MHF 4UI
Unit 1 Polynomials
Day 1 - Dividing Polynomials (2.2)

Dividing a polynomial by another polynomial is similar to performing a division of numbers using long division.

Review of Long Division: Divide 14853 by 6

$$6\overline{)14853}$$

The format for writing remainders is the following:
GIVEN: P(x) = Polynmomial
Q(x) = Quotient
D(x) = Divisor
R(x) = Remainder

$P(x) \div D(x) =$

Example 1: Divide:

$a)\ x^3 + 3x^2 - 5x - 4\ by\ x + 4$

$b)\ x^3 + 2x^2 - x - 2\ by\ x^2 - 1$

Synthetic Division (a shortcut for SOME Long Division)

** Can ONLY use this when dividing by DEGREE 1

E.g. We CAN use it to divide by (x + 2)
We CAN NOT use it to divide by (x^2+2)

Example 2: Divide:

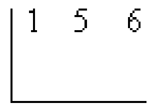
$$a)\ x^3 + 3x^2 - 5x - 4 \ by\ x + 4$$

$$b)\ 2x^3 - x^2 - 4x - 4 \ by\ x - 1$$

Step by Step of Synthetic Division (Summary)

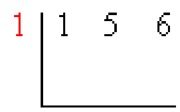
EXAMPLE: Divide $x^2 + 5x + 6$ *by* $x - 1$

First, write the coefficients ONLY inside an upside-down division symbol:

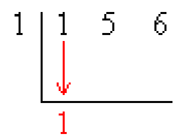


Make sure you leave room inside, underneath the row of coefficients, to write another row of numbers later.

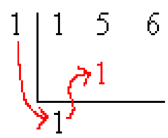
Put the test zero, $x = 1$, at the left:



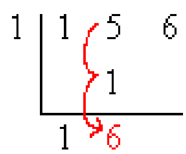
Take the first number inside, representing the leading coefficient, and carry it down, unchanged, to below the division symbol:



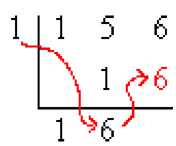
Multiply this carry-down value by the test zero, and carry the result up into the next column:



Add down the column:

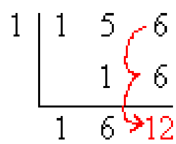


Multiply the previous carry-down value by the test zero, and carry the new result up into the last column:



Add down the column:

This last carry-down value is the remainder.



Step by Step of Long Division (Summary)

For example, divide the polynomial
$x^3 + 13x^2 + 39x + 46$ by $x + 9$

Step 1:

$$\begin{array}{r|l} & x^2 \\ \hline x+9 & x^3+13x^2+39x+46 \end{array}$$

first divide x into x^3 to get x^2

Step 2:

$$\begin{array}{r|l} & x^2 \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \\ & \quad\;\; 4x^2 \end{array}$$

now multiply x^2 by $x + 9$ to get $x^3 + 9x^2$

then subtract $x^3 + 9x^2$ from $x^3 + 13x^2$ to get $4x^2$

Step 3:

$$\begin{array}{r|l} & x^2 + 4x \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \quad \downarrow \\ & \quad\;\; 4x^2 + 39x \end{array}$$

bring down the + 39x

divide $4x^2$ by x to get 4x

Step 4:

$$\begin{array}{r|l} & x^2 + 4x \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \quad \downarrow \\ & \quad\;\; 4x^2 + 39x \\ & \quad\;\; \underline{4x^2 + 36x} \\ & \qquad\qquad 3x \end{array}$$

now multiply 4x by $x + 9$ to get $4x^2 + 36x$

then subtract $4x^2 + 36x$ from $4x^2 + 39x$ to get 3x

Step 5: (Finally)

$$\begin{array}{r|l} & x^2 + 4x + 3 \\ \hline x+9 & x^3+13x^2+39x+46 \\ & \underline{x^3+9x^2} \quad \downarrow \quad \downarrow \\ & \quad\;\; 4x^2 + 39x \quad \downarrow \\ & \quad\;\; \underline{4x^2 + 36x} \quad \downarrow \\ & \qquad\qquad 3x + 46 \\ & \qquad\qquad \underline{3x + 27} \\ & \qquad\qquad\quad 19 \end{array}$$

bring down the + 46
divide 3x by x to get 3

multiply 3 by $x + 9$ to get $3x + 27$

then subtract $3x + 27$ from $3x + 46$ to get 19

Since the remainder has a lower degree than the divisor, the division is now complete. The result can be written as:
$(x + 9)(x^2 + 4x + 3) + 19$

(**NOTE**: *You could check your answer by multiplying out the result.*) ☺