EXAM REVIEW SOLUTIONS

CHAPTER 1: Introduction to Quadratic Functions

1. State the domain and range for the following functions.

(a) $D=\{x\in R\}$
$R=\{y\in R \mid y\le 1\}$

(b) $D=\{x\in R\}$
$R=\{y\in R\}$

2. Determine whether the following relations are functions. State the domain and range.

(a) NOT a function
$D=\{1,5,6\}$
$R=\{2,3\}$

(b) NOT a function
$D=\{2,3,5\}$
$R=\{0,3,8\}$

(c) IS a function
$D=\{0,1,2,3\}$
$R=\{0,2,4,8\}$

3. If $f(x)=3(x-2)^2+1$, determine

(a) $f(-1)$

$$f(x)=3(x-2)^2+1$$
$$f(-1)=3(-1-2)^2+1$$
$$=3(-3)^2+1$$
$$=3(9)+1$$
$$=28$$

(b) $f(x+1)$

$$f(x)=3(x-2)^2+1$$
$$f(x+1)=3(x+1-2)^2+1$$
$$=3(x-1)^2+1$$
$$=3(x^2-2x+1)+1$$
$$=3x^2-6x+3+1$$
$$=3x^2-6x+4$$

4. A relation g is given by $g(x)=3x^2+2x-4$. Evaluate.

(a) $g(-2)$

$$g(x)=3x^2+2x-4$$
$$g(-2)=3(-2)^2+2(-2)-4$$
$$=3(4)-4-4$$
$$=6$$

(b) $g(m)$

$$g(x)=3x^2+2x-4$$
$$g(m)=3m^2+2m-4$$

(c) $g(4a)$

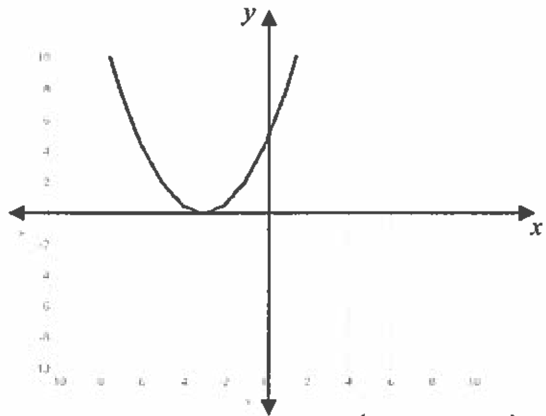
$$g(x)=3x^2+2x-4$$
$$g(4a)=3(4a)^2+2(4a)-4$$
$$=3(16a^2)+8a-4$$
$$=48a^2+8a-4$$

5. In words, describe the transformations to the graph $f(x)=x^2$ to get $g(x)$, if $g(x)=\frac{1}{2}(x+4)^2-3$.

The quadratic function $f(x)=x^2$ has been;

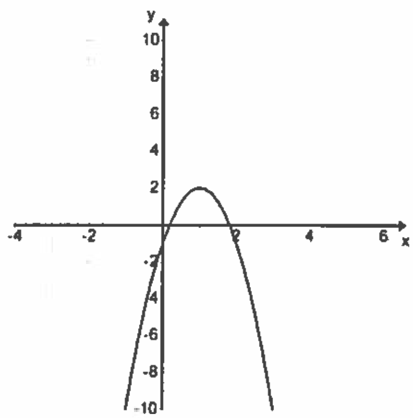
- *horizontally translated 4 units to the left*
- *vertically compressed by a factor of 2*
- *vertically translated down 3 units.*
- *Graph each of the following and then state domain and range.*

6. (a) $y=\frac{1}{2}(x+3)^2$



D = $\{x \in R\}$ R = $\{y \in R \mid y \geq 0\}$

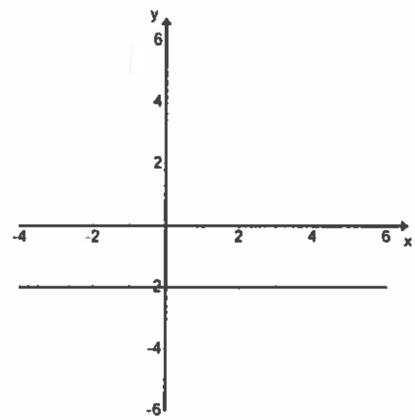
(b) $g(x)=-3(x-1)^2+2$



D = $\{x \in R\}$
R = $\{y \in R \mid y \leq 2\}$

(c) $y=-2$

D = $\{x \in R\}$
R = $\{y \in R \mid y = 2\}$



7. Create a first- and second-difference table for the following data.

x	y	Δy (1st difference)	Δ(Δy) (2nd difference)
-1	1		
		1	
0	2		-6
		-5	
1	-3		-6
		-11	
2	-14		-6
		-17	
3	-31		

(b) What conclusion can be made from the first difference?
The function is NON-LINEAR.

(c) What conclusion can be made from the second difference?
The function is QUADRATIC.

CHAPTER 2: The Algebra of Quadratic Expressions

1. Expand and simplify.

(a) $3(x^2-2)-4x(3x-7)$

$=3x^2-6-12x^2+28x$

$=-9x^2+28x-6$

(b) $-(a-3)^2+3(5a+2)^2$

$=-(a^2-3a-3a+9)+3(25a^2+10a+10a+4)$

$=-(a^2-6a+9)+3(25a^2+20a+4)$

$=-a^2+6a-9+75a^2+60a+12$

$=74a^2+66a+3$

2. Common factor each of the following polynomials.

(a) $12a^3b^3-6a^4b^2+9a^5b^4$

$=3a^3b^2\left(4b-2a+3a^2b\right)$

(b) $5x^2(x+y)-20y^2(-x-y)$

$=5x^2(x+y)+20y^2(x+y)$

$=5(x+y)(x^2+4y^2)$

3. Factor fully.

(a) $-2x^2+8x-10$

$=-2(x^2-4x+5)$

(b) $x^2-4x-32$

$=(x-8)(x+4)$

(c) $63m^2-7n^2$

$=7(9m^2-n^2)$

$=7(3m-n)(3m+n)$

(d) $16a^2-24ab+9b^2$

$=(4a-3b)^2$

(e) $21x^2-13xy+2y^2$

$=(3x-y)(7x-2y)$

(f) $-2x^2+7x+15$

$=-(2x^2-7x-15)$

$=-(x-5)(2x+3)$

4. What are ALL possible integer values, k, such that $x^2+kx-32$ can be factored?

Need two numbers that multiply to -32 and add up to k.

$-32=1\times(-32)\Rightarrow k=-31$

$-32=-1\times 32 \Rightarrow k=31$

$-32=4\times(-8)\Rightarrow k=-4$

$-32=-4\times 8\Rightarrow k=4$

$-32=2\times(-16)\Rightarrow k=-14$

$-32=-2\times 16\Rightarrow k=14$

$\therefore$ *all possible values for* k *are*

$\pm 4,\ \pm 14,\ \pm 31$

5. Factor fully.

(a) $3(b^2-4)+a^2(b^2-4)$

$=(b^2-4)(3+a^2)$ *factor out the common bracket*

$=(b-2)(b+2)(3+a^2)$

(b) $18(2-x)+x^2(x-2)+3x(x-2)$

$=-18(x-2)+x^2(x-2)+3x(x-2)$

$=(x-2)(-18+x^2+3x)$

$=(x-2)(x^2+3x-18)$

$=(x-2)(x+6)(x-3)$

6. Name an integer, k, such that the quadratic $6x^2-22x+k$ can be factored.

Check your answer by factoring and expanding. If you aren't sure, ask the instructor.

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CHAPTER 3: Quadratic Models: Standard & Factored Forms

1. Write each of the following in standard form.

(a) $f(x) = (3x+1)(x-2)$

$f(x) = 3x^2 - 5x - 2$

(b) $f(x) = (2+3x)(x-3)$

$f(x) = 2x - 6 + 3x^2 - 9x$

$= 3x^2 - 7x - 6$

2. Write each of the following in factored form.

(a) $f(x) = x^2 - 16$

$= (x-4)(x+4)$

(b) $f(x) = x^2 + 3x - 18$

$= (x+6)(x-3)$

(c) $f(x) = 5x^2 - 20$

$= 5(x^2 - 4)$

$= 5(x-2)(x+2)$

3. Determine the zeros, the axis of symmetry, and the maximum and minimum value for each of the following quadratic equations. Show your work.

(a) $f(x) = 3x^2 - 3x$

$f(x) = 3x(x-1)$

$\therefore x = 0 \text{ and } x = 1 \text{ are the zeros}$

$\text{axis of symmetry}: x = \frac{1}{2}.$

$$f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)\left(\frac{1}{2} - 1\right)$$

$$= 3\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$= -\frac{3}{4}$$

$\therefore \min = -\frac{3}{4}$

(b) $f(x) = -4x^2 - 12x + 7$

$f(x) = -(4x^2 + 12x - 7)$

$f(x) = -(2x+7)(2x-1)$

$\therefore x = \frac{-7}{2} \text{ and } x = \frac{1}{2} \text{ are the zeros}$

$$\text{axis of symmetry}: x = \frac{\frac{-7}{2} + \frac{1}{2}}{2}$$

$$x = \frac{\frac{-6}{2}}{2}$$

$$x = -\frac{3}{2}$$

$$f\left(-\frac{3}{2}\right) = -4\left(-\frac{3}{2}\right)^2 - 12\left(-\frac{3}{2}\right) + 7$$

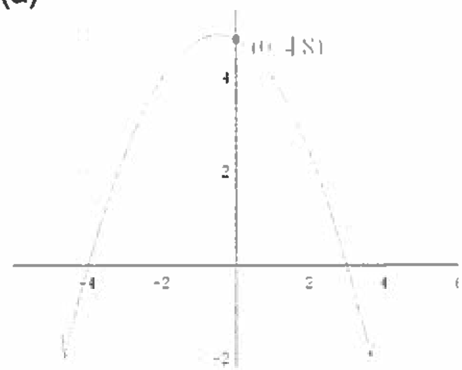
$$= -4\left(\frac{9}{4}\right) + \frac{36}{2} + 7$$

$$= -9 + 18 + 7$$

$\max = 16$

4. Write the corresponding quadratic equation for each of the following functions.
Leave your answer in factored form.

(a)



$$y = a(x-r)(x-s)$$
$$= a(x+4)(x-3)$$
$$sub\,in\,(x,y)=(0,4.8)\,to\,get$$
$$4.8 = a(0+4)(0-3)$$
$$4.8 = a(-12)$$
$$\frac{4.8}{-12} = a \qquad \therefore y = \frac{-2}{5}(x+4)(x-3)$$
$$\frac{48}{-120} = a$$
$$\frac{2}{-5} = a$$

(b)

The function has zeros at $x=2$ and $x=7$ and passes through the point $(0,-4)$

$$y = a(x-r)(x-s)$$
$$= a(x-2)(x-7)$$
$$sub\,in\,(x,y)=(0,-4)\,to\,get$$
$$-4 = a(0-2)(0-7)$$
$$-4 = a(14)$$
$$\frac{-4}{14} = a \qquad \therefore y = \frac{-2}{7}(x-2)(x-7)$$
$$\frac{-2}{7} = a$$

5. Can all quadratic equations be solved by factoring? Explain.
NO. Some quadratics do not pass through the x-axis….meaning there are NO zeroes.

6. Solve for x by factoring. Show your work.

(a) $4x^2+4x-3=0$

$$(2x+3)(2x-1)=0$$
$$\therefore x = \frac{-3}{2} \;\; and \;\; x = \frac{1}{2}$$

(b) $x^2+6x-3=-3$

$$x^2+6x-3=-3$$
$$x^2+6x-3+3=0$$
$$x^2+6x=0$$
$$x(x+6)=0$$
$$\therefore x=0 \;\; and \;\; x=-6$$

7. A firecracker is fired from the ground. The height of the firecracker at a given time is modelled by the function $h(t)=-5t^2+40t$, where $h(t)$ is the height in metres and t is time in seconds.

(a) When will the firecracker hit the ground?

$$h(t) = -5t(t-8)$$
$$\therefore t=0 \;\; and \;\; t=8 \qquad \therefore it\ hits\ the\ ground\ after\ 8\ seconds.$$

(b) What is the maximum height of the firecracker?

$$axis\ of\ symmetry: x = 4$$
$$h(4) = -5(4)^2 + 40(4)$$
$$= 80$$

or

$$h(t) = -5(t^2 - 8t + 16) + 80$$
$$= -5(t-4)^2 + 80$$
$$\therefore the\ \max = 80\ metres$$

(c) When does the firecracker reach a maximum height?

$$the\ vertex = (4, 80)$$
$$\therefore the\ \max\ occurs\ at\ 4\ seconds$$

(d) When will the firecracker reach a height of $75\ m$?

$$75 = -5t^2 + 40t$$
$$0 = -5t^2 + 40t - 75$$
$$0 = -5(t^2 - 8 + 15)$$
$$0 = -5(t-3)(t-5)$$
$$\therefore t = 3\ and\ t = 5$$

$\therefore$ *the rocket reaches* $75\ m$ *at* 3 *seconds (going up)*
and at 5 *seconds (when the rocket is going down).*

8. The population of a city $P(t)$ is modeled by the function $P(t) = 0.5t^2 + 10t + 200$, where $P(t)$ is the population in thousands and t is time in years. NOTE: $t = 0$ represents the year 2000. According to the model,

(a) in what year will the population reach 312 000?

$$312 = 0.5t^2 + 10t + 200$$
$$0 = 0.5t^2 + 10t - 112$$
$$0 = 0.5(t^2 + 20t - 224)$$
$$0 = 0.5(t-8)(t+28)$$
$$t = 8\ \ or\ \ t = -28$$

The population reaches 312 000 in 2008 and in 1972

(b) Will the population reach over 2 million people by the year 2050? Show your work.

$$sub\ \ t = 50$$
$$P(50) = 0.5(50)^2 + 10(50) + 200$$
$$= 1950$$

So the population is 1950000
< 2 *million*

$\therefore$ *No. The population will not exceed* 2 *million by* 2050.

MCF3MI **_Exam Review Solutions_**

CHAPTER 4: Quadratic Models: Standard & Vertex Forms

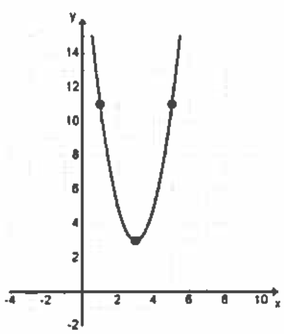
1. Write the function $f(x)=2(x+3)^2-2$ in standard form.

$$\begin{aligned} f(x)&=2(x+3)(x+3)-2\\ &=2(x^2+6x+9)-2\\ &=2x^2+12x+18-2\\ &=2x^2+12x+16 \end{aligned}$$

2. For the function $f(x)=-(x-4)^2+1$, complete the table:

Vertex	(4, 1)
Axis of Symmetry	$x=4$
Max/Min Value	max = 1
Domain	$\{x\in R\}$
Range	$\{y\in R \mid y\le 1\}$

3.

Determine the equation of the parabola . 	$y=a(x-h)^2+k$ $y=a(x-3)^2+3$ $11=a(1-3)^2+3$ $11=a(-2)^2+3$ $11-3=4a$ $8=4a$ $2=a$ $\therefore\ y=2(x-3)^2+3$

4. Write each function in vertex form and state the vertex.

(a) $f(x)=-x^2+6x+7$

$$\begin{aligned} f(x)&=-(x^2-6x)+7\\ &=-(x^2-6x+9-9)+7\\ &=-(x^2-6x+9)+9+7\\ &=-(x-3)^2+16 \end{aligned}$$

$\therefore\ vertex=(3,16)$

(b) $g(x)=2x^2-3x+3.5$

$$\begin{aligned} g(x)&=2(x^2-\frac{3}{2}x)+3.5\\ &=2(x^2-\frac{3}{2}x+\frac{9}{16}-\frac{9}{16})+3.5\\ &=2(x^2-\frac{3}{2}x+\frac{9}{16})-\frac{18}{16}+3.5\\ &=2(x-\frac{3}{4})^2+2.375 \end{aligned}$$

$\therefore\ vertex=(0.75,\ 2.375)$

5. The cost, $C(n)$, of operating a cement-mixing truck is modeled by the function $C(n)=2.2n^2-66n+700$, where n is the number of minutes the truck is running. What is the minimum cost of operating the truck? Show your work.

$$
\begin{aligned}
C(n) &= 2.2(n^2-30n\quad)+700\\
&= 2.2(n^2-30n+225-225)+700\\
&= 2.2(n^2-30n+225)-495+700\\
&= 2.2(n-15)^2+205
\end{aligned}
$$

$$\therefore \min = 205$$

6. Solve using the quadratic formula. State your answers correct to 2 decimal places.

(a) $8x^2-6x+1=0$

$$
\begin{aligned}
&8x^2-6x+1=0\\
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&= \frac{6\pm\sqrt{(-6)^2-4(8)(1)}}{2(8)}\\
&= \frac{6\pm\sqrt{36-32}}{16}\\
&= \frac{6\pm\sqrt{4}}{16}
\end{aligned}
$$

$$\therefore x=\frac{6+2}{16} \quad \textit{and} \quad x=\frac{6-2}{16}$$

$$x=\frac{1}{2} \quad \textit{and} \quad x=\frac{1}{4}$$

(b) $x^2+3x=14$

$$
\begin{aligned}
&x^2+3x=14\\
&x^2+3x-14=0\\
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&= \frac{-3\pm\sqrt{(3)^2-4(1)(-14)}}{2(1)}\\
&= \frac{-3\pm\sqrt{9+56}}{2}\\
&= \frac{-3\pm\sqrt{65}}{2}
\end{aligned}
$$

$$\therefore x=\frac{-3+\sqrt{65}}{2} \quad \textit{and} \quad x=\frac{-3-\sqrt{65}}{2}$$

$$x \approx 2.53 \quad \textit{and} \quad x \approx -5.53$$

7. A theatre company's profit can be modeled by the function $P(x)=-60x^2+700x-1000$ where x is the price of a ticket in dollars. What is the break-even price of the tickets?

$$\textit{Set } P(x)=0$$

$$0=-60x^2+700x-1000$$

$$a=-60,\ b=700,\ c=-1000$$

$$
\begin{aligned}
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\
&= \frac{-700\pm\sqrt{700^2-4(-60)(-1000)}}{2(-60)}\\
&= \frac{-700\pm\sqrt{250000}}{-120}\\
&= \frac{-700\pm 500}{-120}
\end{aligned}
$$

$$\therefore x=\frac{-700+500}{-120} \quad \textit{and} \quad x=\frac{-700-500}{-120}$$

$$\approx 1.67 \qquad\qquad =10.0$$

NOTE: A "break even" point means that you neither made money nor lost money. *ie....* $P(x)=0$

The break even points occur when tickets are sold for \$1.67 and \$10.

8. A model rocket is launched into the air. Its height, $h(t)$, in metres after t seconds is $h(t) = -5t^2 + 40t + 2$.

   (a) When is the rocket at a height of 62 m (correct to 2 decimal places)?

$$62 = -5t^2 + 40t + 2$$
$$0 = -5t^2 + 40t + 2 - 62$$
$$0 = -5t^2 + 40t - 60$$
$$0 = -5(t^2 - 8t + 12)$$
$$0 = -5(t - 6)(t - 2)$$
$$\therefore t = 6 \quad and \quad t = 2$$

The rocket reaches 62m at 2 seconds (going up) and at 6 seconds (coming back down).

   (b) What is the height of the rocket after 6 seconds?

62 metres. (see part (a) above)

   (c) What is the maximum height of the rocket?

$$h(t) = -5(t^2 - 8t \quad\;) + 2$$
$$= -5(t^2 - 8t + 16 - 16) + 2$$
$$= -5(t^2 - 8t + 16) + 80 + 2$$
$$= -5(t - 4)^2 + 82$$

The maximum height of the rocket is 82 metres at 4 seconds.

9. Without solving, determine the number of solutions of each equation. Show your work for full marks.

(a) $x^2 - 5x + 9 = 0$

$$x^2 - 5x + 9 = 0$$
$$b^2 - 4ac = (-5)^2 - 4(1)(9)$$
$$= 25 - 36$$
$$= -11$$
$$< 0$$
$$\therefore ZERO\ real\ roots$$

(b) $3x^2 - 5x - 9 = 0$

$$3x^2 - 5x - 9 = 0$$
$$b^2 - 4ac = (-5)^2 - 4(3)(-9)$$
$$= 25 + 108$$
$$= 133$$
$$> 0$$
$$\therefore TWO\ real\ roots$$

(c) $16x^2 - 8x + 1 = 0$

$$16x^2 - 8x + 1 = 0$$
$$b^2 - 4ac = (-8)^2 - 4(16)(1)$$
$$= 64 - 64$$
$$= 0$$
$$\therefore ONE\ real\ root$$

MCF3MI # Exam Review Solutions

CHAPTER 5: Trigonometry & Acute Angles

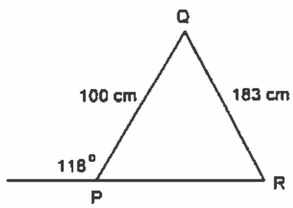
1. Use a calculator to evaluate to four decimal places.

   (a) $\cos 11°$ $=0.9816$ (b) $\tan 83°$ $=8.1443$ (c) $\sin 39°$ $=0.6293$

2. Use a calculator to find θ to the nearest degree.

   (a) $\cos\theta = 0.3862$ $\theta = 67°$ (b) $\tan\theta = 1.2375$ $\theta = 51°$

3. Determine all the interior angles in ΔPQR correct to the nearest degree.

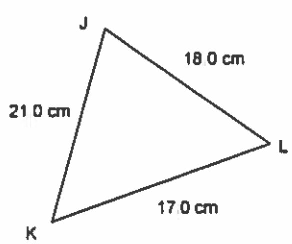


$\angle QPR = 62°(AST)$

$$\frac{\sin 62°}{183} = \frac{\sin R}{100}$$
$$\frac{100\sin 62°}{183} = \sin R$$
$$0.4825 = \sin R$$
$$\angle R \approx 29°$$

$$\therefore \angle Q = 180 - 29 - 62$$
$$= 89°$$

4. Solve ΔJKL where $j = 17.0\ cm$, $k = 18.0\ cm$, and $l = 21.0\ cm$. Include a diagram.



Use the Cosine Law to find angle K

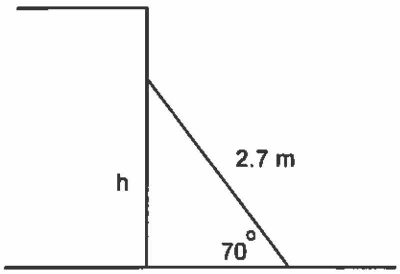
$$k^2 = j^2 + l^2 - 2jl\cos K$$
$$18^2 = 17^2 + 21^2 - 2(17)(21)\cos K$$
$$\frac{324 - 289 - 441}{-714} = \cos K$$
$$\frac{-406}{-714} = \cos K$$
$$0.5686 \approx \cos K$$
$$\angle K = 55.3°$$

Now, use the Sine Law to get $\angle J$.

$$\frac{\sin J}{17} = \frac{\sin 55.3°}{18}$$
$$\sin J = \frac{17\sin 55.3°}{18}$$
$$\sin J = 0.7765$$
$$\angle J = 50.9°$$

$$\therefore \angle L = 180 - 55.3 - 50.9$$
$$= 73.9°\ (AST)$$

5. A $2.7\ m$ ladder can be used safely only at an angle of $70°$ with the horizontal. How high, to the nearest metre, can the ladder reach? Include a diagram.



$$\sin 70° = \frac{h}{2.7}$$

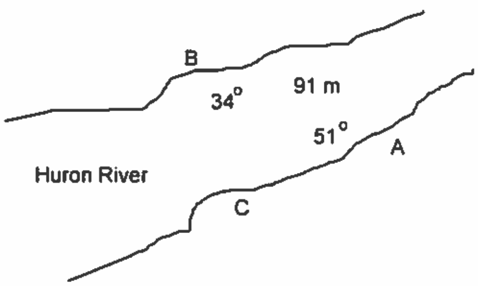
$$2.7 \sin 70° = h$$

$$2.54 = h$$

$$\therefore h \approx 3m$$

$$\therefore \textit{the ladder can reach about } 2.5\,m \textit{ up the wall.}$$

6. A surveyor wants to calculate the distance BC across a river. He selects a position, A, so that BA is $91\ m$, and he measures $\angle ABC$ and $\angle BAC$ as $34°$ and $51°$, respectively. Calculate the distance BC to the nearest tenth of a metre.



$$\angle C = 180 - 51 - 34$$

$$= 95°$$

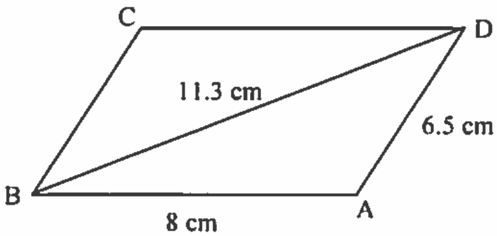
$$\therefore \frac{\sin 95°}{91} = \frac{\sin 51°}{a}$$

$$a = \frac{91 \sin 51°}{\sin 95°}$$

$$a \approx 70.99\,m$$

$$a \approx 71\,m$$

7. Two sides of a parallelogram measure $6.5\,cm$ and $8.0\,cm$. The longer diagonal is $11.3\,cm$ long. How long, to the nearest centimeter, is the other diagonal? (Include a diagram).



Find angle A using cosine law.

$$\cos A = \frac{8^2 + 6.5^2 - 11.3^2}{2(8)(6.5)}$$

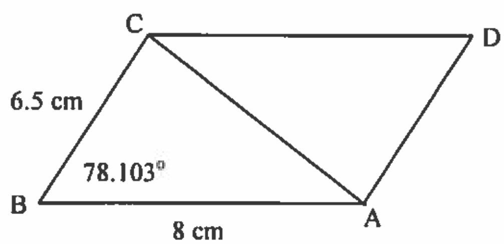
$$\cos A = -\frac{67}{325}$$

$$\cos A = -0.206153846$$

$$A = 101.897^\circ$$

Use the sum of the interior angles of a quadrilateral to find angle B.

A and C are both 101.897
360 = A + B + C + D
360 = 101.897 + B + 101.897 + D
156.206 = B + D
And since B and D are equal angles, they are 156.206/2 = 78.103°.



Find diagonal AC using cosine law.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 6.5^2 + 8^2 - 2(6.5)(8)\cos 78.103$$

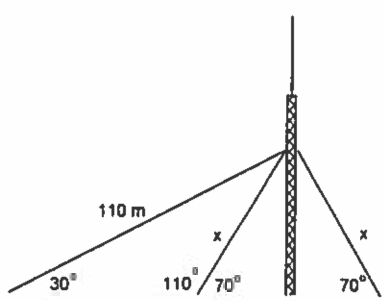
$$b^2 = 106.25 - 21.4399$$

$$b^2 = 84.8101$$

$$b = 9.209$$

Therefore, the other diagonal is approximately 9cm long.

8. A temporary support cable for a radio antenna is $110\ m$ long and has an angle of elevation of $30°$. Two other support cables are already attached, each at an angle of elevation of $70°$. How long, to the nearest centimetre, is each of the shorter cables?



$$\frac{\sin 110^\circ}{110} = \frac{\sin 30^\circ}{x}$$

$$x = \frac{110 \sin 30^\circ}{\sin 110^\circ}$$

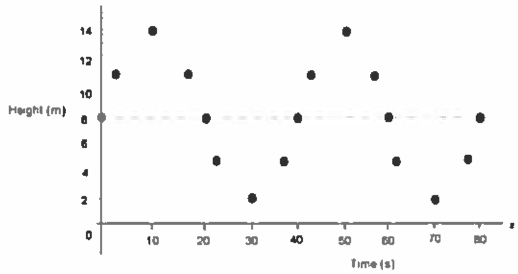
$$x \approx 58.5m$$

Each of the shorter cables is approximately 58. 5 metres long.

Exam Review Solutions

CHAPTER 6: Sinusoidal Functions

1. Information about the movement of a Ferris wheel is shown below.



(a) How long does it take for the Ferris wheel to make <u>five</u> complete rotations?

1 complete turn takes 40 seconds
5 complete turns takes 200 seconds

(b) What is the height of the axle supporting the Ferris wheel?

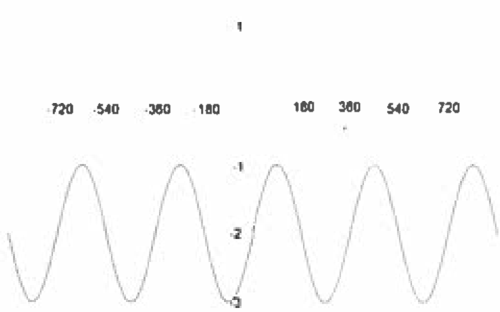
axis = 8 m

(c) Calculate the speed at which the wheel is rotating.

Circumference of the wheel = $2\pi r$
= $2\pi(6)$
= 37.68 m

Speed = distance/time distance travelled is circumference of the wheel
= 37.68/40
= 0.942 m/s

2. Given the following graph, complete the given analysis.



Amplitude: 1

Period: 360°

Range: $\{y \in R \mid -3 \le y \le -1\}$

Number of cycles from -540 to 540: 3

Axis: $y = -2$

3. Describe the transformation $g(x) = -2\sin x + 1$ and then sketch it.

The sinusoidal curve $y = \sin x$ *has been:*

- *vertically stretched by a factor of 2*
- *reflected in the x-axis*
- *vertically translated up 1 unit*

4. What is the range for each of the following sinusoidal functions?

(a) $f(x) = 0.5\sin x - 4$

$\{y \in R \mid -4.5 \le y \le -3.5\}$

(b) $f(x) = \sin(x - 180°)$

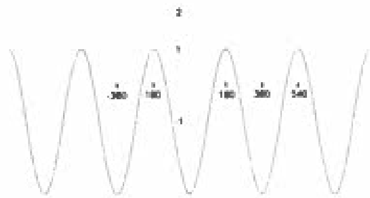
$\{y \in R \mid -1 \le y \le +1\}$

5. The function $f(x) = \sin x$ has been translated $60°$ to the right , vertically stretched by a factor of 3 and reflected in the x-axis. Write the new equation.

$y = -3\sin(x - 60°)$

6. Write the equation for the sinusoidal function.

(a)

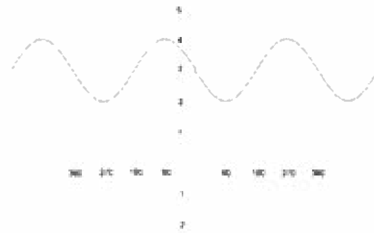


$y = 2\sin(\theta - 90°) - 1$

Other answers exist.

See the teacher

(b)



$y = \sin(\theta + 180°) + 3$

Other answers exist.

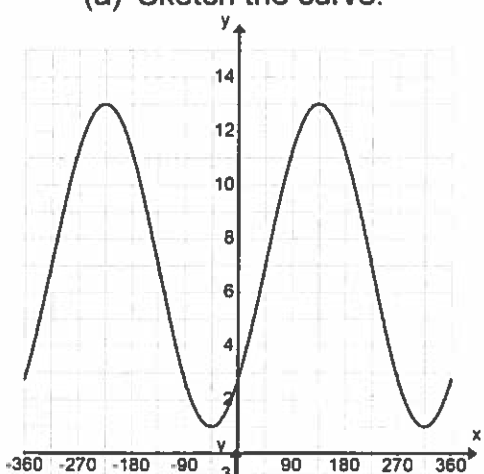
See the teacher

7. Complete the chart below.

Sinusoidal Function	*Maximum*	*Minimum*
(a) $f(x) = 3\sin x$	3	-3
(b) $f(x) = -\sin(x - 45°) + 6$	7	5
(c) $f(x) = -0.25\sin x - 1.5$	-1.25	-1.75

8. The height of a Ferris wheel is modeled by the function $h(x) = 6\sin(x - 45°) + 7$, where $h(x)$ is in metres and x is the number of degrees the wheel has rotated from the boarding position of a rider.

(a) Sketch the curve.



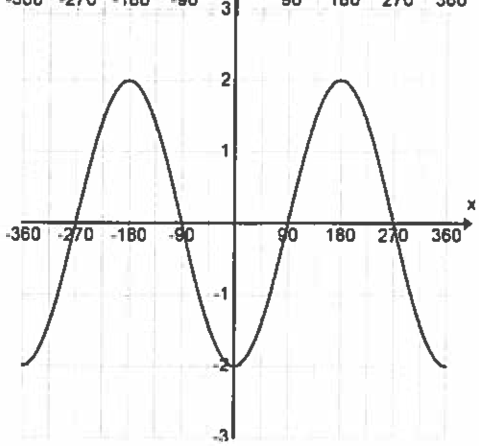
(b) When the rider has rotated 400° from the boarding position, how high above the ground is the rider?

$$\begin{aligned} &sub\ x = 400° \\ &y = 6\sin(400° - 45°) + 7 \\ &\quad = 6\sin(355°) + 7 \\ &\quad = 6(-0.0872) + 7 \\ &\quad = -0.5229 + 7 \\ &\quad = 6.5\,m \end{aligned}$$

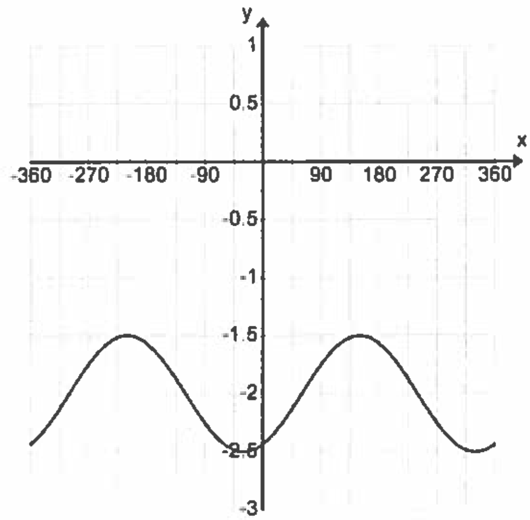
The rider is approx. 6.5 metres above the ground

9. Sketch each sinusoidal function on the grid provided.

(a) $f(x) = 2\sin(x - 90°)$



(b) $f(x) = 0.5\sin(x - 60°) - 2$


y
1
0.5
x
-360
-270
-180
-90
90
180
270
360
-0.5
-1
-1.5
-2
-2.5
-3

Chapter 7 Solutions

CHAPTER 7: Exponential Functions

1. Write as a single power. Express answers with positive exponents. DO NOT EVALUATE.

(a) $4^3 \times 4 \times 4^2$
$= 4^6$

(b) $5(5^3)$
$= 5^4$

(c) $\dfrac{(-4)^6(-4)^3}{((-4)^9)^2}$
$= \dfrac{(-4)^9}{(-4)^{18}}$
$= (-4)^{-9}$
$= \dfrac{1}{(-4)^9}$

(d) $\dfrac{3^4}{(3^2)^3}$
$= \dfrac{3^4}{3^6}$
$= \dfrac{1}{3^2}$

(e) $\dfrac{(20^{-1})^8}{20^2 20^6}$
$= \dfrac{20^{-8}}{20^8}$
$= 20^{-16}$
$= \dfrac{1}{20^{16}}$

(f) $\left(\dfrac{1}{9}\right)^5\left(\dfrac{1}{9}\right)^{-3}$
$= \left(\dfrac{1}{9}\right)^2$
$= \dfrac{1}{9^2}$

2. Evaluate WITHOUT using a calculator.

(a) $256^{\frac{-5}{4}}$
$= \left(\sqrt[4]{256}\right)^{-5}$
$= 4^{-5}$
$= \dfrac{1}{4^5}$
$= \dfrac{1}{1024}$

(b) $\left(-\dfrac{1}{2}\right)^3 + 2^{-3}$
$= -\dfrac{1}{8} + \dfrac{1}{8}$
$= 0$

(c) $4^{-1} + 4^0 + 4^2$
$= \dfrac{1}{4} + 1 + 16$
$= 17.25$

(d) $16^{\frac{3}{2}}$
$= \sqrt{16}^3$
$= 4^3$
$= 64$

(e) $\left(\dfrac{27}{64}\right)^{\frac{-1}{3}}$
$= \left(\dfrac{64}{27}\right)^{\frac{1}{3}}$
$= \dfrac{\sqrt[3]{64}}{\sqrt[3]{27}}$
$= \dfrac{4}{3}$

(f) $\sqrt[5]{-32}$
$= -2$

3. Complete the table.

Exponential Form	***Radical Form***	***Evaluation of Expression***
$81^{\frac{1}{4}}$	$\sqrt[4]{81}$	3
$27^{\frac{4}{3}}$	$\sqrt[3]{27}^4$	81
$7776^{\frac{1}{5}}$	$\sqrt[5]{7776}$	6
$4096^{0.75}$	$\sqrt[4]{4096}^3$	512

4. Use your calculator to evaluate each expression. Express answers to two decimals.

(a) $256^{0.66}$ = 38.85 (b) $15^{\frac{-3}{2}}$ =0.02 (c) $\sqrt[11]{3.7}$ =1.13 (d) $\sqrt[4]{-99}$ not possible

5. Complete the table.

Function	***Exponential Growth or Decay?***	***Initial Value (y-intercept)***	***Growth/Decay rate***
$P(n)=200(1-0.032)^n$	decay	200	3.2%
$A(x)=(2)^x$	growth	1	100%
$Q(x)=0.85(0.77)^x$	decay	0.85	23%

6. Calculate finite differences to classify each function as linear, quadratic, exponential or none of those.

(a)

x	y	Δy	Δ(Δy)
-4	47		
		-21	
-3	26		6
		-15	
-2	11		6
		-9	
-1	2		6
		-3	
0	-1		

Conclusion quadratic

(b)

x	y	Δy	Δ(Δy)
-1	0.125		
		0.125	
0	0.25		1.625
		1.75	
1	2		4.25
		6	
2	8		18
		24	
3	32		

Conclusion none

Formulas:

$P=P_0(1+r)^n$ $\quad P=P_0(1-r)^n$ $\quad I(d)=I_0(1+r)^d$ $\quad N(d)=N_0(1+r)^d$

7. Greg invests $750 in a bond that pays 4.3% per year.

(a) Calculate, to the nearest penny, what Greg's total amount will be after 4 years.

$A=750(1.043)^4$

$A=887.56 \qquad \therefore \textit{the amount will be } \$887.56$

(b) How much money did $750 earn in four years?

$887.56-750=137.56 \qquad \therefore \textit{it earned } \$137.56$

(c) If Greg is planning to enter University in 2018, would his money have doubled by then?

$2018 \ is \ 8 \ years \ from \ now$

$A = 750(1.043)^8$

$A = 1050.35 \qquad \therefore he \ did \ not \ get \ \$1500, \ so \ his \ money \ did \ not \ double$

8. A police diver is searching a harbour for stolen goods. The equation that models the intensity of light per metre of depth is $I(n) = 100(0.92)^n$.

(a) At what rate does the light diminish per metre?

$1 - r = 0.92$

$r = 0.08 \qquad \therefore the \ light \ diminishes \ by \ 8\% \ per \ metre$

(b) Determine the amount of sunlight the diver will have at a depth of 18 m, relative to the intensity at the surface.

$I(18) = 100(0.92)^{18}$

$I(18) = 22.29 \qquad \therefore the \ light \ is \ 22.29\% \ as \ intense \ as \ it \ was \ at \ the \ surface$

9. Ryan purchases a used vehicle for $11, 899. If the vehicle depreciates at a rate of 13% yearly, what will the car be worth, to the nearest dollar, in ten years? Show your work.

$P(10) = 11899(1 - 0.13)^{10}$

$P(10) = 2955.99 \qquad \therefore it \ will \ be \ worth \ \$2955.99 \ in \ 10 \ years$

10. After being filled, a basketball loses 3.2% of its air every day. The initial amount of air in the ball was 840 cm^3

(a) Write an equation to model this situation.

Let $P(t)$ represent the final amount of air left in the ball after t days

$P(t) = 840(1 - 0.032)^t$

(b) Determine the volume after 4 days.

$P(4) = 840(0.968)^4$

$P(4) = 737.53 \qquad \therefore there \ is \ 737.53 \ cm^3 \ of \ air \ left \ in \ the \ ball$

(c) Will this model be valid after 6 weeks? Explain.

We can still use the equation for t = 42 (6 weeks = 42 days). However, the ball probably won't be losing air at the same rate (it will be losing it at a much slower rate that it was when it was first pumped up) so the equation will not model the situation accurately after so long.

11. List 4 characteristics of an exponential function.

Consider the function $f(x)=b^x$, b is positive and not equal to 1

- *domain is $\{x \in R\}$, range is $\{y \in R | y > 0\}$*
- *if b>1, the greater the value, the faster the growth*
- *if 0<b<1, the lesser the value, the faster the decay*
- *horizontal asymptote is y=0 (the x-axis)*
- *y-intercept is 1*

First and second differences are related by a multiplication pattern.

EXTRA QUESTIONS: Chapter 7 p. 526 # 1 – 8

Solutions

CHAPTER 8: Financial Problems Involving Exponential Functions

1. Complete the table (to the nearest penny).

Prinicpal ($)	Annual Interest Rate (%)	Time	Simple Interest Paid ($)	Amount
400	7.25	5 years	*145*	*545*
8098.22	$3\frac{3}{4}\%$	13 months	328.99	*8427.21*
760.60	5.5	*4.3 years*	180.00	940.60

12. Kurtis earned $279.40 in simple interest by investing a principal of $400 in a Treasury bill. If the interest rate was 3.35%/a, for how many years did he have his investment?

$$I = \Pr t$$
$$279.40 = (400)(0.0335)t$$
$$\frac{279.40}{(400)(0.0335)} = t$$
$$20.85 = t$$

Therefore, he had his investment for almost 21 years.

13. Complete the table (correct to 2 decimal places).

Principal ($)	Annual Interest Rate (%)	Years Invested	Compounding Period	Amount ($)	Interest Earned ($)
350	2.75	10	monthly	*460.64*	*110.64*
2500	8.5	2	semi-annually	*2952.87*	*452.87*
267.00	$2\frac{1}{4}\%$	7	annually	315.50	*48.50*
12 000	*3.24%*	7	weekly	15 053.88	*3053.88*

14. Calculate the amount you would end up with if you invested $2500 at $4\frac{1}{2}\%$ /a compounded semi-annually for 8 years?

$P = 2500$

$i = \frac{0.045}{2} = 0.0225$

$n = 2 \times 8 = 16$

$A = P(1+i)^n$

$A = 2500(1.0225)^{16}$

$A = 3569.05$

Therefore, you would end up with $3569.05

15. Johnny borrowed money from a friend. The interest rate was 5.75%/a compounded monthly. If Johnny will repay $5667 over the next 6 years. How much money did Johnny borrow?

$A = 5667$

$i = \frac{0.0575}{12} = 0.004791666$

$n = 6 \times 12 = 72$

$P = A(1+i)^{-n}$

$P = 5667(1.004791666)^{-72}$

$P = 4016.79$

Therefore, Johnny borrowed $4016.79

EXTRA QUESTIONS – Chapter 8 p. 526 #9,10