HF 4UI

Unit 2

al Functions

Day 3

Pumpk

Horizontal and Oblique Asymptotes

Horizontal and Oblique Asymptotes

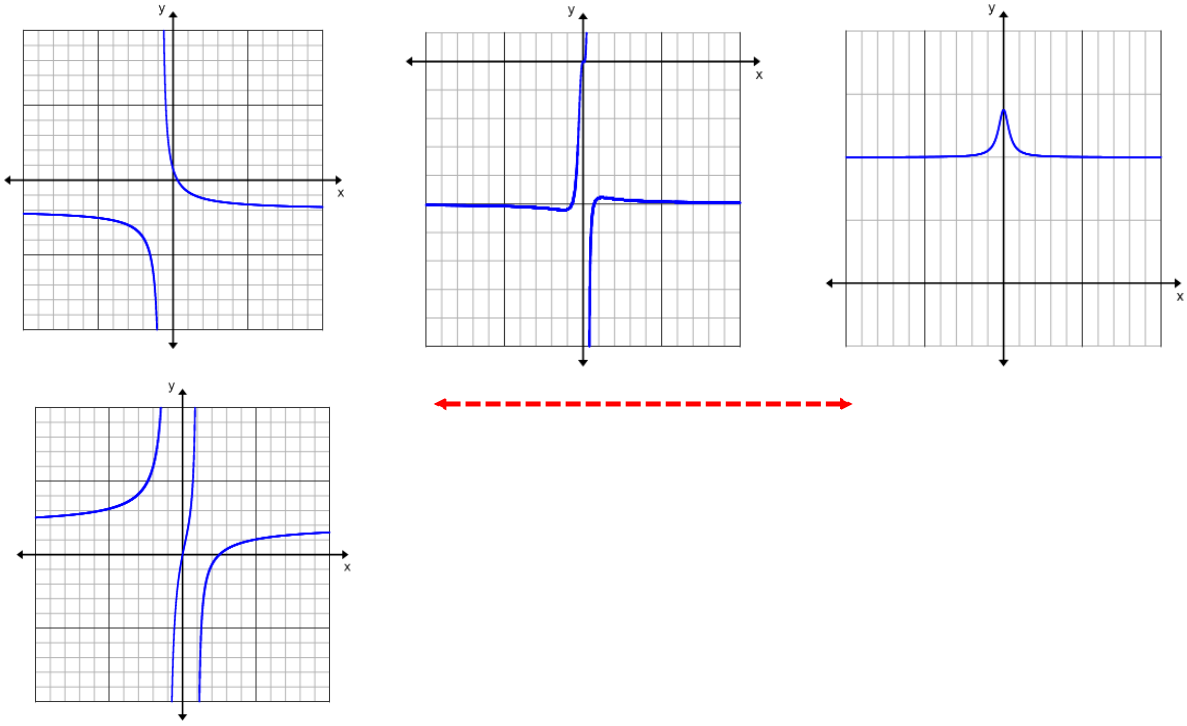
Limits at Infinity

A number L is a limit at infinity for a function *f* if the values of *f(x)* approach L as x gets infinitely large or small.

$$\lim_{x \to \infty} f(x) = L \quad or \quad \lim_{x \to -\infty} f(x) = L$$

The line y = L is called a horizontal asymptote if either of the above are true.

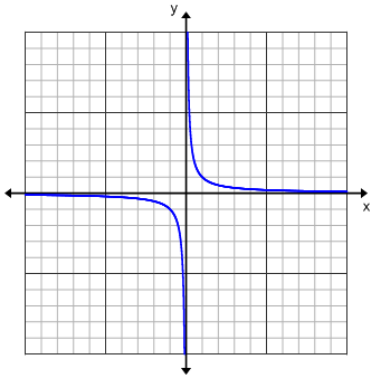
http://www.calculus-help.com/limits-and-infinity/



Evaluating Limits at Infinity

To evaluate limits at infinity we need to understand the following limits:

$$\lim_{x\to\infty}\frac{1}{x}=0 \quad \text{and} \quad \lim_{x\to-\infty}\frac{1}{x}=0$$



Also, if r is a positive integer:

$$\lim_{x\to\infty}\frac{1}{x^r}=0 \quad \text{and} \quad \lim_{x\to-\infty}\frac{1}{x^r}=0$$

SHORTCUT!!:

1. Compare the degrees in the numerator and denominator

2. If the degrees are equal.... the coefficients attached to them are the limit!!!

$$\lim_{x\to\infty}\frac{5x^3-2x^2}{5-x^3}$$

3. If the degrees are not equal

   a) and the degree in the numerator is larger than the denominator, the limit is infinity!!

   ** Unless the degree is EXACTLY one more in the numerator than the denominator... we'll talk about this in a few minutes!!!

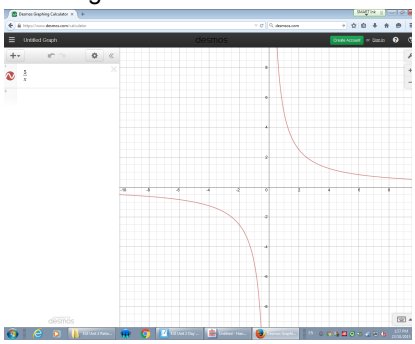
$$\lim_{x\to\infty}\frac{x^5-3x+6}{x^4-x+5}$$

   b) and the degree in the denominator is larger than the degree in the numerator, the limit is zero!!

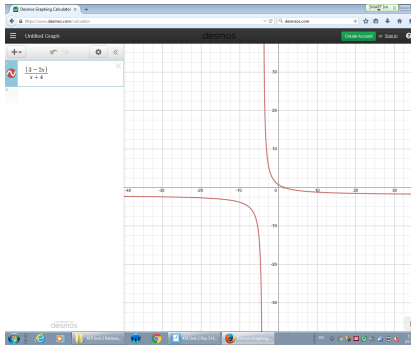
$$\lim_{x\to\infty}\frac{5}{x}$$

Example 1: Evaluate the following limits:

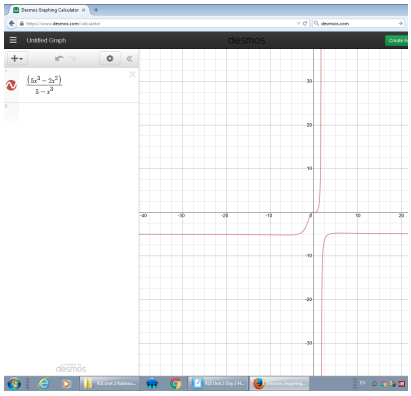
$a)\ \lim_{x\to\infty}\frac{5}{x}$



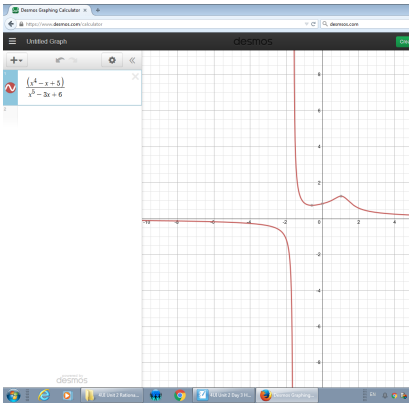
$b)\ \lim_{x\to\infty}\frac{3-2x}{x+4}$



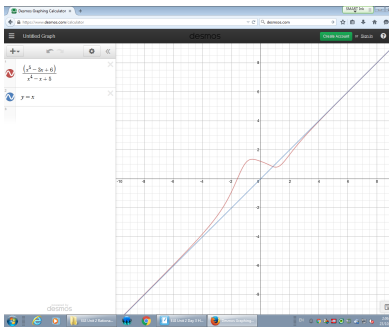
$c)\ \lim_{x\to\infty}\frac{5x^3-2x^2}{5-x^3}$



$d)\ \lim_{x\to\infty}\frac{x^4-x+5}{x^5-3x+6}$



$e)\ \lim_{x\to\infty}\frac{x^5-3x+6}{x^4-x+5}$



Oblique Asymptotes

Oblique Asymptotes occur only if the degree of the numerator is EXACTLY one more than the degree of the denominator.

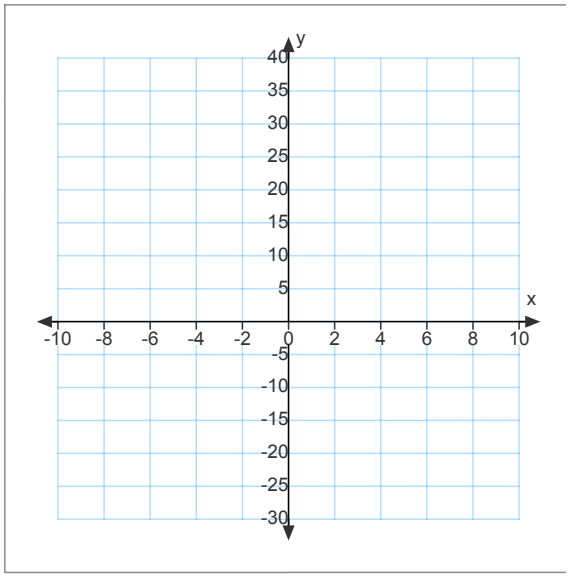
An oblique asymptote has the form y = mx + b

Example 2: Find the oblique asymptotes of the following:

a) $y = \dfrac{5x^2 + 3x - 2}{x - 1}$

Pull

b) $y = \dfrac{x^3 + 4x^2 + 5x + 16}{x^2 - 4}$



SUMMARY:

Vertical Asymptotes:

When Denominator Equals ZERO
Check Left and Right hand limit

Horizontal Asymptotes:

Evaluate $\lim_{x \to \infty} f(x)$

Oblique Asymptotes:

Degree of Numerator is EXACTLY one more than denomi
Divide numerator by denominator
Quotient is equation of oblique asymptote in y=mx+

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Unit 2 Outline – Rational Functions

Date	Text Ref.	Topic	Practice Questions
Fri Oct 26 Mon Oct 29	3.3	Limit of a Function	Pg 153 # 1 – 3, 5 – 8
Tues Oct 30 Wed Oct 31	6.4	Vertical Asymptotes	Pg 347 # , 2ab, 3 – 6
Thurs Nov 1 Fri Nov 2	6.5	Horizontal and Oblique Asymptotes	Pg 359 # 1, 3 – 6 Pg 371 # 1