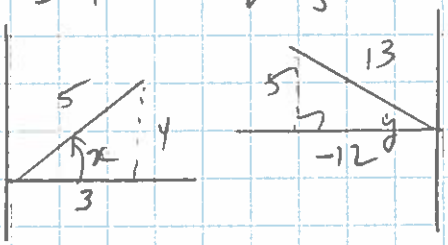
66 Review

(1) $\tan x = \frac{4}{3}$ $\frac{o}{a}$ $\quad \csc y = \frac{13}{5}$ $\rightarrow \sin y = \frac{5}{13}$ $\frac{o}{h}$



a) $\sin(x+y)$

$= \sin x \cos y + \cos x \sin y$

$= \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{-48 + 15}{65}$

$= \frac{-33}{65}$

b) $\tan(x-y)$

$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \frac{\frac{4}{3} - \frac{5}{-12}}{1 + \frac{4}{3}\left(-\frac{5}{12}\right)}$

$= \left[\frac{16}{12} + \frac{5}{12}\right] \div \left[1 - \frac{20}{36}\right]$

$= \frac{21}{12} \div \frac{16}{36}$

$= \frac{21}{12} \times \frac{36}{16}$

$= \frac{63}{16}$

c) $\cos 2(x+y)$

$= 1 - 2\sin^2(x+y)$

$= 1 - 2(\sin(x+y))^2$

$= 1 - 2(\sin x \cos y + \cos x \sin y)^2$

$= 1 - 2\left(\left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)\right)^2$

$= 1 - 2\left(\frac{-48}{65} + \frac{15}{65}\right)^2$

$= 1 - 2\left(\frac{-33}{65}\right)$

$= 1 - 2\left(\frac{1089}{4225}\right)$

$= 1 - \frac{2178}{4225}$

$= \frac{2047}{4225}$

6.6 Review

(2) a) $\sin \frac{13\pi}{12}$

$= \sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + -\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

b) $\cos\left(-\frac{11\pi}{12}\right)$

$= \cos\frac{11\pi}{12}$

$= \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$

c) $-\tan\frac{5\pi}{12}$

$= \pm\tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$

$= \pm\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \pm\left[\frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}\right]$

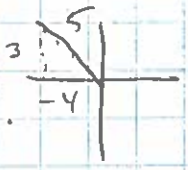
$= \pm\frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}(1)}$

$= \pm\left[\frac{1+\sqrt{3}}{\sqrt{3}} \div \frac{\sqrt{3}-1}{\sqrt{3}}\right]$

$= \pm\left[\frac{1+\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}\right]$

$= \pm\left[\frac{1+\sqrt{3}}{\sqrt{3}-1}\right]$

6.6 #3 $\tan x = -\frac{3}{4}$



a) $\sin 2x$

$= 2\sin x \cos x$

$= 2\left(+\frac{3}{5}\right)\left(-\frac{4}{5}\right)$

$= -\frac{24}{25}$

b) $\cos 2x$

$= \cos^2 x - \sin^2 x$

$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

c) $\tan 2x$

$= \frac{2\tan x}{1 - \tan^2 x}$

$= \frac{2\left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2}$

$= \left(-\frac{6}{4}\right) \div \left[1 - \frac{9}{16}\right]$

$= -\frac{6}{4} \times \frac{16}{7}^{4}$

$= -\frac{24}{7}$

#5 b) $\cos \frac{\pi}{8} =$

$\cos^2\left(\frac{\pi}{8}\right) = \frac{\cos\frac{\pi}{4} + 1}{2}$

$\cos\frac{\pi}{8} = \sqrt{\frac{\frac{1}{\sqrt{2}} + 1}{2}}$

$= \sqrt{\frac{1+\sqrt{2}}{\sqrt{2}} \times \frac{1}{2}}$

$= \frac{\sqrt{1+\sqrt{2}}}{2\sqrt{2}}$

↕ FROM $\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

6.6

#6 a) $\sin 120°$

$= \sin(180 - 120°)$

$= \sin 60°$

$= \frac{\sqrt{3}}{2}$

b) $\cos \frac{11\pi}{6}$

$= \cos \frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

c) $\tan -\frac{7\pi}{3}$

$= -\tan \frac{\pi}{3}$

$= -\frac{\sqrt{3}}{1}$

7 a) $\sin\left(-\frac{7\pi}{6}\right)$

$= -\sin \frac{5\pi}{6}$

$= \sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$= \cos \frac{\pi}{3}$

$= \frac{1}{2}$

b) $\cos 495°$

$= \cos(135°)$

$= \cos(90° + 45°)$

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}}$

c) $\tan \frac{33\pi}{4}$

$= \tan \frac{25\pi}{4}$

$= \tan \frac{17\pi}{4}$

$= \tan \frac{9\pi}{4}$

$= \tan\left(\frac{\pi}{4}\right)$

$= \tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

$= \cot\left(\frac{\pi}{4}\right)$

$= 1$

d) $-\frac{9\pi}{4}$

8 a) $\tan x = \csc 2x - \cot 2x$

L.S.

$= -\tan x$

$= \frac{\sin x}{\cos x}$

R.S.

$= \csc 2x - \cot 2x$

$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$

$= \frac{1 - \cos 2x}{2 \sin x \cos x}$

$= \frac{1 - (1 - 2\sin^2 x)}{2 \sin x \cos x}$

$= \frac{2 \sin^2 x}{2 \sin x \cos x}$

$= \frac{\sin x}{\cos x}$

6.6

8c) $\cos x - \tan y \sin x = \sec y \cos(x+y)$

L.S.

$= \cos x - \frac{\sin y}{\cos y} \sin x$

R.S.

$= \frac{1}{\cos y}(\cos x \cos y - \sin x \sin y)$

$= \cos x - \frac{\sin x \sin y}{\cos y}$

QED

e) $\frac{\sin 4x - \sin 2x}{\sin 2x} = \frac{\cos 3x}{\cos x}$

L.S.

$= \frac{2\sin 2x \cos 2x - \sin 2x}{\sin 2x}$

$= \frac{\sin 2x (2\cos 2x - 1)}{\sin 2x}$

$= 2\cos 2x - 1$

$= 2(2\cos^2 x - 1) - 1$

$= 4\cos^2 x - 2 - 1$

$= 4\cos^2 x - 3$

R.S. a

$= \frac{\cos(2x + x)}{\cos x}$

$= \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$

$= \cos 2x - \frac{2\sin x \cos x \sin x}{\cos x}$

$= \cos 2x - 2\sin^2 x$

$= 2\cos^2 x - 1 - 2[1 - \cos^2 x]$

$= 2\cos^2 x - 1 - 2 + 2\cos^2 x$

$= 4x\cos^2 x - 3$

g) $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$

L.S.

$= \sin x \cos y + \cos x \sin x + \sin x \cos y - \cos x \sin y$

$= 2\sin x \cos y$

R.S.

$= 2\sin x \cos y$

6.6 #

9 a) $2\sin x \cos x = 0$ $\quad 0 \le x \le \pi$

$\sin x = 0$ $\quad$ or $\quad \cos x = 0$

$0, \pi$ $\quad$ $\pi/2$

$\therefore 0, \pi/2, \pi$

c) $\cos^2 x - \cos x = 0$ $\quad 0 \le x \le 2\pi$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$ $\quad$ or $\quad \cos x = 1$

$\pi/2, 3\pi/2$ $\quad$ $0, 2\pi$

$0, \pi/2, 3\pi/2, 2\pi$

e) $\cos^2 2x + 2\cos 2x + 1 = 0$ $\quad -\pi \le x \le \pi$

$(\cos 2x + 1)(\cos 2x + 1) = 0$

$\cos 2x = -1$

$\therefore 2x = \pi$

$\therefore x = \frac{\pi}{2}$ or $x = -\pi/2$

g) $\tan 4x - \tan 2x = 0$ $\quad 0 \le x \le \pi$ change! $\therefore 0 \le 2x \le 2\pi$

$$\frac{\sin 4x}{\cos 4x} - \frac{\sin 2x}{\cos 2x} = 0$$

$$\frac{2\sin 2x \cos 2x}{2\cos^2 2x - 1} - \frac{\sin 2x}{\cos 2x} = 0$$

$$\frac{2\sin 2x \cos 2x (\cos 2x) - \sin 2x (2\cos^2 2x - 1)}{(2\cos^2 2x - 1)(\cos 2x)} = 0$$

$$= \frac{2\sin 2x \cos^2 2x - 2\sin 2x \cos^2 2x + \sin 2x}{(2\cos^2 2x - 1)(\cos 2x)} = 0$$

6.6 #9g (cont'd)

9g) $$\frac{\sin 2x\left(2\cos^2 2x - 2\cos^2 2x + 1\right)}{\left(2\cos^2 2x - 1\right)(\cos 2x)} = 0$$

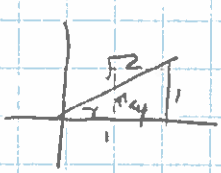
$$\frac{\sin 2x\,(1)}{(2\cos^2 2x - 1)(\cos 2x)} = 0$$

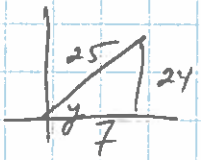
$$\sin 2x = 0$$

$$2x = 0,\ \pi,\ 2\pi$$

$$\therefore\ x = 0,\ \pi/2,\ \pi$$

#10 $\tan x = 1$ $\sin y = \frac{24}{25}$





a) $\csc(x+y)$

$$= \frac{1}{\sin(x+y)}$$

$$= \frac{1}{\sin x\cos y + \cos x \sin y}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{7}{25}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{24}{25}\right)}$$

$$= 1 \div \frac{31}{25\sqrt{2}}$$

$$= 1 \times \frac{25\sqrt{2}}{31}$$

b) $\sec(x-y)$

$$= \frac{1}{\cos(x-y)}$$

$$= \frac{1}{\cos x\cos y + \sin x \sin y}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{7}{25}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{24}{25}\right)}$$

$$= 1 \div \frac{31}{25\sqrt{2}}$$

$$= 1 \times \frac{25\sqrt{2}}{31}$$

$$= \frac{25\sqrt{2}}{31}$$

6.4 # 8b

L.S.

$$= \frac{1-\sin 2x}{\cos 2x}$$

$$= \frac{1-2\sin x\cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x - 2\sin x\cos x}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{(\cos x - \sin x)(\cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

R.S.

$$= \frac{1-\tan x}{1+\tan x}$$

$$= \left[1 - \frac{\sin x}{\cos x}\right] \div \left[1 + \frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x - \sin x}{\cos x} \times \frac{\cos x}{\cos x + \sin x}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

QED

8d) $\sin(\pi + x) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) = -\cot x$

L.S.

$$= -\sin x + \sin x - \cot x$$

$$= -\cot x$$

R.S.

$$= -\cot x$$

QED

f) $\cos x + \cos 2x + \cos 3x = \cos 2x(1 + 2\cos x)$

L.S.

$$= \cos x + \cos 2x + \cos(2x + x)$$
$$= \cos x + \cos 2x + \cos 2x\cos x - \sin 2x\sin x$$
$$= \cos x + \cos 2x + \cos 2x\cos x - 2\sin x\cos x\sin x$$
$$= \cos x + \cos 2x + \cos 2x\cos x - 2\sin^2 x\cos x$$
$$= \cos x + \cos 2x + \cos 2x\cos x - 2(1-\cos^2 x)\cos x$$
$$= \cos x + 2\cos^2 x - 1 + (2\cos^2 x - 1)(\cos x) - 2\cos x + 2\cos^3 x$$
$$= \cos x + 2\cos^2 x - 1 + 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$
$$= 4\cos^3 x + 2\cos^2 x - 2\cos x - 1$$

R.S.

$$= \cos 2x + 2\cos^2 x\cos x$$
$$= 2\cos^2 x - 1 + 2(2\cos^2 x - 1)\cos x$$
$$= 2\cos^2 x - 1 + 4\cos^3 x - 2\cos x$$
$$= 4\cos^3 x + 2\cos^2 x - 2\cos x - 1$$

QED

6.6 # 9b

$\sin^2 x + \sin x = 0$ $\quad -\pi \le x \le \pi$

$\sin x(\sin x + 1) = 0$

$\sin x = 0$ $\quad$ $\sin x = -1$

$-\pi, 0, \pi$ $\quad$ $-\pi/2$

9) $-\pi, -\pi/2, 0, \pi$

1 d) $\sin^2 x - 2\sin x + 1 = 0$ $\quad -2\pi \le x \le 2\pi$

$(\sin x - 1)^2 = 0$

$\sin x = 1$

$x = \pi/2, -3\pi/2$

9f) $\sec^2 2x - 1 = 0$ $\quad -2\pi \le x \le 2\pi$

$\downarrow$

$-4\pi \le 2x \le 4\pi$

$\frac{1}{\cos^2 2x} - 1 = 0$

$1 - \cos^2 2x = 0$

$\cos^2 2x = 1$

$\cos 2x = 1$

$2x = 0, 2\pi, 4\pi, -2\pi, -4\pi$

So $x = 0, \pi, 2\pi, -\pi, -2\pi$

6.6

#4h $\sqrt{3}\cos x + \sin x = 0$ $\quad -2\pi \leq x \leq 0$

$\sqrt{3}\cos x = -\sin x$

$-\frac{\sqrt{3}}{1} = \frac{\sin x}{\cos x}$

$-\sqrt{3} = \tan x$ $\quad x = -\frac{\pi}{3}, -\frac{4\pi}{3}$