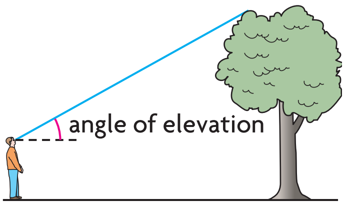
5.1 Applying the Primary Trigonometric Ratios

GOAL

Use primary trigonometric ratios to solve real-life problems.

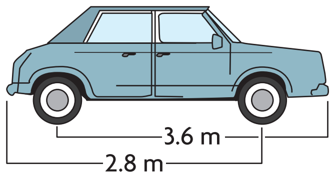
LEARN ABOUT the Math

angle of elevation
the angle between the horizontal and the line of sight when looking up at an object



Eric's car alarm will sound if his car is disturbed, but it is designed to shut off if the car is being towed at an **angle of elevation** of more than 15°. Mike's tow truck can lift a bumper no more than 0.88 m higher than the bumper's original height above ground. Eric's car has these measurements:

- The front bumper is 3.6 m from the rear axle.
- The rear bumper is 2.8 m from the front axle.

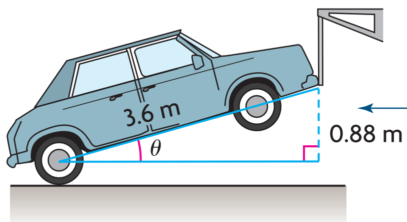


? Will Mike be able to tow Eric's car without the alarm sounding?

EXAMPLE 1 Selecting a strategy to solve a problem involving a right triangle

Determine whether the car alarm will sound.

Jason's Solution: Calculating the Angle of Elevation



If Eric's car is towed from the front, the car forms a right triangle with a hypotenuse of 3.6 m. The side opposite the angle of elevation, θ, is 0.88 m.

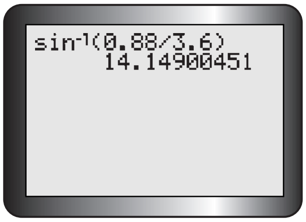
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

In a right triangle, the sine ratio relates an angle to the opposite side and the hypotenuse.

$$\sin \theta = \frac{0.88}{3.6}$$

$$\theta = \sin^{-1}\left(\frac{0.88}{3.6}\right)$$

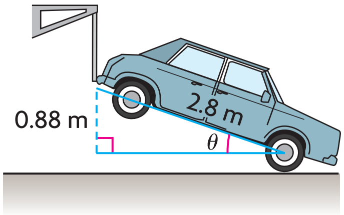
I used the inverse sine ratio to calculate the angle.



$$\theta \doteq 14°$$

I rounded to the nearest degree.

Since the angle is less than 15°, the car has not been lifted enough to shut off the alarm.



If the car is towed from the rear, the car still forms a right triangle. The opposite side is still 0.88, but the hypotenuse is now 2.8 m.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

To determine the angle of elevation, θ, I used the sine ratio, since I knew the lengths of the opposite side and the hypotenuse.

$$\sin \theta = \frac{0.88}{2.8}$$

$$\theta = \sin^{-1}\left(\frac{0.88}{2.8}\right)$$

I used the inverse sine ratio to calculate the angle.

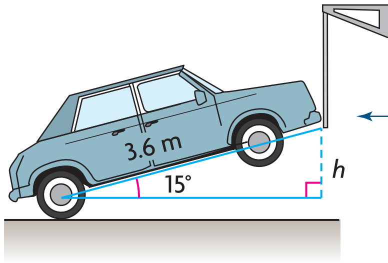
$$\theta \doteq 18°$$

I rounded to the nearest degree.

Since the angle is greater than 15°, the car alarm will shut off.

For the car alarm to shut off, Mike should tow Eric's car from the back.

Monica's Solution: Calculating the Minimum Height



For an angle of elevation of 15°, I wanted to know the height the car must be lifted to shut off the alarm. I started with the front of the car being lifted. I drew a right triangle and labelled the height as h. I knew an angle and the hypotenuse.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

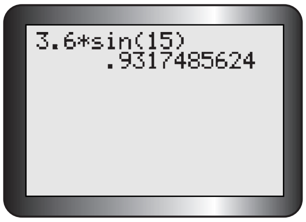
Side h is opposite the 15° angle, so I used the sine ratio to calculate h.

$$\sin 15° = \frac{h}{3.6}$$

$$3.6 \times \sin 15° = \overset{1}{\cancel{3.6}} \times \frac{h}{\underset{1}{\cancel{3.6}}}$$

I solved for h by multiplying both sides of the equation by 3.6.

$$h = 3.6 \times \sin 15°$$



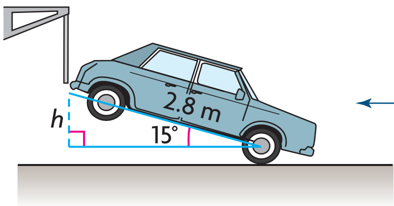
I used a calculator to evaluate and rounded to the nearest hundredth.

$$h \doteq 0.93 \text{ m}$$

When the front of the car is lifted 15°, the front bumper is about 0.93 m above its original height.

Mike can't raise the car high enough from the front to shut off the alarm.

This won't work because Mike can lift the bumper only 0.88 m.



I used the same method, but with the rear of the car being lifted.

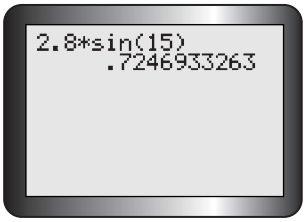
$$\sin 15° = \frac{h}{2.8}$$

I used the sine ratio because I knew the opposite side and the hypotenuse.

$$2.8 \times \sin 15° = \overset{1}{\cancel{2.8}} \times \frac{h}{\underset{1}{\cancel{2.8}}}$$

I solved for *h* by multiplying both sides of the equation by 2.8.

$$h = 2.8 \times \sin 15°$$



I used a calculator to evaluate and rounded to the nearest hundredth.

$$h \doteq 0.72 \text{ m}$$

When the rear of the car is lifted 15°, the rear bumper is about 0.72 m above its original height.

For the alarm to shut off, Mike should tow Eric's car from the rear.

This will work, since Mike can lift a car more than that.

Reflecting

A. Compare the two solutions. How are they the same and how are they different?

B. Which solution do you prefer? Why?

C. Could the cosine or tangent ratios be used instead of the sine ratio to solve this problem? Explain.

APPLY the Math

EXAMPLE 2 Selecting the appropriate trigonometric ratios to determine unknown sides

A hot-air balloon on the end of a taut 95 m rope rises from its platform. Sam, who is in the basket, estimates that the angle of depression to the rope is about 55°.

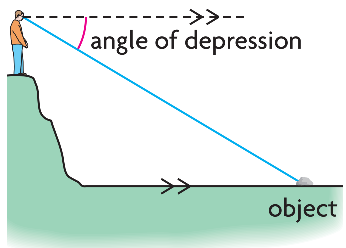
a) How far, to the nearest metre, did the balloon drift horizontally?

b) How high, to the nearest metre, is the balloon above ground?

c) Viewed from the platform, what is the angle of elevation, to nearest degree, to the balloon?

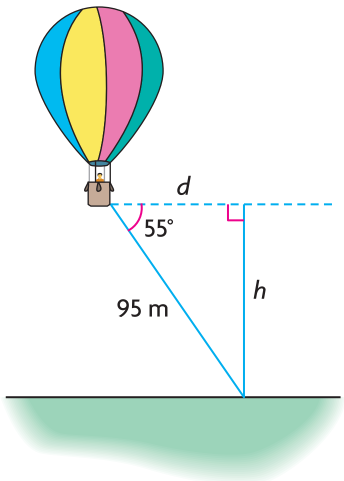
angle of depression

the angle between the horizontal and the line of sight when looking down at an object



Kumar's Solution

a)



I drew a sketch and labelled the angle of depression as 55°. I labelled the horizontal distance the balloon drifted as *d* and the height of the balloon as *h*. Relative to the 55° angle, *d* is the adjacent side and *h* is the opposite side.

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

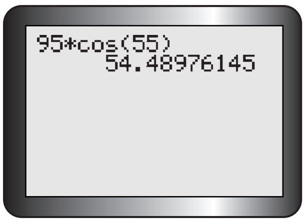
I used the cosine ratio to calculate *d* because in a right triangle, cosine relates an angle to the adjacent side and the hypotenuse.

$$\cos 55° = \frac{d}{95}$$

$$95 \times \cos 55° = \overset{1}{\cancel{95}} \times \frac{d}{\underset{1}{\cancel{95}}}$$

I solved for *d* by multiplying both sides of the equation by 95.

$$d = 95 \times \cos 55°$$



I used a calculator to evaluate.

$$d \doteq 54 \text{ m}$$

The balloon drifted about 54 m horizontally.

b)

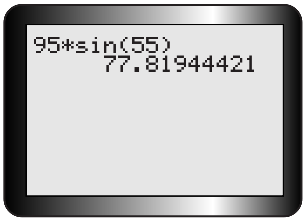
$$\sin 55° = \frac{h}{95}$$

I used the sine ratio to calculate *h* because in a right triangle, sine relates an angle to the opposite side and the hypotenuse.

$$95 \times \sin 55° = \overset{1}{\cancel{95}} \times \frac{h}{\underset{1}{\cancel{95}}}$$

I solved for *h* by multiplying both sides of the equation by 95.

$$h = 95 \times \sin 55°$$

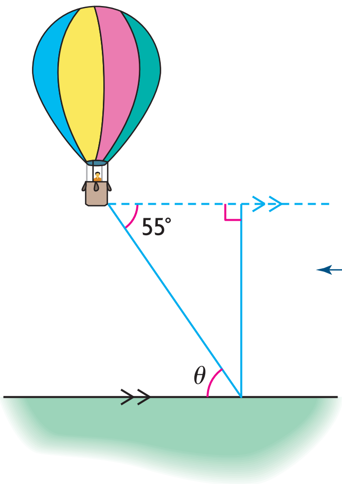


I used a calculator to evaluate.

$h \doteq 78$ m

The balloon is about 78 m above ground.

c)



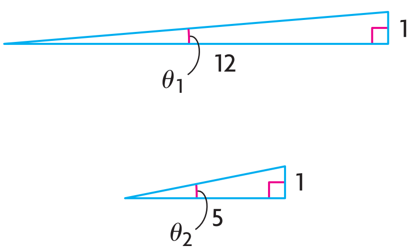
I labelled the angle of elevation to the balloon as θ. The angle of depression $(55°)$ and θ are alternate angles between parallel lines. So both angles are equal.

The angle of elevation to the balloon from the platform is 55°.

EXAMPLE 3 Solving a problem by using a trigonometric ratio

A wheelchair ramp is safe to use if it has a minimum slope of $\frac{1}{12}$ and a maximum slope of $\frac{1}{5}$. What are the minimum and maximum angles of elevation to the top of such a ramp? Round your answers to the nearest degree.

Nazir's Solution



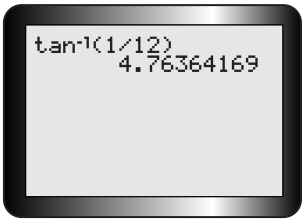
I drew a sketch of each ramp and labelled the angles of elevation as θ_1 and θ_2. Slope is rise divided by run, so, in both triangles, the rise is the vertical side and the run is the horizontal side. Relative to θ_1 and θ_2, these sides are opposite and adjacent.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The tangent ratio relates an angle in a right triangle to the opposite side and the adjacent side.

$$\tan\theta_1 = \frac{1}{12} \qquad \tan\theta_2 = \frac{1}{5}$$

$$\theta_1 = \tan^{-1}\left(\frac{1}{12}\right) \qquad \theta_2 = \tan^{-1}\left(\frac{1}{5}\right)$$



I used the inverse tangent function on my calculator to calculate each angle.

$$\theta_1 \doteq 5° \qquad \theta_2 \doteq 11°$$

The angle of elevation to the top of the wheelchair ramp must be between 5° and 11° to be safe.

In Summary

Key Idea

- The primary trigonometric ratios can be used to determine side lengths and angles in right triangles. Which ratio you use depends on what you want to calculate and what you know about the triangle.

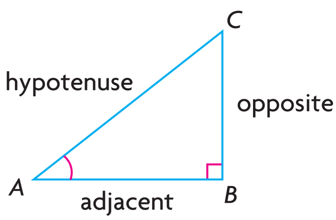
Need to Know

- For any right $\triangle ABC$, the primary trigonometric ratios for $\angle A$ are:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



- Opposite and adjacent sides are named relative to their positions to a particular angle in a triangle.
- The hypotenuse is the longest side in a right triangle and is always opposite the 90° angle.

CHECK Your Understanding

1. Use a calculator to evaluate to four decimal places.

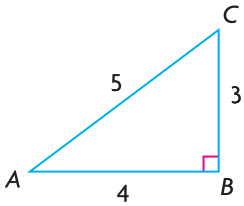
a) $\sin 15°$ **b)** $\cos 55°$

2. Use a calculator to determine the angle to the nearest degree.

a) $\tan^{-1}\left(\frac{2}{5}\right)$ **b)** $\sin^{-1}(0.7071)$

3. State all the primary trigonometric ratios for $\angle A$ and $\angle D$. Then determine $\angle A$ and $\angle D$ to the nearest degree.

a)



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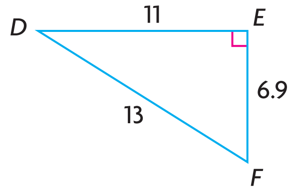
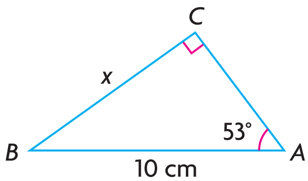


4. For each triangle, calculate x to the nearest centimetre.

a)



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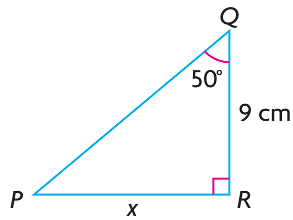
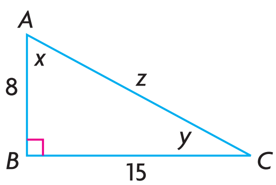


PRACTISING

5. **K** Determine all unknown sides to the nearest unit and all unknown interior angles to the nearest degree.

a)



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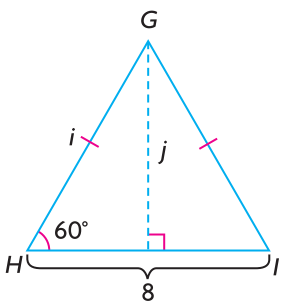

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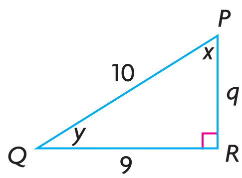
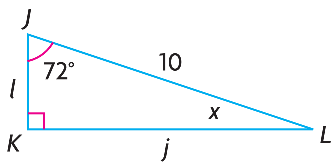
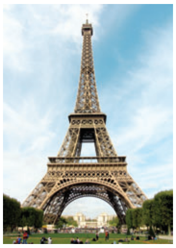
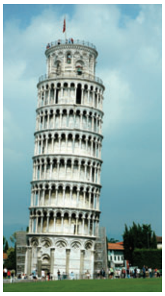
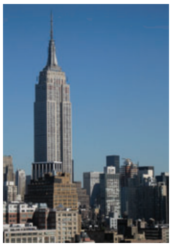


d)




Eiffel Tower


Leaning Tower of Pisa

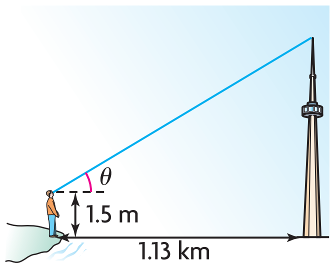

Empire State Building


Big Ben's clock tower

6. Predict the order, from tallest to shortest, of these famous landmarks. Then use the given information to determine the actual heights to the nearest metre.
   a) Eiffel Tower, Paris, France
      68 m from the base, the angle of elevation to the top is 78°.
   b) Empire State Building, New York, New York
      267 m from the base, the angle of elevation to the top is 55°.
   c) Leaning Tower of Pisa, Pisa, Italy
      The distance from a point on the ground to the tallest tip of the tower is 81 m with an angle of elevation of 44°.
   d) Big Ben's clock tower, London, England
      81 m from the base, the angle of elevation is 50°.

7. Manpreet is standing 8.1 m from a flagpole. His eyes are 1.7 m above ground. The top of the flagpole has an angle of elevation of 35°. How tall, to the nearest tenth of a metre, is the flagpole?

8. A The CN Tower is 553 m tall. From a position on one of the Toronto Islands 1.13 km away from the base of the tower, determine the angle of elevation, to the nearest degree, to the top of the tower. Assume that your eyes are 1.5 m above ground.



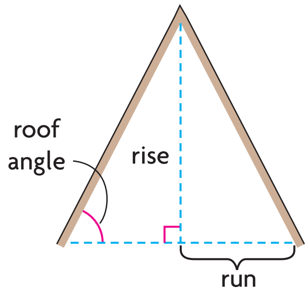
9. Devin wants to estimate the slope of the road near his apartment building. He uses a level that is 1.2 m long and holds it horizontally with one end touching the ground and the other end 39 cm above ground.
   a) What is the slope of the road, to the nearest tenth, at this point?
   b) What angle, to the nearest degree, would represent the slant of the road?

10. A 200 m cable attached to the top of an antenna makes an angle of 37° with the ground. How tall is the antenna to the nearest metre?

11. An underground parking lot is being constructed 8.00 m below ground level.
   a) If the exit ramp is to rise at an angle of 15°, how long will the ramp be? Round your answer to the nearest hundredth of a metre.
   b) What horizontal distance, to the nearest hundredth of a metre, is needed for the ramp?

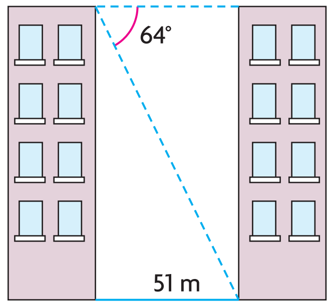
12. The pitch of a roof is the rise divided by the run. If the pitch is greater than 1.0 but less than 1.6, roofers use planks fastened to the roof to stand on when shingling it. If the pitch is greater than 1.6, scaffolding is needed. For each roof angle, what equipment (scaffolding or planks), if any, would roofers need?
T

a) $\theta = 34°$ b) $\theta = 60°$ c) $\theta = 51°$



13. To estimate the width of a river near their school, Charlotte and Mavis have a device to measure angles and a pole of known length. Describe how they might calculate the width of the river with this equipment.

14. Ainsley's and Caleb's apartment buildings are exactly the same height. Ainsley measures the distance between the buildings as 51 m and observes that the angle of depression from the roof of her building to the bottom of Caleb's is about 64°. How tall, to the nearest metre, is each building?



15. To use an extension ladder safely, the base must be 1 m out from the wall for every 2 m of vertical height.

a) What is the maximum angle of elevation, to the nearest degree, to the top of the ladder?

b) If the ladder is extended to 4.72 m in length, how high can it safely reach? Round your answer to the nearest hundredth of a metre.

c) How far out from the wall does a 5.9 m ladder need to be? Round your answer to the nearest tenth of a metre.

16. Martin is installing an array of solar panels 3.8 m high in his backyard. The array needs to be tilted 60° from the ground. Municipal bylaws restrict residents from having any secondary structures taller than 3.0 m. Will Martin be able to build his array? Include calculations in your explanation.
C

Extending

17. When poured into a pile, gravel will naturally form a cylindrical cone with a slope of approximately 34°. If a construction foreman has room only for a pile that is 85 m in diameter, how tall, to the nearest metre, will the pile be?

18. Nalini and Jodi are looking at the top of the same flagpole. They are standing in a line on the same side of the flagpole, 50.0 m apart. The angle of elevation to the top of the pole is 11° from Jodi's position and 7° from Nalini's position. The girls' eyes are 1.7 m above ground. For each question, round your answer to the nearest tenth of a metre.

a) How tall is the flagpole?

b) How far is each person from the base of the flagpole?

c) If Nalini and Jodi were standing in a line on opposite sides of the pole, how tall would the flagpole be? How far would each person be from its base?