

Unit 8: Financial Applications

Day 6: Regular Annuities - Future Value

Today we will...

1. Learn how to use a timeline to visualize an annuity.
2. Learn how to use a formula to calculate the future value of a simple, ordinary annuity.

Some Definitions:

Annuity: A series of equal deposits or payments made at regular intervals.

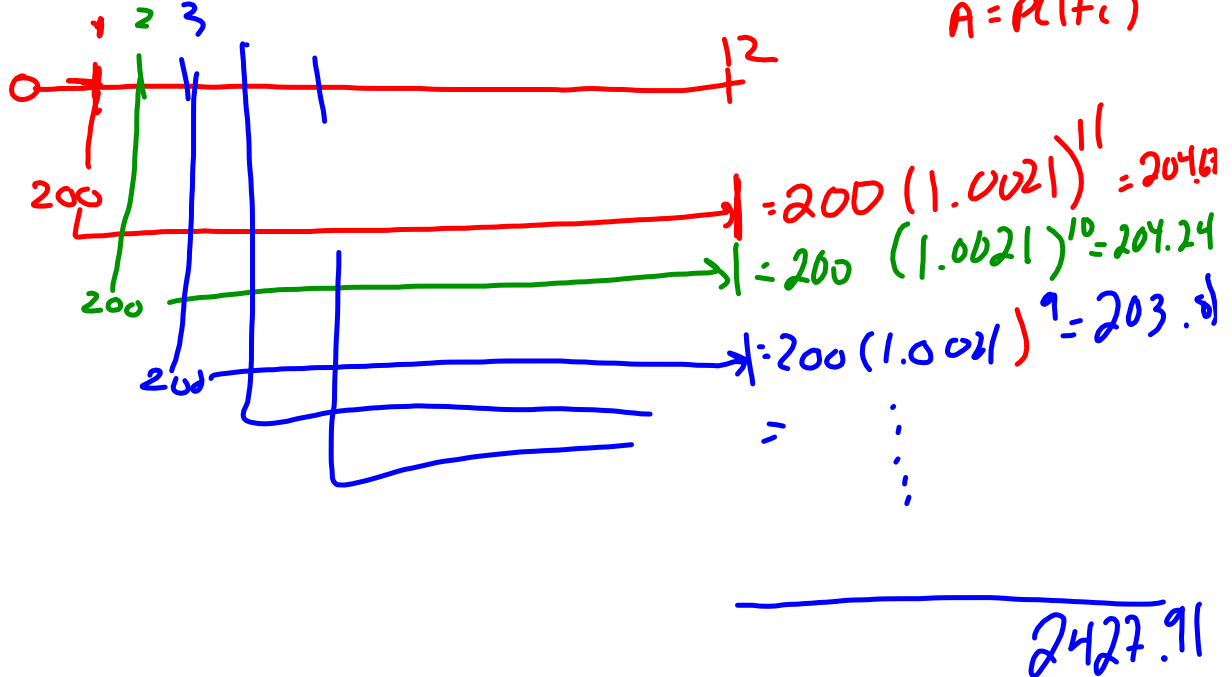
Simple Annuity:

Ordinary Annuity:

In this course, we will assume a simple, ordinary annuity unless otherwise stated.

EX. Kristine has a part time job and is able to save ~~\$200~~ per month. If her bank account pays ~~2.5%~~ p.a. compounded monthly, how much will she have in her account at the end of one year?

METHOD 1: Create a timetable.



METHOD 2: The Annuity Formula

There is a formula for this calculation...

Amount of an Annuity:

$$A = \frac{R[(1+i)^n - 1]}{i}$$

A = Accumulated Value

R = Regular Payment

i = interest rate per comp period.

n = number of compound periods.

Consider the same example as before...

EX. 1. Kristine has a part time job and is able to save \$200 per month. If her bank account pays 2.5% p.a. compounded monthly, how much will she have in her account at the end of one year?

$$\begin{aligned}
 A &= \frac{R[(1+i)^n - 1]}{i} \\
 &= \frac{200[(1.0021)^{12} - 1]}{0.0021} \\
 &= 2427.91
 \end{aligned}
 \quad
 \begin{aligned}
 i &= \frac{0.025}{12} = 0.0021 \\
 n &= 12 \\
 R &= 200
 \end{aligned}$$

\therefore she will have \$2427.91

EX. 2. a) A person finishes college at age 21, debt free and starts an RRSP with monthly contributions of \$50. How much will she have saved by age 50 in an account that pays 3% p.a. compounded monthly?

$$\begin{aligned}
 A &= \frac{R[(1+i)^n - 1]}{i} \\
 &= \frac{50[(1.0025)^{348} - 1]}{0.0025} \\
 &= 27686.42
 \end{aligned}
 \quad
 \begin{aligned}
 i &= \frac{0.03}{12} \\
 &= 0.0025 \\
 n &= 12 \times 29 \\
 &= 348
 \end{aligned}$$

b) How much interest has she earned in her RRSP?

$$\begin{aligned}
 \text{Payments} &= 50 \times 348 \\
 &= 17400 \\
 \therefore \text{Interest} &= \frac{27686.42}{17400} - 1 \\
 &= \$10286.42
 \end{aligned}$$

EX. 3. You want to retire in 30 years with \$1,000,000 in savings. Your current investments are earning, on average, 11% p.a. compounding annually.

a) What annual deposit must you make to reach your savings goal?

$$\begin{aligned}
 A &= \frac{R[(1+i)^n - 1]}{i} \\
 1000000 &= \frac{R[(1.11)^{30} - 1]}{0.11} \\
 11200000 &= R(21.89929657) \\
 1000000 &= R(199.0209779) \\
 \$5024.60 &= R
 \end{aligned}
 \quad
 \begin{aligned}
 i &= 0.11 \\
 n &= 30
 \end{aligned}$$

b) Of the final \$1,000,000, how much is interest?

$$\begin{aligned}
 5024.60 \times 30 &= 150738 \\
 \therefore \text{Interest is } &= \frac{1000000}{150738} - 1 \\
 &= \$849262
 \end{aligned}$$

Homework: p. 498 #3-7, 10, 14