Work Period Questions

6.4 #2-8 (even)
#27-53 (odd)

#2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

LS $(\cos^2 x)^2 - (\sin^2 x)^2$

$= (1-\sin^2 x)^2 - (\sin^2 x)^2$

$= 1 - 2\sin^2 x + \sin^4 x - \sin^4 x$

$= 1 - 2\sin^2 x$

$= RS$ $\quad \therefore$ QED

#4 $\cos^2 x\cos^2 y + \sin^2 x\sin^2 y + \sin^2 x\cos^2 y + \sin^2 y\cos^2 x = 1$

LS $\cos^2 y(\cos^2 x + \sin^2 x) + \sin^2 y(\sin^2 x + \cos^2 x)$

$= \cos^2 y + \sin^2 y$

$= 1$

$= RS$ $\quad \therefore$ QED

#6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = (\tan x)(\tan y)$

LS $\dfrac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}}$

$= \dfrac{\tan x + \tan y}{\frac{\tan y + \tan x}{\tan x \tan y}}$

$= \dfrac{\cancel{(\tan x + \tan y)}(\tan x \tan y)}{\cancel{(\tan y + \tan x)}}$

$= (\tan x)(\tan y) = RS$ $\quad \therefore$ QED

#8. $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$

$\underline{LS}$ $(\cos^2 x)^3 + (\sin^2 x)^3$

$= (\cos^2 x + \sin^2 x)((\cos^2 x)^2 - \cos^2 x \sin^2 x + (\sin^2 x)^2)$

$= (1)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)$

$= (\cos^2 x)^2 - (1 - \sin^2 x)(\sin^2 x) + \sin^4 x$

$= (1 - \sin^2 x)^2 - \sin^2 x + \sin^4 x + \sin^4 x$

$= 1 - 2\sin^2 x + \sin^4 x - \sin^2 x + 2\sin^4 x$

$= 1 - 3\sin^2 x + 3\sin^4 x$

$= RS$ $\therefore$ QED.

#27 $\dfrac{1 + \cos x}{\sin x} = \cot \dfrac{x}{2}$

$\underline{LS}$ $\dfrac{1 + \cos 2(\frac{x}{2})}{\sin 2(\frac{x}{2})}$ $\underline{RS} = \cot \dfrac{x}{2}$

$= \dfrac{1 + 2\cos^2 \frac{x}{2} - 1}{2\sin \frac{x}{2} \cos \frac{x}{2}}$

$= \dfrac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}}$ $\therefore$ QED

$= \dfrac{\cos \frac{x}{2}}{\sin x/2} = \cot \dfrac{x}{2} = RS$

#29 $2\cot 2x = \cot x - \tan x$

LS $\dfrac{2}{\tan 2x}$

$= \dfrac{2}{\dfrac{2\tan x}{1-\tan^2 x}}$

$= \dfrac{2(1-\tan^2 x)}{2\tan x}$

$= \dfrac{1-\tan^2 x}{\tan x}$

$= RS$

RS $\dfrac{1}{\tan x} - \tan x$

$= \dfrac{1-\tan^2 x}{\tan x}$

$= LS$

$\therefore$ QED

31. $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \sec 2x - \tan 2x$

RS

$= \dfrac{1}{\cos 2x} - \dfrac{\sin 2x}{\cos 2x}$

$= \dfrac{1-\sin 2x}{\cos 2x}$

$= \dfrac{1 - 2\sin x \cos x}{\cos^2 x - \sin^2 x}$

$= \dfrac{1 - 2\sin x \cos x}{(\cos x + \sin x)(\cos x - \sin x)}$

$= \dfrac{\sin^2 x + \cos^2 x - 2\sin x \cos x}{(\cos x + \sin x)(\cos x - \sin x)}$

$= \dfrac{\sin^2 x - 2\sin x \cos x + \cos^2 x}{(\cos x + \sin x)(\cos x - \sin x)}$

$= \dfrac{(\sin x - \cos x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$

diff of squares

$(a^2 - b^2) = (a-b)(a+b)$

$\dfrac{\sin y - \cos y}{\cos x + \sin x}$

$= \dfrac{-(\sin x + \cos x)}{\cos x + \sin y} = LS$

$\therefore$ QED

#33 $\cos^6 x - \sin^6 x = \cos 2x\left(1 - \frac{1}{4}\sin^2 2x\right)$

<u>LS</u> $(\cos^2 x)^3 - (\sin^2 x)^3$

<u>RS</u> $\cos 2x\left(1 - \frac{1}{4}\sin^2 2x\right)$

$= (1 - 2\sin^2 x)\left(1 - \frac{1}{4}\sin 2x \sin 2x\right)$

$= (1 - 2\sin^2 x)\left(1 - \frac{1}{4}(2\sin x\cos x \, 2\sin x\cos x)\right)$

$= (1 - 2\sin^2 x)(1 - \sin^2 x\cos^2 x)$

$= 1 - \sin^2 x\cos^2 x - \cancel{2\sin^{2x}} \; 2\sin^4 x\cos^2 x$

$= 1 - \sin^2 x\cos^2 x + 2\sin^4 x\cos^2 x - 2\sin^2 x$

$= 1 - \sin^2 x(1 - \sin^2 x) + 2\sin^4 x(1 - \sin^2 x) - 2\sin^2 x$

$= 1 - \sin^2 x + \sin^4 x + 2\sin^4 x - 2\sin^6 x - 2\sin^2 x$

$= 1 - 3\sin^2 x + 3\sin^4 x - 2\sin^6 x$

$= \underbrace{1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x} - \sin^6 x$

☆ tricky

$= (1 - \sin^2 x)^3 - \sin^6 x$

$= (\cos^2 x)^3 - \sin^6 x$

$= \cos^6 x - \sin^6 x$

$= LS.$

#35 $\sec x - \tan x = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

LS

$$\frac{1}{\cos x} - \tan x$$

$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$= \frac{1 - \sin x}{\cos x}$$

$$= \frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}$$

$$= \frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}$$

$$\frac{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}$$

leaves a (-1)

$$= \frac{\sin\frac{x}{2} - \cos\frac{x}{2}}{(-1)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

* we need tan ∴ divide every term by $\cos\frac{x}{2}$

$$= \frac{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}} - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}} + \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

$$= \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}} = RS \quad \therefore QED$$

RS

$$\tan\frac{\pi}{4} - \frac{x}{2}$$

$$= \frac{\tan\frac{\pi}{4} - \tan\frac{x}{2}}{1 + \tan\frac{\pi}{4}\tan\frac{x}{2}}$$

$$= \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$$

#37. $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$

LS/ $\sin^2 x + \cos^2 x \cos^2 x$

$= \sin^2 x + \cos^2 x (1 - \sin^2 x)$

$= \sin^2 x + \cos^2 x - \cos^2 x \sin^2 x$

$= 1 - \cos^2 x \sin^2 x$

$= 1 - \frac{1}{2}(2 \cos^2 x \sin^2 x)$

$= 1 - \frac{1}{2} \sin 2x$

$= RS$

RS/ $\cos^2 x + \sin^2 x \sin^2 x$

$= \cos^2 x + \sin^2 x (1 - \cos^2 x)$

$= \cos^2 x + \sin^2 x - \sin^2 x \cos^2 x$

$= 1 - \sin^2 x \cos^2 x$

$= 1 - \frac{1}{2}(2 \sin^2 x \cos^2 x)$

$= 1 - \frac{1}{2} \sin 2x$

$= LS$

$\therefore$ QED.

39. $\cos x = \sin x \tan^2 x \cot^3 x$

LS $\cos x$

RS $(\sin x)\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{\cos^3 x}{\sin^3 x}\right)$

$= \frac{\sin^3 x \cos^3 x}{\cos^2 x \sin^3 x}$

$= \cos x$

$= LS$

$\therefore$ QED

41. $\sin^4 x + \cos^4 x = \sin^2 x(\csc^2 x - 2\cos^2 x)$

LS $\sin^4 x + \cos^4 x$

$= \sin^2 x \sin^2 x + \cos^2 x \cos^2 x$

$= (\sin^2 x)(1-\cos^2 x) + \cos^2 x(1-\sin^2 x)$

$= \sin^2 x - \sin^2 x\cos^2 x + \cos^2 x - \sin^2 x\cos^2 x$

$= \sin^2 x + \cos^2 x - \sin^2 x\cos^2 x - \sin^2\cos^2 x$

$= 1 - 2\sin^2 x\cos^2 x$

$= RS$

RS $\sin^2 x\left(\frac{1}{\sin^2 x} - 2\cos^2 x\right)$

$= \cancel{\sin^2 x}\left(\frac{1 - 2\cos^2 x \sin^2 x}{\cancel{\sin^2 x}}\right)$

$= 1 - 2\cos^2 x \sin^2 x$

$= LS$

$\therefore$ QED

43 $\cos\left(\frac{\pi}{12}-x\right)\sec\frac{\pi}{12}-\sin\left(\frac{\pi}{12}-x\right)\csc\frac{\pi}{12}=4\sin x$

LS $\cos\left(\frac{\pi}{12}-x\right)\left(\frac{1}{\cos\frac{\pi}{12}}\right)-\sin\left(\frac{\pi}{12}-x\right)\left(\frac{1}{\sin\frac{\pi}{12}}\right)$

$$=\frac{\cos\frac{\pi}{12}\cos x+\sin\frac{\pi}{12}\sin x}{\cos\frac{\pi}{12}}-\frac{\sin\frac{\pi}{12}\cos x-\cos\frac{\pi}{12}\sin x}{\sin\frac{\pi}{12}}$$

$$=\frac{\cancel{\cos\frac{\pi}{12}\cos x\sin\frac{\pi}{12}}+\sin\frac{\pi}{12}\sin\frac{\pi}{12}\sin x-\cancel{\sin\frac{\pi}{12}\cos\frac{\pi}{12}\cos x}+\cos\frac{\pi}{12}\cos\frac{\pi}{12}\sin x}{\cos\frac{\pi}{12}\sin\frac{\pi}{12}}$$

$$=\frac{\sin^2\frac{\pi}{12}\sin x+\cos^2\frac{\pi}{12}\sin x}{\cos\frac{\pi}{12}\sin\frac{\pi}{12}}$$

$$=\frac{\sin x\left(\sin^2\frac{\pi}{12}+\cos^2\frac{\pi}{12}\right)}{\cos\frac{\pi}{12}\sin\frac{\pi}{12}}$$

$$=\frac{\sin x}{\frac{1}{2}\left(2\cos\frac{\pi}{12}\sin\frac{\pi}{12}\right)}$$

$$=\frac{\sin x}{\frac{1}{2}\sin 2\left(\frac{\pi}{12}\right)}$$

$$=\frac{\sin x}{\frac{1}{2}\sin\frac{\pi}{6}}$$

$$=\frac{\sin x}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}=4\sin x=RS \quad \therefore \text{QED}$$

#45 $\sin 8x = 8\sin x\cos x\cos 2x\cos 4x$

LS/ $\sin 8x$ RS $8\sin x\cos x\,(\cos 2x)(\cos 4x)$

$= \sin 2(4x)$

$= 2\sin 4x\cos 4x$

$= 2[\sin 2(2x)][\cos 4x]$

$= 2[2\sin 2x\cos 2x][\cos 4x]$

$= 4(2\sin x\cos x)(\cos 2x)(\cos 4x)$

$= 8\sin x\cos x\cos 2x\cos 4x$

$= RS$ $\therefore$ QED

#47 $\sin(x+y) + \sin(x-y) = 2\sin x\cos y$

LS/ $\sin x\cos y + \cos x\sin y + \sin x\cos y - \sin x\cos y$

$= 2\sin x\cos y$

$= RS$ $\therefore$ QED

#49 $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} + x\right) = \tan(2\pi - x)$

LS/ $\tan x - \tan x + \dfrac{1}{\tan\left(\frac{\pi}{2}+x\right)}$ RS/ $\tan(2\pi - x)$

$= -\tan x$

$= 0 - \dfrac{1}{\cot x}$

*Related and co-related angles

$= LS$

$= -\tan x$

$= RS$ $\therefore$ QED

#51. $\tan\left(\frac{\pi}{2}-x\right)-\cot\left(\frac{3\pi}{2}-x\right)+\tan(2\pi-x)-\cot(\pi-x)=\frac{4-2\sec^2 x}{\tan x}$

LS/ $\cot x - \tan x - \tan x + \cot x$

$= 2\cot x - 2\tan x$

$= 2\left(\frac{1}{\tan x}\right) - 2\tan x$

$= \frac{2-2\tan^2 x}{\tan x}$

$= \frac{2(1-\tan^2 x)}{\tan x}$

$= \frac{2(2-\sec^2 x)}{\tan x}$

$= \frac{4-2\sec^2 x}{\tan x}$

$=$ RS

$\therefore$ QED

☆ Harder solution.

look here

$1-\tan^2 x$

$= 1-\frac{\sin^2 x}{\cos^2 x}$

$= 1-\frac{(1-\cos^2 x)}{\cos^2 x}$

$= 1-\frac{1}{\cos^2 x}+\frac{\cos^2 x}{\cos^2 x}$

$= 1-\frac{1}{\cos^2 x}+1$

$= 2-\frac{1}{\cos^2 x}$

$= 2-\sec^2 x$

(* Easier Solution)

#51 $\tan\left(\frac{\pi}{2}-x\right)-\cot\left(\frac{3\pi}{2}-x\right)+\tan(2\pi-x)-\cot(\pi-x)=\frac{4-2\sec^2 x}{\tan x}$

LS/ $\cot x-\tan x-\tan+\cot x$

$=2\cot x-2\tan x$

$=2\left(\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}\right)$

$=2\,\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}$

$=\frac{2\cos 2x}{\frac{1}{2}(2\sin x\cos x)}$

$=\frac{4\cos 2x}{\sin 2x}$

$=RS$

RS/ $\frac{2(2-\sec^2 x)}{\tan}$

$=\frac{2\left(2-\frac{1}{\cos^2 x}\right)}{\frac{\sin x}{\cos x}}$

$=\frac{2\left(\frac{2\cos^2 x-1}{\cos^2 x}\right)}{\frac{\sin x}{\cos x}}$

$=\frac{2\cos 2x}{\sin x\cos x}$

$=\frac{2\cos(2x)}{\frac{1}{2}(2\sin x\cos x)}$

$=\frac{4\cos 2x}{\sin 2x}$

$=LS$

$\therefore$ QED

#53 $\csc^2\left(\frac{\pi}{2}-x\right) = 1 + \sin^2 x \csc^2\left(\frac{\pi}{2}-x\right)$

LS $\dfrac{1}{\sin^2\left(\frac{\pi}{2}-x\right)}$

$= \dfrac{1}{\left(\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x\right)^2}$

$= \dfrac{1}{(\cos x - 0)^2}$

$= \dfrac{1}{\cos^2 x}$

$= RS$

RS $1 + \sin^2 x \left(\dfrac{1}{\sin^2\left(\frac{\pi}{2}-x\right)}\right)$

$= 1 + \sin^2 x \dfrac{1}{\left(\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x\right)}$

$= 1 + \dfrac{\sin^2 x}{(\cos x - 0)^2}$

$= 1 + \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= LS$

QED