2 Trig Identities Day 1 Solutions (1

a) $\cos x + \cos(\pi - x) - \cos(\pi + x) - \cos(-x)$

$= \cos x - \cos x + \cos x - \cos x$

$= 0$ ✓

b) $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} - x\right) - \tan(2\pi - x)$

$= \tan x - \tan x + \tan x$? $+ \tan x$

$= 2\tan x$ ✓

c) $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right)$

$= -\sin x + \sin x - \cot x + \cot x$

$= 0$ ✓

d) $\sin\left(\frac{\pi}{2} + x\right) - \cos\left(\frac{3\pi}{2} - x\right) + \sin\left(\frac{3\pi}{2} - x\right)$

$= \cos x + \sin x - \cos x$

$= \sin x$ ✓

e) $\sin\left(\frac{\pi}{2} - x\right) + \sin(\pi - x) + \sin\left(\frac{3\pi}{2} - x\right) + \sin(2\pi - x)$

$= \cos x + \sin x - \cos x - \sin x$

$= 0.$ ✓

7.
a) $$\frac{\cos(\pi+x)\cos(\frac{\pi}{2}+x)}{\cos(\pi-x)} - \frac{\sin(\frac{3\pi}{2}-x)}{\sec(\pi+x)}$$

$$= \frac{(-\cos x)(-\sin x)}{-\cos x} - \frac{-\cos x}{\frac{1}{\cos(\pi+x)}}$$

$$= -\sin x + \cos x(-\cos x)$$

$$= -\sin x - \cos^2 x \quad \checkmark$$

b) $$\frac{\sin(x-\frac{\pi}{2})}{\cos(\pi-x)} + \frac{\tan(x-\frac{3\pi}{2})}{-\tan(\pi+x)}$$

$$= \frac{\sin(-\frac{\pi}{2}+x)}{-\cos x} + \frac{\tan(\frac{-3\pi}{2}+x)}{-\tan x}$$

$$= \frac{\sin(\frac{3\pi}{2}+x)}{-\cos x} + \frac{\tan(\frac{\pi}{2}\cancel{+}x)}{-\tan x}$$

$$= \frac{-\cos x}{-\cos x} + \frac{-\cot x}{-\tan x}$$

$$= +1 + \frac{\cos x}{\sin x}\cdot\frac{\cos x}{\sin x}$$

$$= +1 + \frac{\cos^2 x}{\sin^2 x} \Rightarrow \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} = \mathrm{cs}$$

6.4 # 1-9 (odd #s)

1. $\sin x \tan x = \sec x - \cos x$

LS	RS
$\sin x \left(\frac{\sin x}{\cos x}\right)$	$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$
$= \frac{\sin^2 x}{\cos x}$	$\frac{1-\cos^2 x}{\cos x}$
$= \frac{\sin^2 x}{\cos x}$	$\frac{\sin^2 y}{\cos x}$

LS = RS

∴ QED

3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 y$

LS	RS
$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$	$\left(\frac{1}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$
$\frac{\cos^2 x + \sin^2 x}{\sin^2 y \cos^2 y}$	$\frac{1}{\sin^2 x \cos^2 x}$
$\frac{1}{\sin^2 x \cos^2 x}$	$\frac{1}{\sin^2 x \cos^2 y}$

LS = RS

∴ QED

5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$

LS $= \frac{1}{\cos^2 x} - \frac{1}{\cos^2 y}$

$= \frac{\cos^2 y - \cos^2 x}{\cos^2 x \cos^2 y}$

RS $= \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 y}{\cos^2 y}$

$= \frac{\sin^2 x \cos^2 y - \sin^2 y \cos^2 x}{\cos^2 x \cos^2 y}$

$= \frac{(1-\cos^2 x)\cos^2 y - (1-\cos^2 y)\cos^2 x}{\cos^2 x \cos^2 y}$

$= \frac{\cos^2 y - \cos^2 x \cos^2 y - \cos^2 x + \cos^2 y \cos^2 x}{\cos^2 x \cos^2 y}$

$= \frac{\cos^2 y - \cos^2 x}{\cos^2 x \cos^2 y}$

$=$ LS ∴ QED

L.S. | R.S.

7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1+\tan^2 x}$

$= \left(\dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}\right)\left(\dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x}\right)$

$= \left(\dfrac{\sin^2 x}{\cos x}\right)\left(\dfrac{\cos^2 x}{\sin x}\right)$

$= (\sin x)(\cos x)$

$= \dfrac{\tan x}{1 + \dfrac{\sin^2 x}{\cos^2 x}}$

$= \dfrac{\tan x}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$= \dfrac{\tan x}{\dfrac{1}{\cos^2 x}}$

$= (\tan x)(\cos^2 x)$

$= \dfrac{\sin x}{\cancel{\cos x}}(\cos^2 x)$

$= (\sin x)(\cos x)$

LS = RS

∴ QED

9. $\sec^6 x - \tan^6 x = 1 + 3\tan^2 x \sec^2 x$

Solution #1

LS $\dfrac{1}{\cos^6 x} - \dfrac{\sin^6 x}{\cos^6 x}$

$= \dfrac{1 - \sin^6 x}{\cos^4 x \cos^2 x}$ $1^3 - (\sin^2 x)^3$

diff of cubes $= \dfrac{\cancel{(1-\sin^2 x)}(1+\sin^2 x+\sin^4 x)}{\cos^4 x \cancel{(1-\sin^2 x)}}$

$= \dfrac{1+\sin^2 x+\sin^4 x}{\cos^4 x}$

RS $= 1 + 3\tan^2 x \sec^2 x$

$= 1 + 3\dfrac{\sin^2 x}{\cos^2 x}\left(\dfrac{1}{\cos^2 x}\right)$

$= 1 + \dfrac{3\sin^2 x}{\cos^4 x}$

$= \dfrac{\cos^4 x + 3\sin^2 x}{\cos^4 x}$

$= \dfrac{(\cos^2 x)(\cos^2 x) + 3\sin^2 x}{\cos^4 x}$

$= \dfrac{(1-\sin^2 x)(1-\sin^2 x) + 3\sin^2 x}{\cos^4 x}$

$= \dfrac{1-2\sin^2 x+\sin^4 x+3\sin^2 x}{\cos^4 x}$

$= \dfrac{1+\sin^2 x+\sin^4 x}{\cos^4 x}$

QED

Solution #2

LS

$= (\sec^2 x - \tan^2 x)(\sec^4 x + \sec^2 x \tan^2 x + \tan^4 x)$

$= (\sec^2 x - (1 + \sec^2 x))((\sec^4 x - 2\sec^2 x \tan^2 x + \tan^4 x) + 3\sec^2 x \tan$

$= (1)((\sec^2 x - \tan^2 x)^2 + 3\sec^2 x \tan^2 x))$

$= 1 + 3\sec^2 x \tan^2 x$

$= RS$

$\therefore$ QED.