MHF 4UI UNIT 6
Trigonometric Functions II

Day 2 - Trig Identities (Part II)
(Addition and Subtraction Formulas

Addition and Subtraction Formulas:

cos (a + b) = cosa cosb - sina sin b

cos (a - b) = cosa cosb + sina sin b

sin (a + b) = sina cosb + cosa sin b

sin (a - b) = sina cosb - cosa sin b

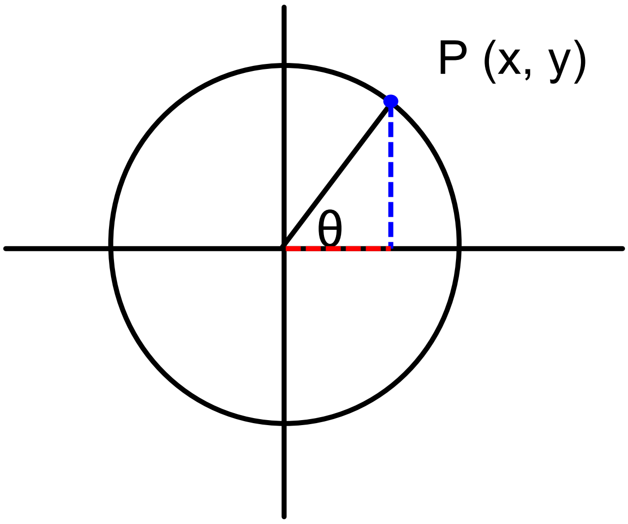
$$\tan (a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan (a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

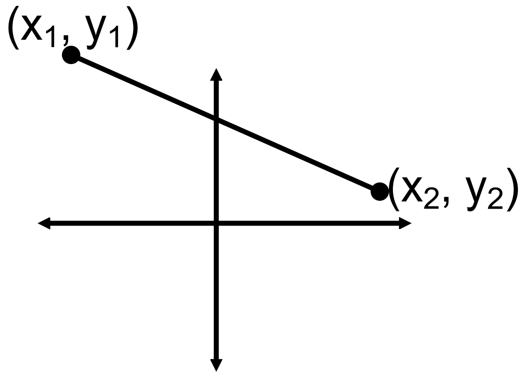
** Do NOT have to memorize these**

Proof of Addition Formula for cosine

Background (Unit Cirlcle):



Background (Length of a Line Segment):



$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Background (Pythagorean Identity):

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2 a + \cos^2 a = 1$$

$$\sin^2 b + \cos^2 b = 1$$

$$\sin^2(a - b) + \cos^2(a - b) = 1 \quad \text{..... etc}$$

Proof of Addition Formula for cosine... cont.

1. Draw unit circle
2. Plot P(1, 0)
3. Plot Q(cosA, sinA)
4. Plot R(cos(A+B), sin(A+B))
5. Plot S(cos(-B), sin(-B))
6. Creat line segments RP and QS

Notice that RP = QS, so we can use the distance formula here!

$$\ell = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$QS = \sqrt{(\cos A - \cos(-B))^2 + (\sin A - \sin(-B))^2}$$

x_2 x_1 y_2 y_1

$$RP = \sqrt{(\cos(A+B) - 1)^2 + (\sin(A+B) - 0)^2}$$

$$\sqrt{(\cos(A+B) - 1)^2 + (\sin(A+B) - 0)^2} = \sqrt{(\cos A - \cos(-B))^2 + (\sin A - \sin(-B))^2}$$

RP QS

$$(\cos(A+B) - 1)^2 + (\sin(A+B))^2 = (\cos A - \cos(-B))^2 + (\sin A - \sin(-B))^2$$

Example 1: Find the exact value of: $\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

Example 2:
a) Evaluate (exact answer) $\cos\left(\frac{\pi}{12}\right)$

b) Evaluate (exact answer) $\sin\frac{17\pi}{12}$

Example 3: If $\sin a = -\frac{4}{5}, \quad \pi \le a \le \frac{3\pi}{2}$

$\cos b = -\frac{5}{13}, \quad \frac{\pi}{2} \le b \le \pi$

find tan (a + b)

Example 4: Prove $1+\cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$