6.2 #1-6, 10 6.4. #11-25 (odd) Day 2 ① Solutions

a) $\cos 2a \cos a - \sin 2a \sin$
$= \cos(2a + a)$
$= \cos(3a)$

b) $\cos x \cos 4x + \sin x \sin 4x$
$= \cos(x - 4x)$
$= \cos(-3x)$
$= \cos(3x)$ related angle

c) $\sin 5 \cos 2 - \cos 5 \sin 2$
$= \sin(5-2)$
$= \sin 3$

d) $\sin 2m \cos m + \cos 2m \sin m$
$= \sin(2m + m)$
$= \sin(3m)$

e) $\dfrac{\tan 2a + \tan 3a}{1 - \tan 2a \tan 3a}$
$= \tan(2a + 3a)$
$= \tan(5a)$

f) $\dfrac{\tan 7 - \tan 9}{1 + \tan 7 \tan 9}$
$= \tan(7-9)$
$= \tan(-2)$ ∡ related angles
$= -\tan(2)$

g) $\cos^2 x - \sin^2 x$
$= \cos x \cos x - \sin x \sin x$
$= \cos(x + x)$
$= \cos(2x)$

h) $\sin a \cos a + \cos a \sin a$
$= \sin(a + a)$
$= \sin 2a$

i) $\dfrac{\tan x + \tan x}{1 - \tan^2 x}$
$= \tan(x + x)$
$= \tan(2x)$

j) $\cos^2 2 + \sin^2 2$
$= 1$ ∡ pythagorean thm

$30 \times \frac{3\pi}{180}$ $\quad \frac{5\pi}{6}$

$120^\circ \quad = 150^\circ \quad = 135^\circ$

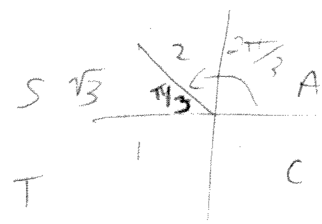
2. a) $\sin \frac{11\pi}{12}$ $\qquad \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{3\pi}{4}, \frac{9\pi}{12}$

$\frac{3\pi}{12} + \frac{8\pi}{12} \qquad \frac{10\pi}{6}$

$= \sin\left(\frac{3\pi}{12} + \frac{8\pi}{12}\right) \qquad \frac{2\pi}{12} \quad \frac{4\pi}{12} \quad \frac{3\pi}{12}$

$\frac{17\pi}{12}$

$= \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$



$= \sin\frac{\pi}{4}\cos\frac{2\pi}{3} + \cos\frac{\pi}{4}\sin\frac{2\pi}{3}$

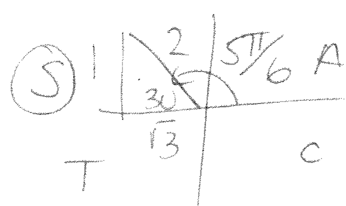
$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{-1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

b) $\cos \frac{13\pi}{12}$

$= \cos\left(\frac{10\pi}{12} + \frac{3\pi}{12}\right)$



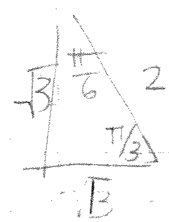
$= \cos\left(\frac{5\pi}{6} + \frac{\pi}{4}\right)$

$= \cos\frac{5\pi}{6}\cos\frac{\pi}{4} - \sin\frac{5\pi}{6}\sin\frac{\pi}{4}$

$= \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

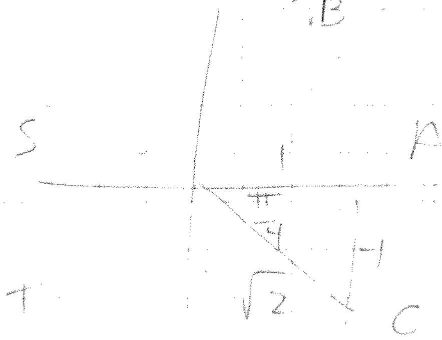
$= \frac{-\sqrt{3}-1}{2\sqrt{2}}$

c) $\tan\left(-\frac{7}{12}\pi\right) = -\tan\left(\frac{7}{12}\pi\right)$



$$= \tan\left(\frac{-3\pi}{12} + \frac{-4\pi}{12}\right)$$

$$= \tan\left(\frac{-\pi}{4} - \frac{\pi}{3}\right)$$



$$= \frac{\tan\frac{-\pi}{4} - \tan\frac{\pi}{3}}{1 + \tan\frac{-\pi}{4}\tan\frac{\pi}{3}}$$

$$= \frac{(-1) - (\sqrt{3})}{1 + (-1)(\sqrt{3})}$$

$$= -\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

d) $\tan\left(-\frac{5}{12}\pi\right)$

$$= \tan\left(\frac{-2\pi}{12} - \frac{3\pi}{12}\right)$$

$$= \tan\left(\frac{-\pi}{6} - \frac{\pi}{4}\right)$$

$$= \frac{\tan\frac{-\pi}{6} - \tan\frac{\pi}{4}}{1 + \tan\frac{-\pi}{6}\tan\frac{\pi}{4}}$$

$$= \frac{\left(\frac{-1}{\sqrt{3}}\right) - 1}{1 + \left(\frac{-1}{\sqrt{3}}\right)(1)}$$

$$= \frac{-1-\sqrt{3}}{\sqrt{3}} \div \frac{\sqrt{3}-1}{\sqrt{3}}$$

$$= \frac{-1-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}$$

$$= \frac{-1-\sqrt{3}}{\sqrt{3}-1}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

#3. a) $\sin\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \dfrac{1-\sqrt{3}}{2\sqrt{2}}$

b) $\cos\left(\frac{-\pi}{6}-\frac{\pi}{4}\right)$

$= \cos\frac{-\pi}{6}\cos\frac{\pi}{4} + \sin\frac{-\pi}{6}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

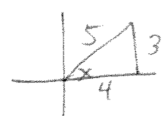
c) $\tan\left(\frac{-3\pi}{4}+\frac{2\pi}{3}\right)$

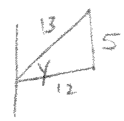
$= \dfrac{\tan\frac{-3\pi}{4} + \tan\frac{2\pi}{3}}{1-\tan\frac{-3\pi}{4}\tan\frac{2\pi}{3}}$

$= \dfrac{(1)-\sqrt{3}}{1-(1)(-\sqrt{3})}$

$= \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

4.





a) $\sin(x-y)$

$= \sin x\cos y - \cos x\sin y$

$= \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{36}{65} - \frac{20}{65}$

$= \frac{16}{65}$ ✓

b) $\cos(x+y)$

$= \cos x\cos y + \sin x\sin y$

$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{48}{65} - \frac{15}{65}$

$= \frac{33}{65}$

c) $\tan(x+y)$

$= \dfrac{\tan x+\tan y}{1-\tan x\tan y}$

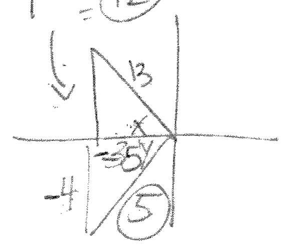
$= \dfrac{(3/4)+(5/12)}{1-(3/4)(5/12)}$

$= \dfrac{36+20}{48-15} = \dfrac{56}{33}$ ✓

5. $x \in \left[\frac{\pi}{2}, \pi\right]$ & $y \in \left[\pi, \frac{3\pi}{2}\right]$

$\cos x = \frac{-5}{13}$ $\tan y = \frac{4}{3}$

$y = \sqrt{13^2-5^2} = 12$



a) $\sin(x+y)$

$= \sin x\cos y + \cos x\sin y$

$= \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right)$

$= \frac{-36}{65} + \frac{20}{65}$

$= \frac{-16}{65}$

b) $\cos(x-y)$

$= \cos x\cos y + \sin x\sin y$

$= \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right)$

$= \frac{15}{65} - \frac{48}{65}$

$= \frac{-33}{65}$

#5 c) $\tan(x-y)$

$$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{\left(\frac{12}{-5}\right) - \left(\frac{4}{3}\right)}{1 + \left(\frac{12}{-5}\right)\left(\frac{4}{3}\right)}$$

$$= \frac{\frac{-36 + 20}{15}}{\frac{15 - 48}{15}}$$

$$= \frac{-56}{-33}$$

$$= \frac{56}{33}$$

#6 b) $\cos\frac{\pi}{7}\cos\frac{4\pi}{21} - \sin\frac{\pi}{7}\sin\frac{4\pi}{21}$

$$= \cos\left(\frac{\pi}{7} + \frac{4\pi}{21}\right)$$

$$= \cos\left(\frac{3\pi}{21} + \frac{4\pi}{21}\right)$$

$$= \cos\left(\frac{7\pi}{21}\right)$$

$$= \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}$$

d) $\sin\frac{5\pi}{36}\cos\frac{5\pi}{18} + \cos\frac{5\pi}{36}\sin\frac{5\pi}{18}$

$$= \sin\left(\frac{5\pi}{36} + \frac{5\pi}{18}\right)$$

$$= \sin\left(\frac{5\pi}{36} + \frac{10\pi}{36}\right)$$

$$= \sin\left(\frac{15\pi}{36}\right)$$

$$= \sin\frac{5\pi}{12}$$

$$= \sin\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

#10 a) $\sin(\pi + x) = -\sin x$

LS $\sin(\pi + x)$

$= \sin\pi\cos x + \cos\pi\sin x$

$= (0)\cos x + (-1)\sin x$

$= -\sin x$

$=$ RS $\therefore$ QED.

b) $\tan(2\pi - x) = -\tan x$

LS $\tan(2\pi - x)$

$= \dfrac{\tan 2\pi - \tan x}{1 + \tan 2\pi \tan x}$

$= \dfrac{(0) - \tan x}{1 + (0)\tan x}$

$= -\tan x$

$=$ RS $\therefore$ QED

c) $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$

LS $\cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x$

$= 0 - (-1)\sin x$

$= \sin x$

$=$ RS $\therefore$ QED

d) $\sin\left(\frac{3\pi}{2}-x\right)=-\cos x$

LS $\sin\left(\frac{3\pi}{2}-x\right)$

$=\sin\frac{3\pi}{2}\cos x+\cos\frac{3\pi}{2}\sin x$

$=(-1)\cos x+0$

$=-\cos x$

$=RS$ $\therefore$ QED

e) $\cos\left(\frac{\pi}{2}+x\right)=-\sin x$

LS $\cos\left(\frac{\pi}{2}+x\right)$

$=\cos\frac{\pi}{2}\cos x-\sin\frac{\pi}{2}\sin x$

$=0-(1)\sin x$

$=-\sin x$

$=RS$ $\therefore$ QED.

f) $\tan\left(\frac{\pi}{2}+x\right)=-\cot x$

* $\tan\frac{\pi}{2}$ is undefined $\therefore$ change to $\frac{\sin}{\cos}$

LS $\frac{\sin\left(\frac{\pi}{2}+x\right)}{\cos\left(\frac{\pi}{2}+x\right)}$

$=\frac{\sin\frac{\pi}{2}\cos x+\cos\frac{\pi}{2}\sin x}{\cos\pi/2\cos x-\sin\frac{\pi}{2}\sin x}$

$=\frac{\cos x}{-\sin x}$ $\therefore$ QED

$=RS$

RS $-\cot x$

$=-\frac{\cos x}{\sin x}$

g) $\sin(x-\pi)=-\sin x$

LS $\sin x\cos\pi+\cos x\sin\pi$

$=-\sin x+0$

$=-\sin x$

$=RS$ $\therefore$ QED

h) $-\tan(-x-\pi)=\tan x$

LS $-\left[\frac{\tan -x-\tan\pi}{1+\tan -x\tan\pi}\right]$

$=\frac{-\tan(-x)}{1}$

$=-(-\tan x)$

$=\tan x$

$=RS$ $\therefore$ QED

6.4 # 11-25 (odd)

11. $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

LS $\cos x\cos^2 y - \sin x\sin y\cos y + \sin x\cos y\sin y + \cos x\sin^2 y$

$= \cos x\cos^2 y + \cos x\sin^2 y - \sin x\sin y\cos y + \sin x\cos y\sin y$

$= \cos x\left[\cos^2 y + \sin^2 y\right] + 0$

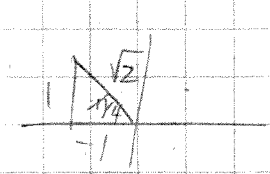
$= \cos x\,[1]$

$= \cos x$

$= 1$

$= RS$ $\therefore$ QED

#13. $\cos\left[\frac{3\pi}{4} + x\right] + \sin\left[\frac{3\pi}{4} - x\right] = 0$



LS $\cos\frac{3\pi}{4}\cos x - \sin\frac{3\pi}{4}\sin x + \sin\frac{3\pi}{4}\cos x - \cos\frac{3\pi}{4}\sin x$

$= \left(\frac{-1}{\sqrt{2}}\right)\cos x - \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$

$= 0 + 0$

$= 0$ $\therefore$ QED

$= RS$

#15 $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$

LS $\left[\sin x\cos y + \cos x\sin y\right]\left[\sin x\cos y - \cos x\sin y\right]$

$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$= (1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)$

$= \cos^2 y - \cos^2 x\cos^2 y - \cos^2 x + \cos^2 x\cos^2 y$

$= \cos^2 y - \cos^2 x$

$= RS$

$\therefore$ QED

#17. $\dfrac{\tan(x-y)+\tan y}{1-\tan(x-y)\tan y} = \tan x$

LS $\tan(x-y+y)$

$= \tan(x)$

$= RS$ $\therefore$ QED

#19. $\sin\left(\frac{\pi}{2}-x\right)\cot\left(\frac{\pi}{2}+y\right) = -\sin y$

LS $\left(\sin\frac{\pi}{2}\cos x-\cos\frac{\pi}{2}\sin x\right)\dfrac{\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$

$= (\cos x-0)\dfrac{\left(\cos\frac{\pi}{2}\cos x-\sin\frac{\pi}{2}\sin x\right)}{\left(\sin\frac{\pi}{2}\cos x-\cos\frac{\pi}{2}\sin x\right)}$

$= \cos x\dfrac{(0-\sin x)}{(\cos x-0)}$

$= \cos x\dfrac{(-\sin x)}{\cos y}$

$= -\sin y$

$= RS$

#21/ $\dfrac{\sin(\pi-x)}{\tan(\pi+x)} \cdot \dfrac{\cot(\frac{\pi}{2}-x)}{\tan(\frac{\pi}{2}+x)} \cdot \dfrac{\cos(2\pi-x)}{\sin(-x)} = \sin x$

LS/ $$\left[\frac{(\sin\pi\cos x - \cos\pi\sin x)}{\frac{\tan\pi + \tan x}{1-\tan\pi\tan x}}\right]\left[\frac{\frac{\cos(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)}}{\frac{\sin(\frac{\pi}{2}+x)}{\cos(\frac{\pi}{2}+x)}}\right]\left[\frac{\cos 2\pi\cos x + \sin 2\pi\sin x}{-\sin x}\right]$$

$$= \left[\frac{0+(+1)\sin x}{\frac{0+\tan x}{1-0}}\right]\left[\frac{\frac{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x}{\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x}}{\frac{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x}}\right]\left[\frac{(1)\cos x + 0}{-\sin x}\right]$$

$$= \left[\frac{+\sin x}{\tan x}\right]\left[\frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{-\sin x}}\right]\left[\frac{\cos x}{-\sin x}\right]$$

$$\left[\frac{+\sin x}{\tan x}\right]\left[\tan x \div \frac{-\cos x}{\sin x}\right]\left[\frac{\cos x}{-\sin x}\right]$$

$$= \left[\frac{+\sin x}{\cos x}\right]\left[\cos x\right]$$

$= +\sin x$

$= RS$

$\therefore$ QED

#23 $\dfrac{\csc(\pi - x)}{\sec(\pi + x)} \cdot \dfrac{\cos(-x)}{\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$

LS

$$= \left[\dfrac{\frac{1}{\sin(\pi - x)}}{\frac{1}{\cos(\pi + x)}}\right]\left[\dfrac{\cos x}{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x}\right]$$

*or use related and corelated angles.

$$= \left[\dfrac{\cos\pi\cos x - \sin\pi\sin x}{\sin\pi\cos x - \cos\pi\sin x}\right]\left[\dfrac{\cos x}{-\sin x}\right]$$

$$= \left[\dfrac{(-1)\cos x}{-(-1)\sin x}\right]\left[\dfrac{\cos x}{-\sin x}\right]$$

$$= \dfrac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x$$

$= RS$ $\therefore$ QED

#25 $\dfrac{\sin(\pi-x)\cos(\pi+x)\tan(2\pi-x)}{\sec\left(\frac{\pi}{2}+x\right)\csc\left(\frac{3\pi}{2}-x\right)\cot\left(\frac{3\pi}{2}+x\right)} = \sin^4 x - \sin^2 x$

LS

$$\frac{(\sin\pi\cos x-\cos\pi\sin x)(\cos\pi\cos x-\sin\pi\sin x)\left(\frac{\tan 2\pi-\tan x}{1-\tan 2\pi\tan x}\right)}{\left(\frac{1}{\cos\left(\frac{\pi}{2}+x\right)}\right)\left(\frac{1}{\sin\left(\frac{3\pi}{2}-x\right)}\right)\left(\frac{\cos\left(\frac{3\pi}{2}+x\right)}{\sin\left(\frac{3\pi}{2}+x\right)}\right)}$$

$$=\frac{(+\sin x)(-\cos x)(-\tan x)}{\left[\frac{1}{\cos\frac{\pi}{2}\cos x-\sin\frac{\pi}{2}\sin x}\right]\left[\frac{1}{\sin\frac{3\pi}{2}\cos x-\cos\frac{3\pi}{2}\sin x}\right]\left[\frac{\cos\frac{3\pi}{2}\cos x-\sin\frac{3\pi}{2}\sin x}{\sin\frac{3\pi}{2}\cos x+\cos\frac{3\pi}{2}\sin x}\right]}$$

$$=\frac{(\sin x)(-\cancel{\cos x})\left(\frac{-\sin x}{\cancel{\cos x}}\right)}{\left(\frac{1}{-\sin x}\right)\left(\frac{1}{-\cos x}\right)\left(\frac{-\sin x}{\cos x}\right)}$$

$$=\frac{\sin^2 x}{-\frac{1}{\cos^2 x}}$$

$=-\sin^2 x\cos^2 x$

$=-\sin^2 x(1-\sin^2 x)$

$=-\sin^2 x+\sin^4 x$

$=RS$

$\therefore$ QED