Unit 7: Exponential Functions

Lesson 6: Review

RECALL: Exponents Rules

1. Multiplying powers $(x^a)(x^b) = x^{a+b}$ *Base stays the same.
2. Dividing powers $\frac{x^a}{x^b} = x^{a-b}$
3. Power of a power $(x^a)^b = x^{a \cdot b}$
4. Power of a product $(xy)^a = x^a y^a$
5. Power of a quotient $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
6. Zero exponents $x^0 = 1$
7. Negative exponent $x^{-a} = \frac{1}{x^a}$ $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$
8. Rational exponents $x^{\frac{1}{n}} = \sqrt[n]{x}$ $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$

or $= \sqrt[n]{x^m}$

EX. 1. Simplify each expression to a single power. Express answers with positive exponents.

a) $x^2 \times x^4 \times x^{\frac{1}{2}} \div x^{-3}$

$= x^{2+4+\frac{1}{2}-(-3)}$

$= x^{\frac{19}{2}} \leftarrow 9\frac{1}{2}$

b) $(a^2 \div a^4)^{-3}$

$= (a^{2-4})^{-3}$

$= (a^{-2})^{-3}$

$= a^6$

c) $(5x^4y^3z^2)^2$

$= 5^2x^8y^6z^4$

$= 25x^8y^6z^4$

d) $\left(\frac{x}{3}\right)^4$

$= \frac{x^4}{3^4}$

$= \frac{x^4}{81}$

e) $\frac{(2m^3n^4)(9m^2n)}{3mn^5}$

$= \frac{18m^5\cancel{n^5}}{3m^1\cancel{n^5}}$

$= 6m^4$

f) $2^4 \times 2^{\frac{3}{4}} \times \sqrt{8}$

$\frac{3}{1} \times \frac{1}{2}$

$= 2^4 \times 2^{\frac{3}{4}} \times (8)^{\frac{1}{2}}$

$= 2^4 \times 2^{\frac{3}{4}} \times (2^3)^{\frac{1}{2}}$

$= 2^4 \times 2^{\frac{3}{4}} \times 2^{\frac{3}{2}}$

$= 2^{\frac{4}{1}+\frac{3}{4}+\frac{3 \times 2}{2 \times 2}}$

$= 2^{\frac{16}{4}+\frac{3}{4}+\frac{6}{4}}$

$= 2^{\frac{25}{4}}$

EX. 2. Write in exponential form.

$\sqrt[3]{x^4} = x^{\frac{4}{3}}$

EX. 3. Write in radical form.

$x^{\frac{3}{5}} = \sqrt[5]{x^3}$ or $= (\sqrt[5]{x})^3$

EX. 4. Evaluate

a) $(\sqrt[4]{81})^2$

$= (3)^2$

$= 9$

b) $(2^4)^{\frac{1}{2}}$

$= (16)^{\frac{1}{2}}$

$= \sqrt{16}$

$= 4$

c) $(\sqrt[5]{-32})^2$

$= (-2)^2$

$= 4$

Difference Tables

Explain how to show that a table of values represents each type of function.

- Linear First diff. are the same.
- Quadratic Second " " " "
- Exponential First Ratios are the same.

EX. 1. Is this linear, quadratic or exponential? If it is exponential, is it an example of exponential growth or decay? What is the constant ratio?

x	y	First Diff	2nd Diff	First Ratios
0	0.25			
		0.75		$\frac{1}{0.25} = 4$
1	1			
		3	9	$\frac{4}{1} = 4$
2	4			
		12	36	$\frac{16}{4} = 4$
3	16			
		48		$\frac{64}{16} = 4$
4	64			

NOT Linear

NOT Quad.

all the same $\therefore$ Exponential

$y = a(4)^x$

$\therefore$ Exp. Growth since base is not a fraction.

Exponential Functions: $y = a(b)^n$

Without graphing, complete the following table.

Function	Domain	Range	Equation of asymptote	y-intercept	Constant ratio	The exponential function is...
$y = (\frac{1}{3})^x$	$x \in \mathbb{R}$	$y \in \mathbb{R}$, $y > 0$	$y = 0$	1	$\frac{1}{3}$	☐ increasing ☒ decreasing ☐ neither
$y = 2(3)^x$	$x \in \mathbb{R}$	$y \in \mathbb{R}$, $y > 0$	$y = 0$	1	3	☒ increasing ☐ decreasing ☐ neither

y=0

y=0

How would adding a *'k'* value to the end of the equation affect things?

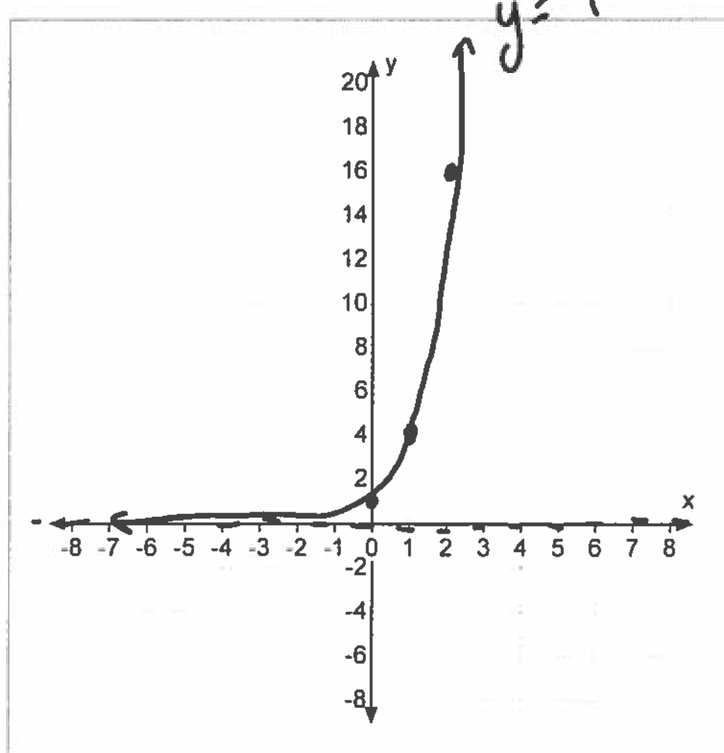
$y = a(b)^n + k$ moves up or down.

Complete the table of values for the function $y = \cancel{4x}\ 4^x$ then graph it.

4^x

x	y
-2	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
-1	$4^{-1} = \frac{1}{4}$
0	1
1	4
2	16
3	64

$y = 4^x$



Exponential Growth & Decay

$y=a(b)^x$

0.75

EX. 1. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers has increased by 75% per year since 1985.

$N(t)=a(b)^t$

a) Write an equation to model this growth where $N(t)$ is the number of subscribers after t years.

$N(t)=285(1+0.75)^t$
$=285(1.75)^t$

b) Determine the number of subscribers in 1998.

$N(13)=285(1.75)^{13}$
$=285(1443.758)$
$=411\,471.247$

∴ 411 471 people owned cell phones in 1998.

c) How many subscribers were there in 1980 (i.e. 5 years ago)?

$N=285(1.75)^{-5}$
$=285(0.0609)$
$=17.364$

∴ there were only 17 cell phones 5 years earlier.

d) After how many years will there be 1,000,000,000 subscribers?

$$\frac{1\,000\,000\,000}{285}=\frac{285(1.75)^t}{285}$$

$3\,508\,771.93=1.75^t$

$1.75^{20}=72\,570$
$1.75^{30}=19\,549\,761$
$1.75^{28}=6\,343\,595$
$1.75^{27}=3\,647\,768$

after 1985 (2012)

∴ after 27 years the 1 000 000 000 mark is passed.

EX. 2. The number of participants in a local tennis tournament can be modeled by the function $P(t) = 128\,(0.5)^t$ where $P(t)$ is the number of participants after t rounds of play.

a) How many participants entered in the tournament?

b) At what rate are participants eliminated after each round?

c) After how many rounds of play will the number of participants be half the original amount?

Practice: p. 444 #1-13