

6.3 #1-7, 18

Day 5 Solutions

a)  $\cos 2(2x)$

$= 2\cos^2 2x - 1$

b)  $\sin 3x$

$= \sin 2\left(\frac{3}{2}x\right)$

$= 2\sin \frac{3}{2}x \cos \frac{3}{2}x$

c)  $\tan 6x$

$= \tan 2(3x)$

$= \frac{2 \tan 3x}{1 - \tan^2 3x}$

d)  $\sin \frac{1}{2}x$

$= \sin 2\left(\frac{1}{4}x\right)$

$= 2\sin \frac{1}{4}x \cos \frac{1}{4}x$

e)  $\cos \frac{2}{3}x$

$= \cos 2\left(\frac{2}{6}x\right)$

$= \cos 2\left(\frac{1}{3}x\right)$

$= \cos^2 \frac{1}{3}x - \sin^2 \frac{1}{3}x$

f)  $\tan(-7x)$

$= \tan 2\left(-\frac{7}{2}x\right)$

$= \frac{-2 \tan \frac{7}{2}x}{1 - \tan^2 \frac{7}{2}x}$

2. a)  $2\sin 3\theta \cos 3\theta$

$= \sin 2(3\theta)$

$= \sin 6\theta$

b)  $6\sin \theta \cos \theta$

$= 3(2\sin \theta \cos \theta)$

$= 3\sin 2\theta$

c)  $\frac{1}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2}$

$= \frac{1}{4} (2\sin \frac{\theta}{2} \cos \frac{\theta}{2})$

$= \frac{1}{4} \sin 2\left(\frac{\theta}{2}\right)$

$= \frac{1}{4} \sin \theta$

d)  $\cos^2 \frac{3\theta}{2} - \sin^2 \frac{3\theta}{2}$

$= \cos 2\left(\frac{3\theta}{2}\right)$

$= \cos 3\theta$

e)  $1 - 2\sin^2 \frac{\theta}{4}$

$= \cos 2\left(\frac{\theta}{4}\right)$

$= \cos \frac{\theta}{2}$

f)  $2\cos^2\left(\frac{7\theta}{2}\right) - 1$

$= \cos 2\left(\frac{7\theta}{2}\right)$

$= \cos 7\theta$

g)  $8\sin^2 2\theta - 4$

$= -4(-2\sin^2 2\theta + 1)$

$= -4(\cos 2(2\theta))$

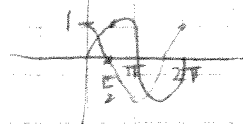
$= -4 \cos 4\theta$

h)  $1 - 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= \cos 2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

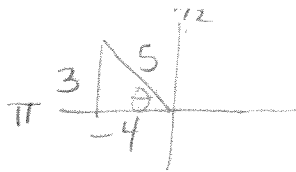
$= \cos\left(\frac{\pi}{2} - x\right)$  \* cos subtraction formula.

$= \cos \frac{\pi}{2} \cos x - \sin \frac{\pi}{2} \sin x = \sin x$



$$\#3. \cos \theta = -\frac{4}{5}$$

$$\frac{\pi}{2} \leq \theta \leq \pi$$



$$\sin 2\theta$$

$$= 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{3}{5} \right) \left( -\frac{4}{5} \right)$$

$$= -\frac{24}{25}$$

$$\cos 2\theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 2 \left( \cos \theta \right)^2 - 1$$

$$= 2 \left( -\frac{4}{5} \right)^2 - 1$$

$$= 2 \left( \frac{16}{25} \right) - \frac{25}{25}$$

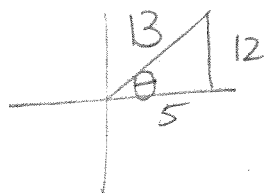
$$= \frac{32 - 25}{25}$$

$$= \frac{7}{25}$$

$$2\theta$$
$$= 2(143)$$
$$= 286^\circ$$

$\therefore$  Quad. 4

$$\#4. \sin \theta = \frac{12}{13}, \quad 0 \leq \theta \leq \frac{\pi}{2}$$



$$\sin 2\theta$$

$$= 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{12}{13} \right) \left( \frac{5}{13} \right)$$

$$= \frac{120}{169}$$

$$\cos 2\theta$$

$$= 1 - 2 \sin^2 \theta$$

$$= 1 - 2 \left( \frac{12}{13} \right)^2$$

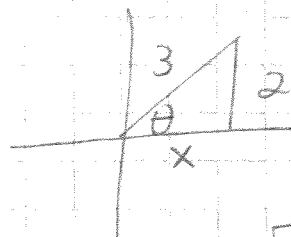
$$= 1 - 2 \left( \frac{144}{169} \right)$$

$$= \frac{169 - 288}{169}$$

$$= -\frac{119}{169}$$

$$2\theta$$
$$= 2(67)$$
$$= 135^\circ$$
$$= \text{Quad II}$$

#5.  $\sin \theta = \frac{2}{3}$   $0 \leq \theta \leq \frac{\pi}{2}$



$$x = \sqrt{3^2 - 2^2}$$

$$= \sqrt{5}$$

$$\sin 4\theta$$

$$= \sin 2(2\theta)$$

$$= 2 \sin 2\theta \cos 2\theta$$

$$= 2 \left[ 2 \sin \theta \cos \theta \right] \left[ 1 - 2 \sin^2 \theta \right]$$

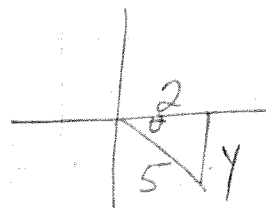
$$= 2 \left[ 2 \left( \frac{2}{3} \right) \left( \frac{\sqrt{5}}{3} \right) \right] \left[ 1 - 2 \left( \frac{2}{3} \right)^2 \right]$$

$$= \left( \frac{6\sqrt{5}}{9} \right) \left( 1 - 2 \left( \frac{4}{9} \right) \right)$$

$$= \left( \frac{6\sqrt{5}}{9} \right) \left( \frac{9-8}{9} \right)$$

$$= \frac{6\sqrt{5}}{81}$$

#6.  $\cos \theta = \frac{2}{5}$   $\frac{3\pi}{2} \leq \theta \leq 2\pi$



$$y = \sqrt{5^2 - 2^2}$$

$$= \sqrt{21}$$

$$\csc 2\theta$$

$$\sec 2\theta$$

$$= \frac{1}{\sin 2\theta}$$

$$= \frac{1}{\cos 2\theta}$$

$$= \frac{1}{2 \sin \theta \cos \theta}$$

$$= \frac{1}{2 \cos^2 \theta - 1}$$

$$= \frac{1}{2 \left( \frac{\sqrt{21}}{5} \right) \left( \frac{2}{5} \right)}$$

$$= \frac{1}{2 \left( \frac{2}{5} \right)^2 - 1}$$

$$= \frac{-25}{4\sqrt{21}}$$

$$= \frac{2 \left( \frac{4}{25} \right) \cdot 25}{25}$$

$$= \frac{25}{-17}$$

#7  $\tan a = \frac{1}{2}$ ,  $0 \leq a \leq \frac{\pi}{2}$

$\tan 2a$

$= \frac{2 \tan a}{1 - \tan^2 a}$

$= \frac{2 \left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2}$

$= \frac{1}{\frac{4}{4} - \frac{1}{4}}$

$= \frac{4}{3}$

#18 If  $\cos \theta + \sin \theta = \frac{1 + \sqrt{3}}{2}$  and  $\cos \theta - \sin \theta = \frac{1 - \sqrt{3}}{2}$  find

$\sin 2\theta$

$= 2 \sin \theta \cos \theta$

\*need to know what each are

$= 2 \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2}$

$\therefore \sin \theta = \frac{1 + \sqrt{3}}{2} - \cos \theta$        $\cos \theta - \frac{1 - \sqrt{3}}{2} = \sin \theta$

$\frac{1 + \sqrt{3}}{2} - \cos \theta = \cos \theta - \frac{1 - \sqrt{3}}{2}$

$\frac{1 + \sqrt{3}}{2} + \frac{1 - \sqrt{3}}{2} = 2 \cos \theta$

$\frac{2}{2} = 2 \cos \theta$

$\frac{1}{2} = \cos \theta$

