadratic relation with vertex (-1, -4), passing through (3, 4).

33. Graph the parabola $y=x^2$. Then use transformations to graph the following parabolas:

a) $y=3x^2$ b) $y=-x^2+5$ c) $y=-(x-1)^2-3$

d) $y=3(x+4)^2$ e) $y=2(x+3)^2-4$ f) $y=-\frac{1}{2}(x+7)^2+2$

Additional Practice: page 281 #4, 5, page 422 #9, page 424 #12bc

Unit 6: Solving Quadratic Equations

34. Solve by factoring:

a) $x^2 - 8x - 9 = 0$ b) $x^2 + 7x + 6 = 0$ c) $x^2 - 121 = 0$ d) $x^2 - 6x + 9 = 0$

e) $2x^2 + 3x = 0$ f) $4x^2 - 9 = 0$ g) $3x^2 + 8x - 3 = 0$

35. Solve using the quadratic formula. Round answers to two decimal places.

a) $x^2 - 4x - 1 = 0$ b) $2x^2 - x - 3 = 0$ c) $5x^2 - 6x - 2 = 1$ d) $m^2 - 5m + 3 = 0$

e) $3w^2 + 8w + 2 = 0$ f) $-3x^2 + 12x - 7 = 0$

Additional Practice: page 335 #16, page 403 #4

Unit 6: Quadratic Word Problems

36. The sum of the squares of three consecutive integers is 194. What are the integers?

37. The hypotenuse of a right triangle is 26 *cm*. The sum of the other two sides is 34 *cm*. Find the lengths of the other two sides of the triangle.

38. A rectangular field is to be enclosed by 800 *m* of fencing.
a) What dimensions will give the maximum area? b) What is the maximum area?

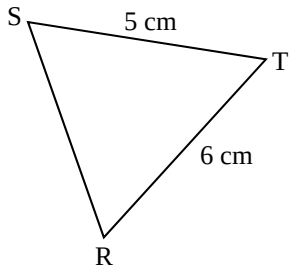
39. A rectangular building that measures 100 *m* by 80 *m* is to be surrounded by a lawn of uniform width. The area of the lawn must equal the area of the building. Find the width of the lawn to the nearest *metre*.

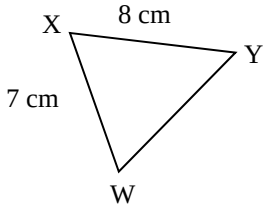
40. The path of a baseball after a batter hit a pop-up can be modeled by the following equation: $h = -0.07(d - 10)^2 + 8$ where *h* is the height of the ball in *metres* and *d* is the horizontal distance of the ball from home plate, where it was hit.
a) What was the maximum height of the ball?
b) What was the horizontal distance of the ball from home when it reached its maximum height?
c) What was the height of the ball when it was hit?
d) If the ball was caught 19.5 *m* from home plate, how far off the ground was the infielder's glove when the ball was caught, to the nearest tenth of a *metre*?

Additional Practice: page 308 #12, page 391 #14, page 405 # 16, 18

Unit 7: Congruent and Similar Triangles

41. $\Delta RST \sim \Delta WXY$. Find the lengths of the unknown sides.





42. Carlos is trying to determine the height of a mature tree. A young tree's height is 5.0 *m* and it casts a 2.0 *m* shadow. At the same time, the mature tree casts a 3.5 *m* shadow. How tall is the mature tree?

Additional Practice: page 518 # 3, 4, page 580 #3

Unit 7: Trigonometry

43. Solve each of the following triangles. Round each side length to the nearest tenth and each angle to the nearest degree.
    a) In ΔABC, $\angle B = 90^\circ$, $\angle A = 47^\circ$ and $b = 15$ *cm*.
    b) In ΔDEF, $\angle D = 90^\circ$, $e = 8$ *m* and $f = 12$ *m*.

44. From a point 35 *m* from the base of a building, the angle of elevation of the top of building is 65°. How tall is the building, to the nearest *metre*?

45. The sides of a triangle measure 15 *cm*, 17 *cm*, and 18 *cm*. Find the measure of the largest angle, to the nearest degree.

46. Solve each of the following acute triangles. Round each side length to the nearest tenth and each angle to the nearest degree.
    a) In ΔWXY, $\angle W = 52^\circ$, $\angle X = 70^\circ$ and $w = 20$ *cm*
    b) In ΔPQR, $\angle R = 68^\circ$, $r = 15$ *m* and $q = 16$ *m*
    c) In ΔABC, $\angle B = 55^\circ$, $a = 11$ *cm* and $c = 20$ *cm*
    d) In ΔJKL, $j = 23$ *m*, $k = 27$ *m* and $l = 29$ *m*

47. A plot of land is in the shape of a triangle. Two of the angles measure 48° and 74°. The length of the side between them is 90 *m*.
    a) Calculate the perimeter of the plot to the nearest *metre*.
    b) Calculate the area of the plot to the nearest square *metre*.

Additional Practice: page 524, #18, page 526 # 1c, 5b, page 582 #4, 5, 6, page 583 # 7, 8, 9

ANSWER

1. a)	-22	1. b)	-19/72	1. c)	71	1. d)	-7/6
1 e)	-35/12	1. f)	16	1. g)	4/25	1. h)	16/9
1. i)	1	1. j)	65/8	1. k)	1/25	2.	(2, 3)
3. a)	(26/7, 1/7)	3. b)	(5/6, 14/3)	4. a)	(-1, 0)	4. b)	(2, 1)
4. c)	(2, -1)	4. d)	(-3, 2)	5.	(16 *cm*, 9 *cm*)	6.	($12000, $8000)
7.	(24 *t*, 32 *t*)	8.	(300 *km*, 180 *km*)	9. a)	zero solutions	9. b)	one solution
9. c)	many solutions	10. a)	3.6 *units*	10. b)	1.4 *units*	11. a)	(1, 5)
11. b)	(-3, 3)	12.	(3, -1)	13.	$x^2 + y^2 = 225$	14.	$r = 10$
15. a)	$P = 26$ *units*	15. b)	$A = 36$ *units*^{2}	16. a)	$y = \frac{-7}{2}x - \frac{1}{2}$	16. b)	y = -4x-1
16. c)	y = -4x	17.	5 units	18.	5 units	20. a)	$9x + 7$
20. b)	$x^2 - 7x + 5$	21. a)	$7x + 3$	21. b)	$-4t + 53$	21. c)	$5x^2 - 17x$
21. d)	$2x^2 - 11x + 22$	21. e)	$18x^2 - 8$	21. f)	$8x^2 - 36x - 20$	21. g)	$5y^2 - 9y - 26$
21. h)	$x - 26$	21. i)	$x^2 + 4x + 4$	21. j)	$9x^2 - 6x + 1$	22. a)	$7(x + 6)$
22. b)	$4x(x - 7)$	22. c)	Does not factor	22. d)	$7rt(2r - 1 + 3t)$	23. a)	$(x + 4)(x + 3)$
23. b)	$(y + 3)(y - 2)$	23. c)	Does not factor	23. d)	$(x - 5)(x + 3)$	23. e)	$(y + 7)(y + 3)$
23. f)	$(x - 9)(x - 2)$	24. a)	$2(x + 1)(x - 4)$	24. b)	$3(x + 3)(x + 1)$	24. c)	$(2x - 3)(t - 3)$ Hint: factor by grouping
24. d)	$(x - 2)(y + 1)$	25. a)	$(x + 2)(3x - 1)$	25. b)	$(2y + 3)(y + 4)$	25. c)	Does not factor
25. d)	Does not factor	26. a)	$(x - 4)(x + 4)$	26. b)	$(y + 5)^2$	26. c)	Does not factor
26. d)	$(x - 3)^2$	27. a)	$2(x - 1)(2x + 5)$	27. b)	$6(x - 1)(3x - 2)$	27. c)	$7(x - 1)(x + 1)$
27. d)	$5(x^2 - 2x + 4)$	28. a)	zeros: 2 and -4, vertex: (-1, -9)	28. b)	zeros: -1, 3, vertex: (1, 8)	28. c)	zeros: 0, -6 vertex: (-3, -9)
29. a)	$y = 2(x + 2)(x - 7)$	29. b)	(5/2, -81/2)	30. a)	vertex: (2, 1), axis: $x = 2$, opens: up	30. b)	vertex: (-4, 0), axis: $x = -4$, opens: down
31. a)	$y = (x + 3)^2 - 12$	31. b)	$y = \frac{1}{2}(x + 5)^2 - 19.5$	32.	$y = \frac{1}{2}(x + 1)^2 - 4$	33. a)	stretched by a factor of 3
33. b)	reflection in x-axis, up 5 units	33. c)	reflection in x-axis, right 1 unit, down 3 units	33. d)	stretched by a factor of 3, left 4 units	33. e)	stretched by a factor of 2, left 3 units, down 4 units
33. f)	compressed by a factor of 2, reflection in x-axis, left 7 units, up 2 units	34. a)	$x = -1,\ x = 9$	34. b)	$x = -1,\ x = -6$	34. c)	$x = \pm 11$
34. d)	$x = 3$	34. e)	$x = 0,\ x = -3/2$	34. f)	$x = \pm 3/2$	34. g)	$x = 1/3,\ x = -3$
		35. a)	$x = 4.24,\ -0.24$	35. b)	$x = 1.5,\ -1$	35. c)	$x = 1.58,\ -0.38$
35. d)	$m = 4.30,\ 0.70$	35. e)	$w = -0.28,\ -2.39$	35. f)	$x = 0.71,\ 3.29$	36.	7, 8, 9 or -9, -8, -7
37.	10 *cm*, 24 *cm*	38. a)	200 *m* by 200 *m*	38. b)	40,000 m^2	39.	18.4 *m*
40. a)	8 *m*	40. b)	10 *m*	40. c)	1 *m*	40. d)	1.7 *m*
41.	RS = 4.4 *cm*, WY = 9.6 *cm*	42.	8.75 *m*	43. a)	$a = 11\,cm,\ c = 10.2$ *cm*, $\angle C = 43^\circ$	43. b)	$d = 14.4$ *mm*, $\angle F = 56^\circ,\ \angle E = 34^\circ$
44.	75.1 *m*	45.	68°	46. a)	$\angle Y = 58^\circ, x = 23.8cm$ $y = 21.5$ *cm*	46. b)	$\angle Q = 81^\circ$, $\angle P = 31^\circ,\ p = 8.3$ *m*
46. c)	$\angle A = 33^\circ,\ \angle C = 92^\circ,$ $b = 16.4$ *cm*	46. d)	$\angle J = 49^\circ,\ \angle K = 61^\circ,$ $\angle L = 70^\circ$	47. a)	270.9 *m*	47. b)	3411.9 m^2