Exercise 6.1 Functions of Related Values (related angle identities and co-related angle identities)

3. Simplify.
   (a) $\cos x + \cos(\pi - x) - \cos(\pi + x) - \cos(-x)$
   (b) $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} - x\right) - \tan(2\pi - x)$
   (c) $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right)$
   (d) $\sin\left(\frac{\pi}{2} + x\right) - \cos\left(\frac{3\pi}{2} - x\right) + \sin\left(\frac{3\pi}{2} - x\right)$
   (e) $\sin\left(\frac{\pi}{2} - x\right) + \sin(\pi - x) + \sin\left(\frac{3\pi}{2} - x\right) + \sin(2\pi - x)$

7. Simplify.
   (a) $\dfrac{\cos(\pi + x)\cos\left(\frac{\pi}{2} + x\right)}{\cos(\pi - x)} - \dfrac{\sin\left(\frac{3\pi}{2} - x\right)}{\sec(\pi + x)}$
   (b) $\dfrac{\sin\left(x - \frac{\pi}{2}\right)}{\cos(\pi - x)} + \dfrac{\tan\left(x - \frac{3\pi}{2}\right)}{-\tan(\pi + x)}$

Exercise 6.2 Compound Angle Formulas (addition and subtraction identities)

A 1. Express as a single trigonometric function.
   (a) $\cos 2a \cos a - \sin 2a \sin a$ (b) $\cos x \cos 4x + \sin x \sin 4x$
   (c) $\sin 5 \cos 2 - \cos 5 \sin 2$
   (d) $\sin 2m \cos m + \cos 2m \sin m$
   (e) $\dfrac{\tan 2a + \tan 3a}{1 - \tan 2a \tan 3a}$ (f) $\dfrac{\tan 7 - \tan 9}{1 + \tan 7 \tan 9}$
   (g) $\cos^2 x - \sin^2 x$ (h) $\sin a \cos a + \cos a \sin a$
   (i) $\dfrac{\tan x + \tan x}{1 - \tan^2 x}$ (j) $\cos^2 2 + \sin^2 2$

2. Evaluate using formulas developed in this section.
   (a) $\sin \frac{11\pi}{12}$ (b) $\cos \frac{13\pi}{12}$
   (c) $\tan\left(-\frac{7}{12}\pi\right)$ (d) $\tan\left(-\frac{5}{12}\pi\right)$

3. Find the value of each of the following.
   (a) $\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$ (b) $\cos\left(-\frac{\pi}{6} - \frac{\pi}{4}\right)$ (c) $\tan\left(-\frac{3\pi}{4} + \frac{2\pi}{3}\right)$

4. If x and y are in the interval $\left(0, \frac{\pi}{2}\right)$ and $\sin x = \frac{3}{5}$ and $\cos y = \frac{12}{13}$ evaluate each of the following.
   (a) $\sin(x - y)$ (b) $\cos(x + y)$ (c) $\tan(x + y)$

5. If x is in the interval $\left(\frac{\pi}{2}, \pi\right)$ and y is in the interval $\left(\pi, \frac{3\pi}{2}\right)$ and $\cos x = -\frac{5}{13}$ and $\tan y = \frac{4}{3}$, evaluate each of the following.
   (a) $\sin(x + y)$ (b) $\cos(x - y)$ (c) $\tan(x - y)$

6. Find the exact value of each of the following.
   (b) $\cos \frac{\pi}{7} \cos \frac{4\pi}{21} - \sin \frac{\pi}{7} \sin \frac{4\pi}{21}$
   (d) $\sin \frac{5\pi}{36} \cos \frac{5\pi}{18} + \cos \frac{5\pi}{36} \sin \frac{5\pi}{18}$

EXERCISE 6.1 (Answers)

3. (a) 0 (b) $2 \tan x$ (c) 0 (d) $\sin x$ (e) 0

7. (a) $-\sin x - \cos^2 x$ (b) $\csc^2 x$

EXERCISE 6.2 (Answers)

1. (a) $\cos 3a$ (b) $\cos 3x$ (c) $\sin 3$ (d) $\sin 3m$ (e) $\tan 5a$ (f) $-\tan 2$ (g) $\cos 2x$ (h) $\sin 2a$ (i) $\tan 2x$ (j) 1

2. (a) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ (b) $\frac{-1 - \sqrt{3}}{2\sqrt{2}}$ (c) $-\frac{1 + \sqrt{3}}{1 - \sqrt{3}}$ (d) $\frac{\sqrt{3} + 1}{1 - \sqrt{3}}$ (e) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ (f) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$

3. (a) $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ (b) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ (c) $\frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

4. (a) $\frac{16}{65}$ (b) $\frac{33}{65}$ (c) $\frac{56}{33}$

5. (a) $-\frac{16}{65}$ (b) $-\frac{33}{65}$ (c) $\frac{56}{33}$

6. (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3} - 1}{\sqrt{3} + 1}$ (d) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$

9. Use the Addition Formula for Tangent to prove the Subtraction Formula for Tangent.

10. Prove each of the following.

(a) $\sin(\pi + x) = -\sin x$ (b) $\tan(2\pi - x) = -\tan x$

(c) $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$ (d) $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

(e) $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$ (f) $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$

(g) $\sin(x - \pi) = -\sin x$ (h) $-\tan(-x - \pi) = \tan x$

Exercise 6.3 Double Angle Formulas

1. Use a Double Angle Formula to rewrite each expression.

(a) $\cos 2(2x)$ (b) $\sin 3x$ (c) $\tan 6x$

(d) $\sin \frac{1}{2}x$ (e) $\cos \frac{2}{3}x$ (f) $\tan(-7x)$

2. Express as a single sine or cosine function.

(a) $2 \sin 3\theta \cos 3\theta$ (b) $6 \sin \theta \cos \theta$

(c) $\frac{1}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ (d) $\cos^2 \frac{3\theta}{2} - \sin^2 \frac{3\theta}{2}$

(e) $1 - 2\sin^2 \frac{\theta}{4}$ (f) $2\cos^2\left(\frac{7}{2}\theta\right) - 1$

(g) $8 \sin^2 2\theta - 4$ (h) $1 - 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

3. If $\cos \theta = -\frac{4}{5}, \frac{\pi}{2} \leq \theta \leq \pi$, find the value of $\sin 2\theta$ and $\cos 2\theta$. Determine the quadrant of angle 2θ.

4. If $\sin \theta = \frac{12}{13}, 0 \leq \theta \leq \frac{\pi}{2}$, evaluate $\sin 2\theta$ and $\cos 2\theta$. Determine the quadrant of angle 2θ.

5. If $\sin \theta = \frac{2}{3}, 0 \leq \theta \leq \frac{\pi}{2}$, find the value of $\sin 4\theta$.

6. If $\cos \theta = \frac{2}{5}, \frac{3\pi}{2} \leq \theta \leq 2\pi$, find the values of $\csc 2\theta$ and $\sec 2\theta$.

7. If $\tan a = \frac{1}{2}, 0 \leq a \leq \frac{\pi}{2}$, find the value of $\tan 2a$.

8. If $\tan a = 2, -2\pi \leq a \leq -\frac{3\pi}{2}$, evaluate $\tan 4a$.

9. Develop formulas for

(a) $\sin 3\theta$ in terms of $\sin \theta$. (b) $\cos 3\theta$ in terms of $\cos \theta$.

(c) $\tan 3\theta$ in terms of $\tan \theta$. (d) $\cos 4\theta$ in terms of $\cos \theta$.

18. If $\cos \theta + \sin \theta = \frac{1 + \sqrt{3}}{2}$ and $\cos \theta - \sin \theta = \frac{1 - \sqrt{3}}{2}$ find the value of $\sin 2\theta$.

EXERCISE 6.3 Answers

1. (a) $\cos^2 2x - \sin^2 2x$ or $1 - 2\sin^2 2x$ or $2\cos^2 2x - 1$ (b) $2 \sin \frac{3}{2}x \cos \frac{3}{2}x$ (c) $\frac{2 \tan 3x}{1 - \tan^2 3x}$ (d) $2 \sin \frac{1}{4}x \cos \frac{1}{4}x$ (e) $\cos^2 \frac{1}{3}x - \sin^2 \frac{1}{3}x$ (f) $\frac{-2 \tan \frac{7}{2}x}{1 - \tan^2 \frac{7}{2}x}$

2. (a) $\sin 6\theta$ (b) $3 \sin 2\theta$ (c) $\frac{1}{4} \sin \theta$ (d) cos (e) $\cos \frac{\theta}{2}$ (f) $\cos 7\theta$ (g) $-4 \cos 4\theta$ (h) sin

3. $\sin 2\theta = -\frac{24}{25}$, $\cos 2\theta = \frac{7}{25}$, 4th quadrant

4. $\sin 2\theta = \frac{120}{169}$, $\cos 2\theta = -\frac{119}{169}$, 2nd quadrant

5. $\frac{8\sqrt{5}}{81}$

6. $\csc 2\theta = \frac{25}{-4\sqrt{21}}$, $\sec 2\theta = \frac{25}{-17}$

7. $\frac{4}{3}$ 8. $\frac{24}{7}$

9. (a) $3 \sin \theta - 4 \sin^3 \theta$ (b) $4 \cos^3 \theta - 3 \cos \theta$ (c) $\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$ (d) $8 \cos^4 \theta - 8 \cos^2 \theta + 1$

18. $\frac{\sqrt{3}}{2}$

ise 6.4 Trigonometric Identities

The following identities involve the reciprocal, quotient, and Pythagorean relationships. Prove each one.

1. $\sin x \tan x = \sec x - \cos x$
2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$
3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$
4. $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$
5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$
6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = (\tan x)(\tan y)$
7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$
8. $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$
9. $\sec^6 x - \tan^6 x = 1 + 3\tan^2 x \sec^2 x$

The following involve the addition and subtraction formulas.

10. $1 + \cot x \tan y = \dfrac{\sin(x + y)}{\sin x \cos y}$
11. $\cos(x + y)\cos y + \sin(x + y)\sin y = \cos x$
12. $\sin x - \tan y \cos x = \dfrac{\sin(x - y)}{\cos y}$
13. $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right) = 0$
14. $\dfrac{\tan\left(\frac{\pi}{4} + x\right) - \tan\left(\frac{\pi}{4} - x\right)}{\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right)} = 2\sin x \cos x$
15. $\sin(x + y)\sin(x - y) = \cos^2 y - \cos^2 x$
16. $\tan(x + y)\tan(x - y) = \dfrac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$
17. $\dfrac{\tan(x - y) + \tan y}{1 - \tan(x - y)\tan y} = \tan x$
18. $\sin 5x = \sin x(\cos^2 2x - \sin^2 2x) + 2\cos x \cos 2x \sin 2x$

The following involve related and co-related angles.

19. $\sin\left(\dfrac{\pi}{2} - x\right)\cot\left(\dfrac{\pi}{2} + x\right) = -\sin x$
20. $\cos(-x) + \cos(\pi - x) = \cos(\pi + x) + \cos x$
21. $\dfrac{\sin(\pi - x)}{\tan(\pi + x)} \dfrac{\cot\left(\frac{\pi}{2} - x\right)}{\tan\left(\frac{\pi}{2} + x\right)} \dfrac{\cos(2\pi - x)}{\sin(-x)} = \sin x$
22. $\dfrac{\sin(-x)}{\sin(\pi + x)} - \dfrac{\tan\left(\frac{\pi}{2} + x\right)}{\cot x} + \dfrac{\cos x}{\sin\left(\frac{\pi}{2} + x\right)} = 3$
23. $\dfrac{\csc(\pi - x)}{\sec(\pi + x)} \dfrac{\cos(-x)}{\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$
24. $\dfrac{\cos\left(\frac{\pi}{2} + x\right)\sec(-x)\tan(\pi - x)}{\sec(2\pi + x)\sin(\pi + x)\cot\left(\frac{\pi}{2} - x\right)} = -1$
25. $\dfrac{\sin(\pi - x)\cos(\pi + x)\tan(2\pi - x)}{\sec\left(\frac{\pi}{2} + x\right)\csc\left(\frac{3\pi}{2} - x\right)\cot\left(\frac{3\pi}{2} + x\right)} = \sin^4 x - \sin^2 x$

The following involve the double angle formulas.

26. $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$
27. $\dfrac{1 + \cos x}{\sin x} = \cot \dfrac{x}{2}$
28. $2\csc 2x = \sec x \csc x$
29. $2\cot 2x = \cot x - \tan x$
30. $\dfrac{\cos 2x}{1 + \sin 2x} = \tan\left(\dfrac{\pi}{4} - x\right)$
31. $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \sec 2x - \tan 2x$
32. $\dfrac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$
33. $\cos^6 x - \sin^6 x = \cos 2x\left(1 - \frac{1}{4}\sin^2 2x\right)$
34. $4(\cos^6 x + \sin^6 x) = 1 + 3\cos^2 2x$
35. $\sec x - \tan x = \tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$
36. $\dfrac{\sin 2x}{1 + \cos 2x} \dfrac{\cos x}{1 + \cos x} = \tan \dfrac{x}{2}$

The following involve a variety of formulas and identities.

37. $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$

38. $\tan x - \cot x = (\tan x - 1)(\cot x + 1)$

39. $\cos x = \sin x \tan^2 x \cot^3 x$

40. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

41. $\sin^4 x + \cos^4 x = \sin^2 x(\csc^2 x - 2\cos^2 x)$

42. $\sin^3 x + \cos^3 x = (1 - \sin x \cos x)(\sin x + \cos x)$

43. $\cos\left(\frac{\pi}{12} - x\right)\sec\frac{\pi}{12} - \sin\left(\frac{\pi}{12} - x\right)\csc\frac{\pi}{12} = 4\sin x$

44. $\tan(x - y) + \tan(y - z) = \dfrac{\sec^2 y\,(\tan x - \tan z)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$

45. $\sin 8x = 8 \sin x \cos x \cos 2x \cos 4x$

46. $\sin x = 1 - 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

47. $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

48. $\dfrac{\sin(x - y)}{\sin x \sin y} + \dfrac{\sin(y - z)}{\sin y \sin z} + \dfrac{\sin(z - x)}{\sin z \sin x} = 0$

49. $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} + x\right) = \tan(2\pi - x)$

50. $\sin\left(\frac{\pi}{2} + x\right)\cos(\pi - x)\cot\left(\frac{3\pi}{2} + x\right)$
$= \sin\left(\frac{\pi}{2} - x\right)\sin\left(\frac{3\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right)$

51. $\tan\left(\frac{\pi}{2} - x\right) - \cot\left(\frac{3\pi}{2} - x\right) + \tan(2\pi - x) - \cot(\pi - x)$
$= \dfrac{4 - 2\sec^2 x}{\tan x}$

52. $\tan(x + y + z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan x \tan z - \tan y \tan z}$

53. $\csc^2\left(\frac{\pi}{2} - x\right) = 1 + \sin^2 x \csc^2\left(\frac{\pi}{2} - x\right)$

54. $\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = 2\sec 2x$

55. $\dfrac{1 - \sin 2x}{\cos 2x} = \dfrac{\cos 2x}{1 + \sin 2x}$

56. $\dfrac{\sin 4x}{1 - \cos 4x} \times \dfrac{1 - \cos 2x}{\cos 2x} = \tan x$

MHF 4UI

Trigonometry
Solving Trig Equations

1. Solve for θ in radians to two decimal places. $0 \le \theta \le \frac{\pi}{2}$

a) $\sin\theta = 0.82$ b) $\cos\theta = 0.75$ c) $\tan\theta = 3.158$ d) $\csc\theta = 5.682$
e) $\sec\theta = 2.142$ f) $\cot\theta = 1.502$ g) $\sin\theta = 0.2552$ h) $\cos\theta = 0.1123$

2. Solve for θ in radians to two decimal places. $0 \le \theta \le 2\pi$

a) $\sin\theta = 0.2671$ b) $\cos\theta = 0.8923$ c) $\tan\theta = 0.324$ d) $\sec\theta = 1.167$
e) $\csc\theta = 8.487$ f) $\cot\theta = 9.34$ g) $\cos\theta = 0.44$ h) $\sin\theta = 0.6805$
i) $\tan\theta = 2.671$ j) $\cos\theta = -0.1067$ k) $\sin\theta = -0.804$ l) $6\cos\theta + 1 = 0$
m) $3\sin\theta + 1 = 2$ n) $6\sin^2\theta + 5\sin\theta - 6 = 0$ o) $4\cos^2\theta + 5\cos\theta - 6 = 0$

3. Solve for θ in radians $0 \le \theta \le 2\pi$ (exact answers)

a) $\cos q = \frac{\sqrt{3}}{2}$ b) $\sin q = \frac{-1}{\sqrt{2}}$ c) $\sin q = \frac{1}{2}$ d) $\tan q = 1$
e) $\sin\theta = 0$ f) $\cos\theta = -1$ g) $\sec\theta = 1$ h) $\cot\theta = 0$

4. Solve for θ in radians $0 \le \theta \le 2\pi$ (exact answers)

a) $2\cos\theta = 1$ b) $2\sin\theta + 1 = 0$ c) $3\cos\theta + 4 = 0$
d) $\tan\theta + \sqrt{3} = 0$ e) $\cos\theta + 2 = 3\cos\theta$ f) $\tan\theta - 1 = 0$

5. Solve for θ in radians $0 \le \theta \le 2\pi$ (exact answers)

a) $2\cos^2\theta = 2$ b) $\sin^2 q = \frac{3}{4}$ c) $4\sin^2\theta - 1 = 0$ d) $4\cos^2\theta - 3 = 0$
e) $\cos^2\theta - 1 = 0$ f) $\cos^2 q = \frac{1}{4}$ g) $\sin^2\theta - 1 = 0$ h) $4\sin^2\theta - 3 = 0$
i) $4\cos^2\theta - 1 = 0$ j) $\tan^2\theta + \tan\theta = 0$ k) $\sin^2\theta + \sin\theta = 0$ l) $\cos^2\theta - \cos\theta = 0$
m) $\sin^2\theta + 2\sin\theta - 3 = 0$ n) $2\sin^2\theta - 3\sin\theta + 1 = 0$ o) $\cos^2\theta - 2\cos\theta - 3 = 0$
p) $2\cos^2 q + (2 - \sqrt{3})\cos q - \sqrt{3} = 0$ q) $2\sin^2\theta + \sin\theta = 0$ r) $2\sin^2\theta + \sin\theta - 1 = 0$
s) $\sin^2\theta + 5\sin\theta + 6 = 0$ t) $2\sec^2 - 3\sec\theta - 2 = 0$ u) $2\cos^2\theta + \cos\theta = 0$
v) $\sin^2\theta - 2\sin\theta - 3 = 0$ w) $\cos^2\theta - 3\cos\theta - 4 = 0$

6. Solve for θ in radians $0 \le \theta \le 2\pi$ (exact answers)

a) $\cos^3\theta + 2\cos^2\theta - \cos\theta - 2 = 0$ b) $2\cos^3\theta + 7\cos^2\theta + 2\cos\theta - 3 = 0$
c) $3 - 3\sin\theta - 2\cos^2\theta = 0$

7. Solve for θ

a) $\sin 2\theta = \frac{1}{2}$ $0 \le 2\theta \le 2\pi$ b) $\cos 2\theta = \frac{-\sqrt{3}}{2}$ $0 \le 2\theta \le 2\pi$

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c) $\tan 2\theta = \sqrt{3}$, $0 \le 2\theta \le 2\pi$ d) $\cos 2\theta = \cos^2\theta$, $-\pi \le \theta \le \pi$
e) $\sin 2\theta = \cos\theta$, $-\pi \le 2\theta \le \pi$ f) $\cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta = 0$, $0 \le 2\theta \le \pi$
g) $\tan 2\theta = 8\cos^2\theta - \cot\theta$ $0 \le \theta \le \frac{\pi}{2}$ 7h) $\tan 6 + \sec 26 = 1$ $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$
7i) $2(\sin^4\theta + \cos^4\theta) = 1$, $-\pi \le \theta \le \pi$

Answers:

1. a) 0.96 b) 0.72 c) 1.26 d) 0.18 e) 1.09 f) 0.59 g) 0.26 h) 1.46

2. a) 0.27, 2.87 b) 0.47, 5.81 c) 0.31, 3.45 d) 0.54, 5.74 e) 0.12, 3.02 f) 0.11, 3.25
g) 1.12, 5.17 h) 0.75, 2.39 i) 1.21, 4.35 j) 1.68, 4.61 k) 4.08, 5.35 l) 1.74, 4.54
m) 0.34, 2.8 n) 0.73, 2.41 o) 0.72, 5.56

3. a) $\frac{\pi}{6}, \frac{11\pi}{6}$ b) $\frac{5\pi}{4}, \frac{7\pi}{4}$ c) $\frac{\pi}{6}, \frac{5\pi}{6}$ d) $\frac{\pi}{4}, \frac{5\pi}{4}$ e) $0, \pi, 2\pi$ f) π g) $0, 2\pi$ h) $\frac{\pi}{2}, \frac{3\pi}{2}$

4. a) $\frac{\pi}{3}, \frac{5\pi}{3}$ b) $\frac{7\pi}{6}, \frac{11\pi}{6}$ c) no solution d) $\frac{2\pi}{3}, \frac{5\pi}{3}$ e) $0, 2\pi$ f) $\frac{\pi}{4}, \frac{5\pi}{4}$

5. a) $0, \pi, 2\pi$ b) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ c) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ d) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ e) $0, \pi, 2\pi$
f) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ g) $\frac{\pi}{2}, \frac{3\pi}{2}$ h) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ i) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
j) $0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$ k) $0, \pi, \frac{3\pi}{2}, 2\pi$ l) $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$ m) $\frac{\pi}{2}$ n) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ o) π
p) $\frac{\pi}{6}, \pi, \frac{11\pi}{6}$ q) $0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ r) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ s) no solution t) $\frac{\pi}{3}, \frac{5\pi}{3}$
u) $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}, \frac{4\pi}{3}$ v) $\frac{3\pi}{2}$ w) π

6. a) $0, \pi, 2\pi$ b) $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$ c) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

7. a) $\frac{\pi}{12}, \frac{5\pi}{12}$ b) $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{5\pi}{3}, \frac{7\pi}{12}$ c) $\frac{\pi}{6}, \frac{2\pi}{3}$ d) $-\pi, 0, \pi$ e) $\frac{-\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}$
f) $\frac{\pi}{8}$ g) $\frac{\pi}{24}, \frac{5\pi}{24}$ h) $\frac{-\pi}{8}, 0, \frac{3\pi}{8}$ i) $\frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

6.6 REVIEW EXERCISE

1. If x is in the interval $\left[0, \frac{\pi}{2}\right]$ and y is in the interval $\left[\frac{\pi}{2}, \pi\right]$ and $\tan x = \frac{4}{3}$ and $\csc y = \frac{13}{5}$, evaluate.
   (a) $\sin(x + y)$ (b) $\tan(x - y)$ (c) $\cos 2(x + y)$
2. Evaluate.
   (a) $\sin \frac{13\pi}{12}$ (b) $\cos\left(\frac{-11\pi}{12}\right)$ (c) $-\tan\left(-\frac{5\pi}{12}\right)$
   (d) $\sin 15°$ (e) $\cos(-75°)$ (f) $-\tan 105°$
3. If $\tan x = -\frac{3}{4}, \frac{\pi}{2} \leq x \leq \pi$, evaluate.
   (a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$
4. If $\sin \frac{x}{2} = \frac{2}{3}, 0 \leq x \leq \frac{\pi}{2}$, evaluate.
   (a) $\cos x$ (b) $\tan x$ (c) $\sin \frac{x}{4}$
5. Find the value of
   (a) $\sin 112\frac{1}{2}°$ (b) $\cos \frac{\pi}{8}$ (c) $\tan \frac{3\pi}{16}$
6. Express each of the following as a function of its related acute angle and evaluate.
   (a) $\sin 120°$ (b) $\cos \frac{11\pi}{6}$ (c) $\tan\left(-\frac{7}{3}\pi\right)$
7. Express each of the following as a function of its co-related acute angle and evaluate.
   (a) $\sin\left(-\frac{7\pi}{6}\right)$ (b) $\cos 495°$ (c) $\tan \frac{33\pi}{4}$
8. Prove the following identities.
   (a) $\tan x = \csc 2x - \cot 2x$ (b) $\frac{1 - \sin 2x}{\cos 2x} = \frac{1 - \tan x}{1 + \tan x}$
   (c) $\cos x - \tan y \sin x = \sec y \cos(x + y)$
   (d) $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) = -\cot x$
   (e) $\frac{\sin 4x - \sin 2x}{\sin 2x} = \frac{\cos 3x}{\cos x}$
   (f) $\cos x + \cos 2x + \cos 3x = \cos 2x\,(1 + 2 \cos x)$
   (g) $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$
9. Solve.
   (a) $2 \sin x \cos x = 0, 0 \leq x \leq \pi$
   (b) $\sin^2 x + \sin x = 0, -\pi \leq x \leq \pi$
   (c) $\cos^2 x - \cos x = 0, 0 \leq x \leq 2\pi$
   (d) $\sin^2 x - 2 \sin x + 1 = 0, -2\pi \leq x \leq 2\pi$
   (e) $\cos^2 2x + 2 \cos 2x + 1 = 0, -\pi \leq x \leq \pi$
   (f) $\sec^2 2x - 1 = 0, -2\pi \leq x \leq 2\pi$
   (g) $\tan 4x - \tan 2x = 0, 0 < x < \pi$
   (h) $\sqrt{3} \cos x + \sin x = 0, -2\pi \leq x \leq 0$
10. If $\tan x = 1$ and $\sin y = \frac{24}{25}$ with x and y in the interval $\left[0, \frac{\pi}{2}\right]$, evaluate.
    (a) $\csc(x + y)$ (b) $\sec(x - y)$
11. Express $\cos 12a$ in terms of $\cos 3a$ and in terms of $\sin 3a$.
12. Use reflections to prove
    (a) $\sin(b - \pi) = -\sin b, \pi < b < \frac{3\pi}{2}$
    (b) $\cos\left(b - \frac{3}{2}\pi\right) = -\sin b, \frac{\pi}{2} < b < \pi$

6.6 REVIEW EXERCISE Answers

1. (a) $-\frac{33}{65}$ (b) $\frac{63}{16}$ (c) $\frac{2047}{4225}$
2. (a) $\frac{-\sqrt{3} + 1}{2\sqrt{2}}$ (b) $\frac{-\sqrt{3} - 1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ (d) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ (e) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ (f) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$
3. (a) $-\frac{24}{25}$ (b) $\frac{7}{25}$ (c) $-\frac{24}{7}$
4. (a) $\frac{1}{9}$ (b) $4\sqrt{5}$ (c) $\sqrt{\frac{3 - \sqrt{5}}{6}}$
5. (a) $\sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}$ (b) $\sqrt{\frac{1 + \sqrt{2}}{2\sqrt{2}}}$ (c) $\frac{-1 + \sqrt{4 + 2\sqrt{2}}}{1 + \sqrt{2}}$
6. (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $-\sqrt{3}$
7. (a) $\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$ (c) -1
9. (a) $0, \frac{\pi}{2}, \pi$ (b) $-\pi, -\frac{\pi}{2}, 0, \pi$ (c) $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$ (d) $-\frac{3\pi}{2}, \frac{\pi}{2}$ (e) $\pm\frac{\pi}{2}$
   (f) $0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \pm 2\pi$
   (g) $0, \frac{\pi}{2}, \pi$ (h) $-\frac{4\pi}{3}, -\frac{\pi}{3}$
10. (a) $\frac{25\sqrt{2}}{31}$ (b) $\frac{25\sqrt{2}}{31}$